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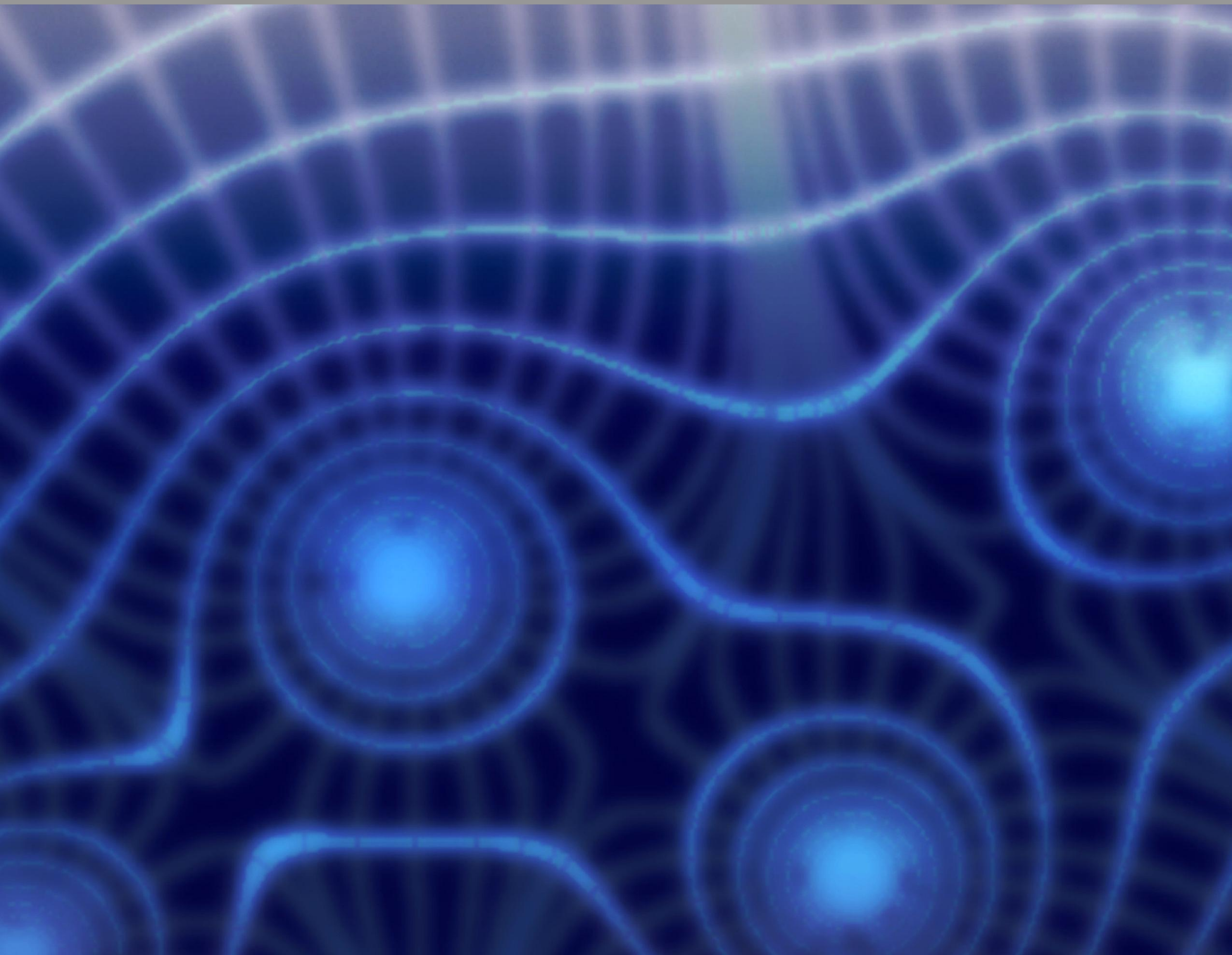

People’s Physics Book Version 2

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Contents

CHAPTER 1 Units and Problem Solving Version 2

Chapter Outline

1.1 The Big Idea

Since physics depends fundamentally on measurements that are interpreted through math, the first distinction we have to make is between different types of measurements and their properties. First, all measurements must have units. Units identify what a specific number refers to. For instance, the number 42 can be used to represent 42 miles, 42 pounds, or 42 elephants! Numbers are mathematical objects, but units give them physical meaning. Keeping track of units can help you avoid mistakes when you work out problems.

1.2 Units Concepts

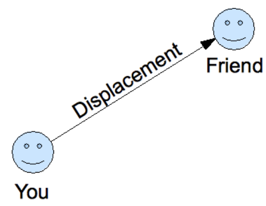
- Every answer to a physics problem must include units. Even if a problem explicitly asks for a speed in meters per second (m/s), the answer is 5 m/s, not 5.
- When you're not sure how to approach a problem, you can often get insight by considering how to obtain the units of the desired result by combining the units of the given variables. For instance, if you are given a distance (in meters) and a time (in hours), the only way to obtain units of speed (meters/hour) is to divide the distance by the time. This is a simple example of a method called *dimensional analysis*, which can be used to find equations that govern various physical situations without any knowledge of the phenomena themselves. To use dimensional analysis, assume that the answer to a problem consists of a product of all the variables given raised to various powers. Many times, there will be only one such combination that gives the desired result.
- This textbook uses *SI units* (La Système International d'Unités), the most modern form of the metric system.
- When converting speeds from metric to American units, remember the following rule of thumb: a speed measured in *mi/hr* is about double the value measured in *m/s (i.e.,* 10 m/s is equal to about 20 MPH). Remember that the speed itself hasn't changed, just our representation of the speed in a certain set of units.
- If a unit is named after a person, it is capitalized. So you write "10 Newtons," or "10 N," but "10 meters," or "10 m."

Scalars

The simplest kind of measurement is a single number, or *scalar*. Scalars are all one needs to describe temperature, density, length, and many other phenomena in physics. The mathematics used in the manipulation of scalars – addition, subtraction, multiplication, and division – come naturally to humans, and, to a large extent, to other animals. Many mammals have an innate ability to divide a pile of food into relatively equal pieces, to distinguish between objects of different size, and to perform other tasks that seemingly require intelligence. It would seem crazy to suggest that the animals are performing mathematical operations based on formal logic, but that is not the point. Much more likely is the idea that formal mathematics is an extension of our natural abilities. In fact, the way math has been taught throughout history and across the world – think of your own elementary and middle school classes – seems to reflect this underlying property of human nature.

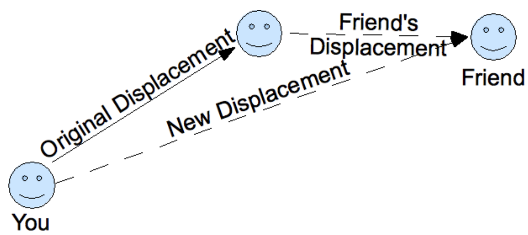
Vectors

The first new concept introduced here is that of a vector: a scalar magnitude with a direction. In a sense, we are almost as good at natural vector manipulation as we are at adding numbers. Consider, for instance, throwing a ball to a friend standing some distance away. To perform an accurate throw, one has to figure out both where to throw and how hard. We can represent this concept graphically with an arrow: it has an obvious direction, and its length can represent the distance the ball will travel in a given time. Such a vector (an arrow between the original and final location of an object) is called a displacement:

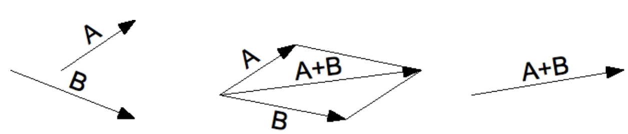


Vector Addition and Subtraction

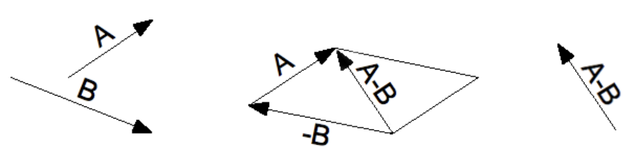
Like scalars, vectors have a branch of mathematics dedicated to them; and again, the basics can be considered an extension of our natural abilties, while the more advanced parts are quite foreign to our intuition. The first concept is that of vector addition. Think about throwing a pass, but this time to a moving target. If we use our original arrow, the target will have moved by the time the ball reaches its endpoint. To be accurate, we need to consider the displacement of the target and add it to the original arrow. The picture can be presented this way:



It should be apparent that if we throw the ball according to the dashed arrow, we will hit the target. This third vector is the sum of the first two displacements, and of course, also a displacement vector. This is how vectors are added graphically: if the end of the first vector is drawn at the beginning of the second, the arrow linking the beginning of the first with the end of the second will be their sum. Alternatively, the two vectors can be moved to become the legs of a parallelogram. Their sum is then the diagonal:

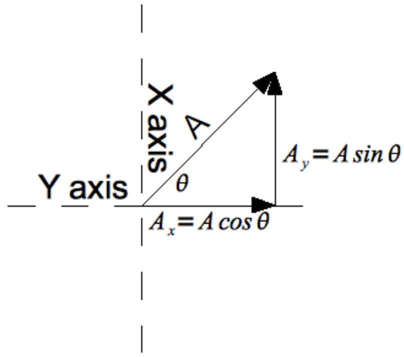


To subtract vectors, you can simply flip the vector you are subtracting by 180 degrees and add them. This is essentially the vector version of saying that subtracting a positive number is the same as adding a negative one:



Vector Components

From the above examples, it should be clear that two vectors add to make another vector. Sometimes, the opposite operation is useful: we often want to represent a vector as the sum of two other vectors. This is called breaking a vector into its components. When vectors point along the same line, they essentially add as scalars. If we break vectors into components along the same lines, we can add them by adding their components. The lines we pick to break our vectors into components along are often called a **basis**. Any basis will work in the way described above, but we usually break vectors into *perpendicular* components, since it will frequently allow us to use the Pythagorean theorem in time-saving ways. Specifically, we usually use the x and y axes as our basis, and therefore break vectors into what we call their x and y components:



A final reason for breaking vectors into perpendicular components is that they are in a sense independent: adding vectors along a component perpendicular to an original component one will *never* change the original component, just like changing the y-coordinate of a point can never change its x-coordinate.

1.3 Frequently Used Measurements, Greek Letters, and Prefixes

Measurements

TABLE 1.1: **Types of Measurements**

Type of measurement	*Commonly used symbols*	*Fundamental units*
length or position	d, x, L	meters (m)
time	t	seconds (s)
velocity or speed	v, u	meters per second (m/s)
mass	m	kilograms (kg)
force	$\mathbf{F}$	Newtons (N)
energy	E, K, U, Q	Joules (J)
power	P	Watts (W)
electric charge	q, e	Coulombs (C)
temperature	T	Kelvin (K)
electric current	I	Amperes (A)
electric field	$\mathbf{E}$	Newtons per Coulomb (N/C)
magnetic field	$\mathbf{B}$	Tesla (T)

Prefixes

TABLE 1.2: **Prefix Table**

SI prefix	In Words	Factor
nano (n)	billionth	$1 * 10^{-9}$
micro (μ)	millionth	$1 * 10^{-6}$
milli (m)	thousandth	$1 * 10^{-3}$
centi (c)	hundreth	$1 * 10^{-2}$
deci (d)	tenth	$1 * 10^{-1}$
deca (da)	ten	$1 * 10^{1}$
hecto (h)	hundred	$1 * 10^{2}$
kilo (k)	thousand	$1 * 10^{3}$
mega (M)	million	$1 * 10^{6}$
giga (G)	billion	$1 * 10^{9}$

Greek Letters

TABLE 1.3: Frequently used Greek letters.

μ "mu"	τ "tau"	Φ "Phi"*	ω "omega"	ρ "rho"
θ "theta"	π "pi "	Ω "Omega"*	λ "lambda"	Σ "Sigma"*
α "alpha"	β "beta"	γ "gamma"	Δ "Delta"*	ε "epsilon"

1.4 Applications and Examples

Here are some situations where the ideas covered in the chapter are useful.

Question: The lengths of the sides of a cube are doubling each second. At what rate is the volume increasing?

Solution:The cube side length, x, is doubling every second. Therefore after 1 second it becomes $2x$. The volume of the first cube of side x is $x \times x \times x = x^3$. The volume of the second cube of side $2x$ is $2x \times 2x \times 2x = 8x^3$. The ratio of the second volume to the first volume is $8x^3/x^3 = 8$. Thus the volume is increasing by a factor of 8 every second.

Fermi Questions

The late great physicist Enrico Fermi used to solve problems by making educated guesses. Say you want to *guess*timate the number of cans of soda drunk in San Francisco in one year. You'll come pretty close if you guess that there are about 800,000 people in S.F. and that one person drinks on average about 100 cans per year. So, around 80,000,000 cans are consumed every year. Sure, this answer is not exactly right, but it is likely not off by more than a factor of 10 (i.e., an "order of magnitude"). That is, even though we guessed, we're going to be in the *ballpark* of the right answer. This is often the first step in working out a physics problem.

Dimensional Analysis

Question: find (up to a proportionality constant) the period of a pendulum hanging on a string; that is, find how long it takes such a pendulum to swing through one cycle, knowing that it depends only the acceleration due to gravity $g = 9.8\text{m}/\text{s}^2$ and its length, l, which is measured in meters.

Solution: since the period is in units of time, the answer needs to have units of time (seconds). The only way to obtain seconds from the given quantities is to take the square root of the reciprocal of g (which will have units of seconds over square root of meters) and multiply it by the square root of l, which has units of square root of meters — this will get rid of the meters altogether. In other words, the period will be proportional to the square root of l divided by the square root of g:

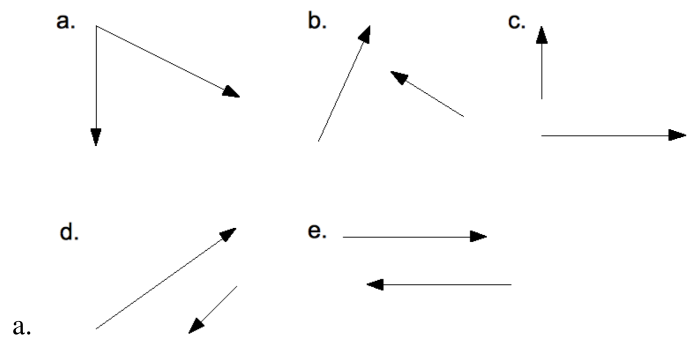
$$T \propto \sqrt{\frac{l}{g}}$$

1.5 Units and Problem Solving Problem Set

1. Estimate or measure your height.
   a. Convert your height from feet and inches to meters.
   b. Convert your height from feet and inches to centimeters $(100 \text{ cm} = 1 \text{ m})$
2. Estimate or measure the amount of time that passes between breaths when you are sitting at rest.
   a. Convert the time from seconds into hours
   b. Convert the time from seconds into milliseconds (ms)
3. Convert the French speed limit of 140 km/hr into mi/hr.
4. Estimate or measure your weight.
   a. Convert your weight in pounds into a mass in kg
   b. Convert your mass from kg into μg
   c. Convert your weight into Newtons
5. Find the *SI* unit for pressure.
6. An English lord says he weighs 12 stone.
   a. Convert his weight into pounds (you may have to do some research online)
   b. Convert his weight in stones into a mass in kilograms
7. If the speed of your car increases by 10 mi/hr every 2 seconds, how many mi/hr is the speed increasing every second? State your answer with the units mi/hr/s.
8. A tortoise travels 15 meters (m) west, then another 13 centimeters (cm) west. How many meters total has she walked?

9. A tortoise, Bernard, starting at point A travels 12 m west and then 150 millimeters (mm) east. How far west of point A is Bernard after completing these two motions?
10. $80 \text{ m} + 145 \text{ cm} + 7850 \text{ mm} = X \text{ mm}$. What is X ?
11. A square has sides of length 45 mm. What is the area of the square in mm^2?
12. A square with area 49 cm^2 is stretched so that each side is now twice as long. What is the area of the square now? Include a sketch.
13. A rectangular solid has a square face with sides 5 cm in length, and a length of 10 cm. What is the volume of the solid in cm^3? Sketch the object, including the dimensions in your sketch.
14. As you know, a cube with each side 4 m in length has a volume of 64 m^3. Each side of the cube is now doubled in length. What is the *ratio* of the new volume to the old volume? Why is this ratio **not** simply 2? Include a sketch with dimensions.
15. What is the ratio of the mass of the Earth to the mass of a single proton? (See equation sheet.)
16. A spacecraft can travel 20 km/s. How many km can this spacecraft travel in 1 hour (h)?
17. A dump truck unloads 30 kilograms (kg) of garbage in 40 s. How many kg/s are being unloaded?
18. The lengths of the sides of a cube are doubling each second. At what rate is the volume increasing?

19. Estimate the number of visitors to Golden Gate Park in San Francisco in one year. Do your best to get an answer that is correct within a factor of 10.
20. Estimate the number of water drops that fall on San Francisco during a typical rainstorm.
21. What does the formula $a = \frac{F}{m}$ tell you about the units of the quantity a (whatever it is)?
22. Add the following vectors using the parallelogram method.



Answers to Selected Problems

1. a. A person of height 5 ft. 11 in. is 1.80 m tall
2. b. The same person is
3. 180 cm
4. a. 3 seconds $= 1/1200$ hours b. $3x10^3$ ms
5. 87.5 mi/hr
6. c. if the person weighs 150 lb. this is equivalent to 668 N
7. Pascals (Pa), which equals N/m^2
8. 168 lb., 76.2 kg
9. 5 mi/hr/s
10. 15.13 m
11. 11.85 m
12. 89,300 mm
13. f. 2025 mm^2
14. b. 196 cm^2
15. c. 250 cm^3
16. 8 : 1, each side goes up by 2 cm, so it will change by 2^3
17. $3.5 \times 10^{51} : 1$
18. 72,000 km/h
19. 0.75 kg/s
20. 8×2^N cm^3/ sec; N is for each second starting with 0 seconds for 8 cm^3
21. About 12 million
22. About $1\frac{1}{2}$ trillion (1.5×10^{12})
23. $[a] = \text{N/kg} = \text{m/s}^2$

CHAPTER 2 Energy ConservationVersion 2

Chapter Outline

2.1 The Big Idea

Energy, a scalar quantity measured in Joules, is a measure of the amount of, or potential for, dynamical activity in something. The total amount of energy in the universe is constant. This kind of symmetry is called a conservation law. Conservation of energy is actually just one of five conservation laws that have been identified by scientists.

The concept of energy conservation is one of the most important in physics, since according to our best model of the world, everything in the universe — even mass — is a form of energy. Any group of things (we'll use the word *system* for this concept in the book) has a certain amount of energy. Energy can be added to a system: when chemical bonds in a burning log break, they release heat. A system can also lose energy: when a spacecraft "burns up" its energy of motion during re-entry, it releases energy to the surrounding atmospher in the form of heat. A *closed* system is one for which the energy is constant, or *conserved*. In this chapter, we will often consider closed systems; although the total amount of energy stays the same, it can transform from one kind to another. We will consider transfers of energy *between* systems – known as *work* – in more detail in Chapter 8. Needless to say, the universe as a whole is a closed system in this sense.

2.2 Types of Energy

Understanding how various processes change energy from one form to another is equivalent to understanding physics. In this class, we will present an overview of various forms of energy but will mainly focus on three: kinetic, gravitational potential, and electrical potential. We will focus on the first two in this chapter; electrical potential energy is covered in later chapters.

Kinetic Energy

The first is Kinetic Energy, or the energy of motion. Any moving object — from the earth to an individual gas molecule — has some kinetic energy, which can be calculated by using the following formula:

$$K = \frac{1}{2}mv^2 \text{ [1]}$$

The m refers to the object's mass, while the v is its speed.

Gravitational Potential Energy

The second type of energy is due to gravity and is therefore called gravitational potential energy. Things with mass have noticeable gravitational potential energy when they are near another object of significant mass, such as the earth, the sun, a black hole, etc. This energy is different from kinetic energy in that it represents potential for motion, rather than motion itself. If I lift a rock away from the surface of the earth to some height and then let it drop, it will gain velocity as it travels downwards. According to the last paragraph, this means it also gains kinetic energy. Assuming no energy is lost to air resistance, there will be a one to one correspondence between gravitational potential energy lost and kinetic energy gained. Near the surface of a planet, the gravitational potential energy gained by an object of mass m when raised a height Δh from its original position *perpendicular to the surface of the planet* is just

$$E_g = mg\Delta h \text{ [2]}$$

The constant g will vary from planet to planet, star to star. On earth, the acceleration due to gravity is 9.8m/s^2, often rounded to 10m/s^2. This is the formula you will likely use the majority of the time. However, there is a way to express the gravitational potential energy of *any two objects in the universe* — any number of objects, in fact. For the two object case, if we call their masses m_1 and m_2 and the distance between their *centers of mass* r, the formula is:

$$E_G = \frac{Gm_1m_2}{r} \text{ [3]}$$

In fact, equation [2] is a **special case** of equation [3]. That is, it is a version of equation [3] that holds under specific circumstances. See the appendix to this chapter for a derivation.

2.3 Key Concepts

- Any object in motion has kinetic energy. Kinetic energy increases as the square of the velocity, so faster objects have much more kinetic energy than slower ones.
- The energy associated with gravity is called gravitational potential energy. Near the surface of the earth, an object's gravitational potential energy increases linearly with its height.
- Molecules store chemical potential energy in the bonds between electrons; when these bonds are broken the released energy can be transferred into kinetic and/or potential energy. 1KCal (1 food Calorie) is equal to 4180 Joules of stored chemical potential energy.
- Energy can be transformed from one kind into another and exchanged between systems; if there appears to be less total energy in a system at the end of a process then at the beginning, the "lost" energy has been transferred to another system, often by heat or sound waves.

2.4 Key Applications

- In "roller coaster" problems, a cart's gravitational potential energy at the top of one hill is transformed into kinetic energy at the next valley. It turns back into potential energy as cart climbs the next hill, and so on. In reality a fraction of the energy is lost to the tracks and air as heat, which is why the second rise is rarely as big as the first in amusement parks.

- In "pole-vaulter" problems, the athlete's body breaks down the food molecules to change some of the bonding energy into energy that is used to power the body. This energy is transformed into kinetic energy as the athlete gains speed. The kinetic energy can be changed into potential energy as the athlete gains height.

- In "pendulum" problems, the gravitational potential energy of the pendulum at its highest point changes to kinetic energy as it swings to the bottom and then back into potential energy as it swings up. At any in-between point there is a combination of kinetic energy and potential energy, but the total energy remains constant.

2.5 Examples

Example 1

Question A roller coaster begins at rest 120m above the ground at point A. Assume no energy is lost. The radius of the loop is 40 ft

a) Find the speed of the roller coaster at point B, D, F, and H.

b) At point G the roller coaster's speed is 22m/s. How high off the ground is point G?

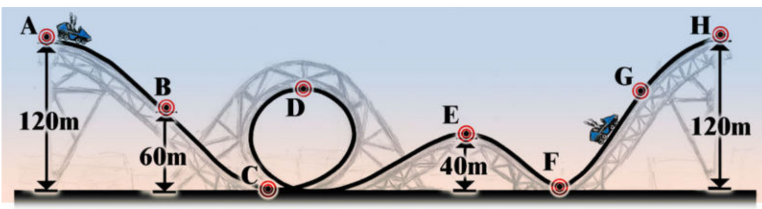


FIGURE 2.1

Roller coaster for problem 8.

Solution a) To solve for the speed at any point on the roller coaster, we use conservation of energy: the cart's total energy, equal to its initial energy (all potential) is split between kinetic and potential energy at all points during the trip. Therefore, at any point

$$mgh+\frac{1}{2}mv^2 = mg\times 120$$

Solving for v:

$$120mg = \frac{1}{2}mv^2+mgh \Rightarrow 120g-gh = \frac{1}{2}mv^2 \Rightarrow g(120-h) = \frac{1}{2}mv^2 \Rightarrow \sqrt{2g(120-h)} = v$$

Therefore

B: $\sqrt{2gh} = \sqrt{2\times 9.8\text{m/s}^2\times(120-60\text{m})} = 34\text{m/s}$

D: $\sqrt{2gh} = \sqrt{2\times 9.8\text{m/s}^2\times(120-80\text{m})} = 28\text{m/s}$

F: $\sqrt{2gh} = \sqrt{2\times 9.8\text{m/s}^2\times(120-0\text{m})} = 48\text{m/s}$

H: $\sqrt{2gh} = \sqrt{2\times 9.8\text{m/s}^2\times(120-120\text{m})} = 0\text{m/s}$

b) As in part a), we start with the equation

$$mgh+\frac{1}{2}mv^2 = mg\times 120$$

but this time we will solve for h.

$$120mg = \frac{1}{2}mv^2+mgh \Rightarrow 120g-\frac{1}{2}v^2 = gh \Rightarrow \frac{120g}{g}-\frac{v^2}{2g} = h \Rightarrow 120-\frac{v^2}{2g} = h$$

Now simply input the known variables to solve for h.

$$h = 120 - \frac{v^2}{2g} = 120 - \frac{(22\text{m/s})^2}{2 \times 9.8\text{m/s}^2} = 25\text{m}$$

2.6 Energy Conservation Problem Set

1. A stationary bomb explodes into hundreds of pieces. Which of the following statements best describes the situation?

   a. The kinetic energy of the bomb was converted into heat.
   b. The chemical potential energy stored in the bomb was converted into heat and gravitational potential energy.
   c. The chemical potential energy stored in the bomb was converted into heat and kinetic energy.
   d. The chemical potential energy stored in the bomb was converted into heat, sound, kinetic energy, and gravitational potential energy.
   e. The kinetic and chemical potential energy stored in the bomb was converted into heat, sound, kinetic energy, and gravitational potential energy.

2. You hike up to the top of Granite Peak in the Trinity Alps to think about physics.

   a. Do you have more potential or kinetic energy at the top of the mountain than you did at the bottom? Explain.
   b. Do you have more, less, or the same amount of energy at the top of the mountain than when you started? (Let's assume you did not eat anything on the way up.) Explain.
   c. How has the total energy of the Solar System changed due to your hike up the mountain? Explain.
   d. If you push a rock off the top, will it end up with more, less, or the same amount of energy at the bottom? Explain.
   e. For each of the following types of energy, describe whether you *gained* it, you *lost* it, or it stayed the same during your hike:

      a. Gravitational potential energy
      b. Energy stored in the atomic nuclei in your body
      c. Heat energy
      d. Chemical potential energy stored in the fat cells in your body
      e. Sound energy from your footsteps
      f. Energy given to you by a wind blowing at your back

3. Just before your mountain bike ride, you eat a 240 Calorie exercise bar. (You can find the conversion between food Calories and Joules in the chapter.) The carbon bonds in the food are broken down in your stomach, releasing energy. About half of this energy is lost due to inefficiencies in your digestive system.

   (a) Given the losses in your digestive system how much of the energy, in Joules, can you use from the exercise bar?

   After eating, you climb a 500 m hill on your bike. The combined mass of you and your bike is 75 kg.

   (b) How much gravitational potential energy has been gained by you and your bike?
   (c) Where did this energy come from?
   (d) If you ride quickly down the mountain without braking but losing half the energy to air resistance, how fast are you going when you get to the bottom?

4. You find yourself on your bike at the top of Twin Peaks in San Francisco. You are facing a 600 m descent. The combined mass of you and your bicycle is 85 kg.

(a) How much gravitational potential energy do you have before your descent?
(b) You descend. If all that potential energy is converted to kinetic energy, what will your speed be at the bottom?
(c) Name two other places to which your potential energy of gravity was transferred besides kinetic energy. How will this manifest itself in your speed at the bottom of the hill? (No numerical answer is needed here.)

5. Before a run, you eat an apple with $1,000,000$ Joules of binding energy.
   a. $550,000$ Joules of binding energy are wasted during digestion. How much remains?
   b. Some 95% of the remaining energy is used for the basic processes in your body (which is why you can warm a bed at night!). How much is available for running?
   c. Let's say that, when you run, you lose 25% of your energy overcoming friction and air resistance. How much is available for conversion to kinetic energy?
   d. Let's say your mass is 75 kg. What could be your top speed under these idealized circumstances?
   e. But only 10% of the available energy goes to KE, another 50% goes into heat exhaust from your body. Now you come upon a hill if the remaining energy is converted to gravitational potential energy. How high do you climb before running out of energy completely?
6. A car goes from rest to a speed of v in a time t. Sketch a schematic graph of kinetic energy vs. time. You do not need to label the axes with numbers.
7. A 1200 kg car traveling with a speed of 29 m/s drives horizontally off of a 90 m cliff.
   a. Sketch the situation.
   b. Calculate the potential energy, the kinetic energy, and the total energy of the car as it leaves the cliff.
   c. Make a graph displaying the kinetic, gravitational potential, and total energy of the car at each 10 m increment of height as it drops
8. A roller coaster begins at rest 120 m above the ground at point A, as shown above. Assume no energy is lost from the coaster to frictional heating, air resistance, sound, or any other process. The radius of the loop is 40 m .

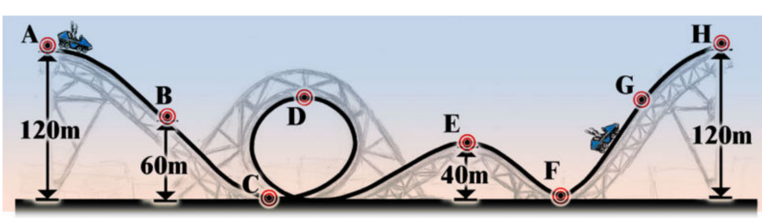


FIGURE 2.2

Roller coaster for problem 8.

a. Find the speed of the roller coaster at points B, C, D, E, F, and H.
b. At point G the speed of the roller coaster is 22 m/s. How high off the ground is point G?

9. A pendulum has a string with length 1.2 m. You hold it at an angle of 22 degrees to the vertical and release it. The pendulum bob has a mass of 2.0 kg.

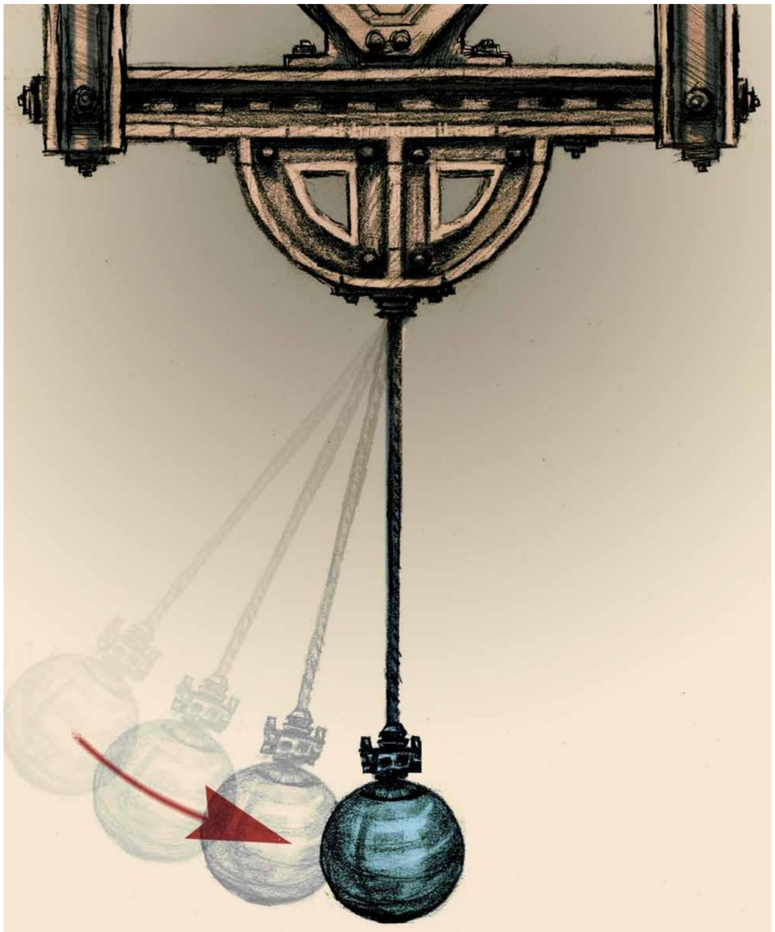


FIGURE 2.3
Pendulum for problem 9.

a. What is the potential energy of the bob before it is released? (*Hint: use geometry to determine the height when released.*)
b. What is its speed when it passes through the midpoint of its swing?
c. Now the pendulum is transported to Mars, where the acceleration of gravity g is 2.3 m/s^2. Answer parts (a) and (b) again, but this time using the acceleration on Mars.

10. On an unknown airless planet an astronaut drops a 4.0 kg ball from a 60 m ledge. The mass hits the bottom with a speed of 12 m/s.

a. What is the acceleration of gravity g on this planet?
b. The planet has a twin moon with exactly the same acceleration of gravity. The difference is that this moon has an atmosphere. In this case, when dropped from a ledge with the same height, the 4.0 kg ball hits bottom at the speed of 9 m/s. How much energy is lost to air resistance during the fall?

11. A 1500 kg car starts at rest and speeds up to 3.0 m/s.

a. What is the gain in kinetic energy?
b. We define efficiency as the ratio of output energy (in this case kinetic energy) to input energy. If this car's efficiency is 0.30, how much input energy was provided by the gasoline?

   c. If 0.15 gallons were used up in the process, what is the energy content of the gasoline in Joules per gallon?

12. A pile driver's motor expends $310,000$ Joules of energy to lift a 5400 kg mass. The motor is rated at an efficiency of 0.13 (see 11b). How high is the mass lifted?

Answers to Selected Problems

1. d
2. (discuss in class)
    1. 5.0×10^5 J
    2. 3.7×10^5 J
    3. Chemical bonds in the food.
    4. 99 m/s

    1. 5.0×10^5 J
    2. 108 m/s

    1. $450,000$ J
    2. $22,500$ J
    3. $5,625$ J
    4. 21.2 m/s
    5. 9.18 m

3. .
4. b. $KE = 504,600$ J; $U_g = 1,058,400$ J; $E_{total} = 1,563,000$ J
    1. 34 m/s at B; 28 m/s at D, 40 m/s at E, 49 m/s at C and F; 0 m/s at H
    2. 96 m

    1. 1.7 J
    2. 1.3 m/s
    3. 0.4 J, 0.63 m/s

    1. 1.2 m/s^2
    2. 130 J

    1. 6750 J
    2. 2.25×10^5 J
    3. 1.5×10^5 J/gallon of gas

5. 0.76 m

2.7 References

1. . Roller Coaster Illustration. CC-BY-SA 3.0
2. . Roller coaster. CC-BY-SA 3.0
3. . Pendulum. CC-BY-SA 3.0

CHAPTER 3 Energy Conservation Appendix

Chapter Outline

3.1 Equivalence between [2] and [3]

The formulas above look pretty different, but the main conceptual split is that the g in the first formula above ([2]) varies from planet to planet, but the G in [3] constant throughout the universe — in fact, it's called the **Gravitational Constant**. You might think that this makes the second formula more fundamental than the first, and you would be right. The first is actually a "special case" of the second. That is, the [3] always holds, but [2] only holds when certain conditions are met: that is, you are at the surface of a spherical body. In this case they are equivalent, but [2] is obviously simpler.

It is important to see the relationship between [2] and [3], since it is typical of the stuff of physics. If [3] is the more fundamental equation, we should be able to start with it and derive [2]. First, we will make a minor simplification: we will assume that the "object of interest" starts at the surface of the earth, and not some arbitrary height near it (now we don't have to deal with the deltas and hairier details without sacrificing any of the content). Then the question we want to answer is: if we raise this object from the surface to a height h, what will its gravitational potential energy change by?

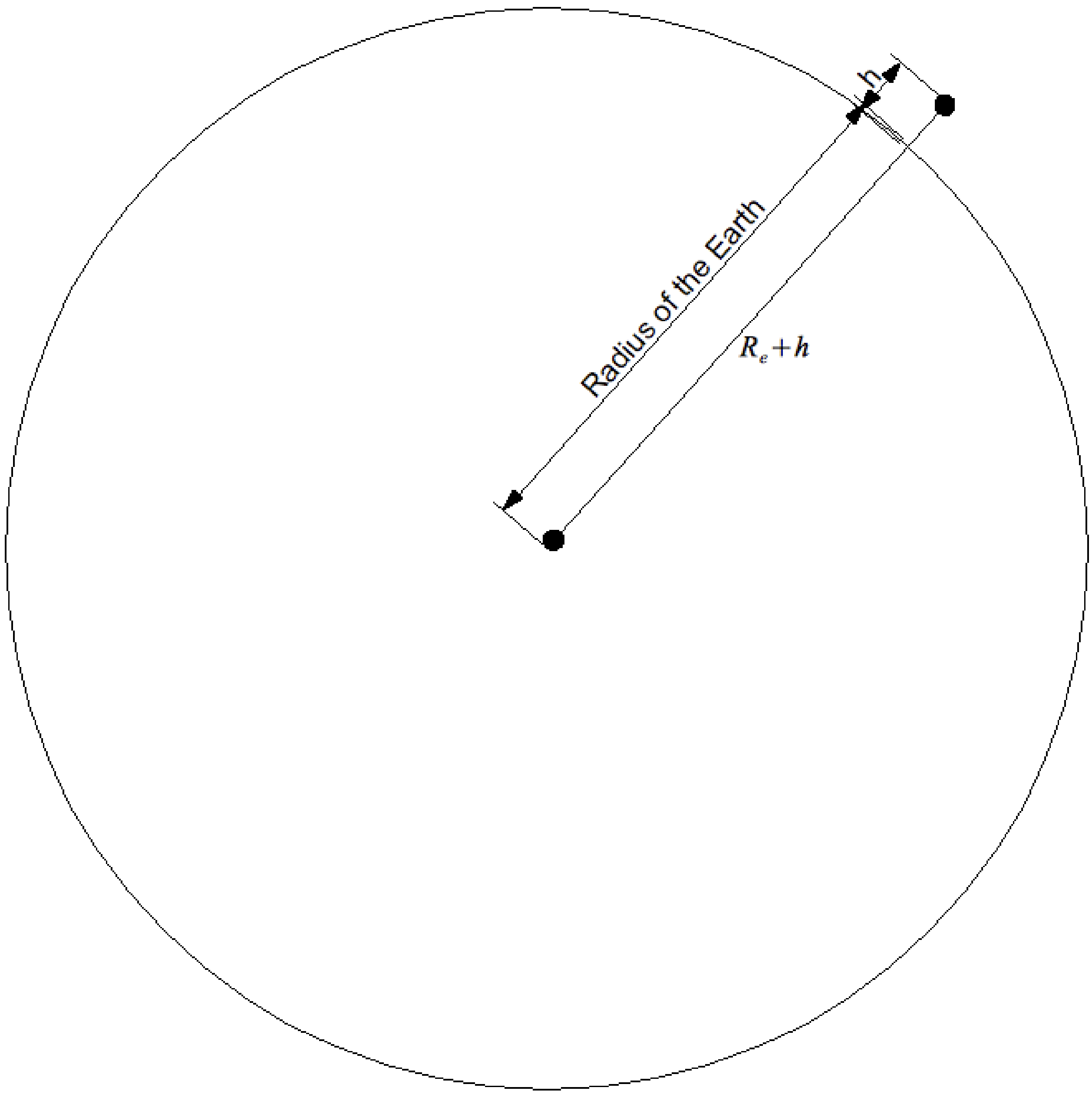


FIGURE 3.1
Illustration for the Equivalence Derivation

Since equation [3] always holds exactly (as far as we're concerned), we can certainly use it. Before it was raised, the object was at the surface, so its potential energy from Earth's gravity was given by:

$$E_{Gi} = \frac{Gm_1m_2}{R_{\text{earth}}} \text{ [4]}$$

This held because the r in [3] is the distance between the two objects' centers of mass. The center of mass of the earth is, predictably, at its center, so we can replace r above with R_{earth} for objects on its surface. After the object is raised, its distance from the center of the Earth increases by h. In other words, its new energy is

$$E_{Gf} = \frac{Gm_1m_2}{R_{\text{earth}} + h} \text{ [5]}$$

Now we will rewrite equation [5] in the following way:

$$E_{Gf} = \frac{Gm_1m_2}{R_{\text{earth}}} \times \frac{R_{\text{earth}}}{R_{\text{earth}} + h}$$

which, if we divide the denominator and numerator of the second fraction on the right of the above equation by the Earth's radius, gives us:

$$E_{Gf} = \frac{Gm_1m_2}{R_{\text{earth}}} \times \frac{1}{1 + \frac{h}{R_{\text{earth}}}} = \frac{Gm_1m_2}{R_{\text{earth}}} \times 1 \times \underbrace{(1 + \frac{h}{R_{\text{earth}}})^{-1}}_{\text{of the form } (1+x)^{-1}} \text{ [6]}$$

If we explore the quantities of the type $(1+x)^{-1}$, we will see that when $|x| \ll 1$, $(1+x)^{-1} \approx 1 - x$. This kind of adjustment is called a Taylor approximation. To prove this to yourself, use your calculator to try a bunch of different numbers and see what the error is.

The ratio of h, the distance the object was raised, to the radius of the earth is miniscule. So, we can use the theorem above, remembering that it is this ratio that plays the role of x above. This ratio is only small for large objects, such as planets; and an object's center of mass may not always be at its geographic center. Both have to be true for our results to hold. Hence, special case.

Then we find that [6] reduces to

$$E_{Gf} = \frac{Gm_1m_2}{R_{\text{earth}}} \times (1 - \frac{h}{R_{\text{earth}}}) \text{ [7]}$$

Therefore, we can now rewrite the object's final potential energy as:

$$E_{Gf} = \frac{Gm_1m_2}{R_{\text{earth}}} - \frac{hGm_1m_2}{{R_{\text{earth}}}^2} \text{ [8]}$$

The last step is to finally calculate the change in potential energy due to the movement. This is straightforward, since

$$\Delta E = E_{Gi} - E_{Gf} = (\frac{Gm_1m_2}{R_{\text{earth}}}) - (\frac{Gm_1m_2}{R_{\text{earth}}} - \frac{hGm_1m_2}{{R_{\text{earth}}}^2}) = \frac{hGm_1m_2}{{R_{\text{earth}}}^2}$$

This is the equation we have been looking for. Although it doesn't look like it, it is completely equivalent to the formula $E_g = mgh$. The only variable in this formula is h, everything else — the radius and mass of Earth and G — are constants. If we rearrange the formula, we see that:

$$\frac{hGm_1m_2}{{R_{\text{earth}}}^2} = \underbrace{m1}_{m} \times \underbrace{\frac{Gm_2}{{R_{\text{earth}}}^2}}_{g} \times \underbrace{h}_{h}$$

The quantity labeled g, if calculated with the appropriate radius and mass, will give the effective acceleration due to gravity near the surface. When Earth's mass and radius are used. the result is 9.8 m/s, but feel free to check.

3.2 References

1. Alex Zaliznyak. IllustrationfortheEquivalenceDerivation. Public Domain

CHAPTER 4 One Dimensional Motion Version 2

Chapter Outline

4.1 The Big Idea

One dimensional motion describes objects moving in straight lines. Speed is a scalar measure of how quickly an object is moving along this line: units of length per one unit of time. If an object's speed changes, it is said to be accelerating (or decelerating). As we will see, understanding an object's acceleration is the key to understanding its motion. At this point, we are not worried about *where* the acceleration is coming from — we will deal with that question later.

In general, position, displacement, velocity and acceleration have directions and are therefore vectors. It's important to note, however, that in one dimension there are only two possible directions for vectors to point in, and these are usually labeled $+$ and $-$. We will therefore typically avoid calling one dimensional quantities vectors, since their direction can be represented with a sign. The quantities labeled vectors below can and will be treated as scalars with signs in one dimensional situations throughout the book.

4.2 Key Definitions

$$\text{Symbols}\begin{cases}\Delta(\text{anything}) & \text{Final value }-\text{ initial value}\\ \text{anything}_0 & \text{Value at time 0}\end{cases}$$

$$\text{Scalars}\begin{cases}t & \text{Time in seconds, s}\\ d=|\vec{\Delta x}_1|+|\vec{\Delta x}_2| & \text{Distance (in meters, m)}\\ v=|\vec{v}| & \text{Speed (in meters per second, m/s)}\end{cases}$$

$$\text{Vectors}\begin{cases}\vec{x}=\vec{x}(t) & \text{Position}\\ \vec{\Delta x}=\vec{x}_f-\vec{x}_i & \text{Displacement}\\ \vec{v}_i & \text{Initial velocity}\\ \vec{v}_f & \text{Final velocity}\\ \vec{\Delta v}=\vec{v}_f-\vec{v}_i & \text{Change in velocity}\\ \vec{a}=\frac{\Delta\vec{v}}{\Delta t} & \text{Acceleration}\end{cases}$$

4.3 Deriving the Kinematics Equations

The simplest case of one dimensional motion is an object at rest. A slightly more difficult problem is that of an object moving at a constant velocity. Such an object's position at time t is given by the familiar $d = rt$ formula, that is, distance equals rate times time. In our language, this would be:

$$\Delta x = x_f - x_i = vt \qquad \text{When velocity is constant [1]}$$

If an object is undergoing an acceleration that changes with time, it is in general quite difficult to find its position and velocity as a function of time. However, it's always true that over a period of time Δt average velocity and average acceleration are given by:

$$v_{avg} = \frac{\Delta x}{\Delta t} \qquad \text{Always [2]}$$

$$a_{avg} = \frac{\Delta v}{\Delta t} \qquad \text{Always [3],}$$

In other words,

$$\Delta x = v_{avg} \Delta t \qquad \text{Always [4]}$$

$$\Delta v = a_{avg} \Delta t \qquad \text{Always [5],}$$

Therefore, finding an object's position or velocity can be reduced to finding the average velocity or average acceleration, respectively. Usually, this is just as difficult as the problem mentioned above, but in one very common and specific case — *constant acceleration* — these formulas are very useful. In this case, *velocity* changes at a linear rate with time, that is:

$$v_f = at + v_i \qquad \text{When acceleration constant [6]}$$

You should realize this is just another version of equation [1], which in fact describes anything changing at a linear rate. Since the average of a linear function over some time is just the average of its endpoints (figure 1), we have:

$$v_{avg} = \frac{v_f + v_i}{2} \qquad \text{When acceleration constant [7]}$$

Now, we

Start with equation [7] $\qquad v_{avg} = \frac{v_f + v_i}{2}$

Plug in equation [6] $\qquad v_{avg} = \frac{at + v_i + v_i}{2}$

And end up with $\qquad v_{avg} = \frac{at}{2} + v_i$

Finally, since $\Delta x(t) = v_{avg} t$ $\qquad x(t) = x_i + v_i t + \frac{1}{2}at^2$[8]

We have obtained the equations of uniformly, accelerated motion, also known as the

Big Three Equations

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \qquad [8]$$

$$v(t) = v_0 + at \qquad [6]$$

$$v_f{}^2 = v_0{}^2 + 2ax \qquad \text{(Derivation left to reader) [9]}$$

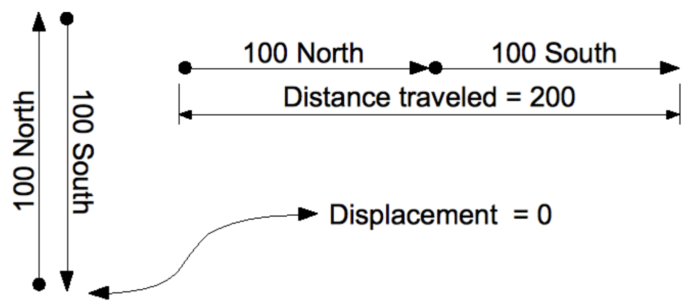
4.4 Hints for Problems

- When beginning a one dimensional problem, define a positive direction. The other direction is then taken to be negative. Traditionally, "positive" is taken to mean "to the right"; however, any definition of direction used consistency throughout the problem will yield the right answer.
- Be sure you understand the difference between average velocity (measured over a period of time) and instantaneous velocity (measured at a single moment in time).
- Gravity near the Earth pulls an object toward the surface of the Earth with an acceleration of $9.8\text{m}/\text{s}^2 \approx 10\text{m}/\text{s}^2$. In the absence of air resistance, all objects will fall with the same acceleration. Air resistance can cause low-mass, large area objects to accelerate more slowly.
- *Deceleration* is the term used when an object's *speed* is decreasing due to an acceleration in the opposite direction of its velocity.
- The Big Three equations define the graphs of position and velocity as a function of time. When there is no acceleration (constant velocity), position increases linearly with time – distance equals rate times time. Under constant acceleration, velocity increases linearly with time but distance does so at a quadratic rate. The slopes of the position and velocity graphs will give instantaneous velocity and acceleration, respectively.
- At first, you might get frustrated trying to figure out which of the Big Three equations to use for a certain problem, but don't worry, this comes with practice. Making a table that identifies the variables given in the problem and the variables you are looking for can sometimes help.

4.5 One Dimensional Examples

Example 1

Question: If Bob walks 100 meters north, turns around, and walks 100 meters south, what is his total distance d and displacement $\Delta\vec{x}$?



Solution: If Bob walked 100 meters in north, turned around, and walked back to his starting position his total distance traveled would be:

$$d = |\Delta\vec{x}_1| + |\Delta\vec{x}_2| = |100\text{m}| + |-100\text{m}| = 200\text{m}.$$

However, his displacement would be:

$$\text{f - \_i = 0 m + 0 m = 0 m. } \Delta\vec{x} = \vec{x}$$

As you can see, if Bob returns to his original starting position his displacement (0 meters) and his total distance traveled (200 meters) are very different.

Example 2

Question: Joe throws a ball straight up with a initial velocity of $70\frac{\text{m}}{\text{s}}$. For this problem ignore Joe's height.

a) How high does the ball go?

b) For how many seconds is the ball in the air?

c) Joe is now standing underneath a ceiling that is 237m high. How fast will the ball be traveling when it strikes the ceiling?

Solution:

a) We know the initial velocity ($70\text{m}/\text{s}^2$) and the acceleration (the only acceleration that acts upon the ball after it has left Joe's hand is the acceleration of gravity, which is $-9.8\text{m}/\text{s}^2$). We can figure out the final velocity by realizing that when the ball is at the highest point, the velocity will be $0\text{m}/\text{s}^2$. To find the height of the ball we will use the equation ${v_f}^2 = {v_i}^2 + 2ax$ and solve for x.

$$v_f{}^2 = v_i{}^2 + 2ax \Rightarrow \frac{v_f{}^2 - v_i{}^2}{2a} = x$$

We can now substitute in the known values to get x.

$$\frac{v_f{}^2 - v_i{}^2}{2a} = \frac{(0\text{m/s} - 70^2\text{m/s})}{2 \times (-9.8\text{m/s}^2)} = 250\text{m}$$

b) To solve for the total time the ball is in the air we will for the time that the ball is traveling up and double it (the trip down will take the same time as the trip up). We know the initial velocity is 70m/s and the acceleration due to gravity is -9.8m/s^2. We also know that the final velocity is 0m/s. We will use the equation $v_f = v_i + at$ and solve for t.

$$v_f = v_i + at \Rightarrow \frac{v_f - v_i}{a} = t$$

We can now substitute what we know into the equation to solve for t.

$$\frac{v_f - v_i}{a} = \frac{0\text{m/s} - 70\text{m/s}}{-9.8\text{m/s}^2} = 7\text{sec}$$

Remember that this is only the trip up though. To solve for the total time the ball in the air we simply double the answer.

$$7\text{sec} \times 2 = 14\text{sec}$$

c) We know the ceiling height (237m), the initial velocity (70m/s), and the acceleration (-9.8m/s^2). Using the equation $v_f{}^2 = v_i{}^2 + 2ax$, we can solve for v.

$$v_f{}^2 = v_i{}^2 + 2ax \Rightarrow v_f = \sqrt{v_i{}^2 + 2ax}$$

We can now find the velocity.

$$\sqrt{v_i{}^2 + 2ax} = \sqrt{70^2\text{m/s} + 2 \times (-9.8\text{m/s}^2) \times (237\text{m})} = 16\text{m/s}$$

Example 3

Question: Two cars are heading toward each other, traveling at 50km/hr (car A) and 70km/hr (car B). They are $12km$ apart. How much time do they have before they collide?

Answer: To find the answer, we must find the proportion of the distance that each car travels. In the time that car A travels 50km, car B will travel 70km. This can be made into the ratio $\frac{50}{70}$ which can then be simplified into $\frac{5}{7}$. This means that car A travels $\frac{5}{12}$ (which is 5km) of the distance and car B travels $\frac{7}{12}$ (which is 7km) of the distance. Car B takes

$$50\text{km/hr} \times t = 5\text{km} \Rightarrow t = \frac{5\text{km}}{50\text{km/hr}} \Rightarrow t = .1\text{hr} = 6\text{min}$$

to cover 5km. Car A takes

$$70\text{km/hr} \times t = 7\text{km} \Rightarrow t = \frac{7\text{km}}{70\text{km/hr}} \Rightarrow t = .1\text{hr} = 6\text{min}$$

to cover 7km. Therefore the time before the collision is 6 minutes.

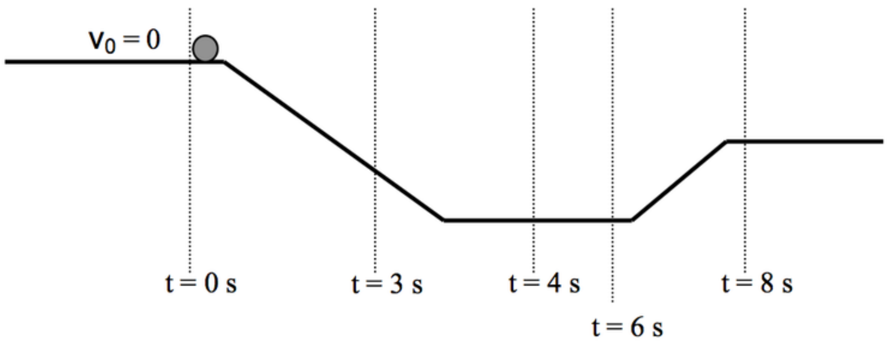
4.6 One-Dimensional Motion Problem Set

1. Answer the following questions about one-dimensional motion.
   a. What is the difference between distance d and displacement $\triangle x$ Write a few sentences explaining this.
   b. Does the odometer reading in a car measure distance or displacement?
   c. Imagine a fox darting around in the woods for several hours. Can the displacement $\triangle x$ of the fox from his initial position ever be larger than the total distance d he traveled? Explain.
   d. What is the difference between acceleration and velocity? Write a paragraph that would make sense to a 5^{th} grader.
   e. Give an example of a situation where an object has an upward velocity, but a downward acceleration.
   f. What is the difference between average and instantaneous velocity? Make up an example involving a trip in a car that demonstrates your point.
   g. If the position of an object is increasing linearly with time (i.e., $\triangle x$ is proportional to t), what can we say about its acceleration? Explain your thinking.
   h. If the position of an object is increasing non-linearly with time (i.e., $\triangle x$ is not proportional to t), what can we say about its velocity? Explain your thinking.
2. A cop passes you on the highway. Which of the following statements must be true at the instant he is passing you? You may choose more than one answer.
   a. Your speed and his speed are the same.
   b. Your position x along the highway is the same as his position x along the highway.
   c. Your acceleration and his acceleration are the same.
3. If a car is slowing down from 50 MPH to 40 MPH, but the x position is increasing, which of the following statements is true? You may choose more than one.
   a. The velocity of the car is in the $+x$ direction.
   b. The acceleration of the car is in the same direction as the velocity.
   c. The acceleration of the car is in the opposite direction of the velocity.
   d. The acceleration of the car is in the $-x$ direction.
4. A horse is galloping forward with an acceleration of 3 m/s^2. Which of the following statements is necessarily true? You may choose more than one.
   a. The horse is increasing its speed by 3 m/s every second, from 0 m/s to 3 m/s to 6 m/s to 9 m/s.
   b. The speed of the horse will triple every second, from 0 m/s to 3 m/s to 9 m/s to 27 m/s.
   c. Starting from rest, the horse will cover 3 m of ground in the first second.
   d. Starting from rest, the horse will cover 1.5 m of ground in the first second.
5. Below are images from a race between Ashaan (above) and Zyan (below), two daring racecar drivers. High speed cameras took four pictures in rapid succession. The first picture shows the positions of the cars at $t = 0.0$. Each car image to the right represents times $0.1, 0.2$, and 0.3 seconds later.

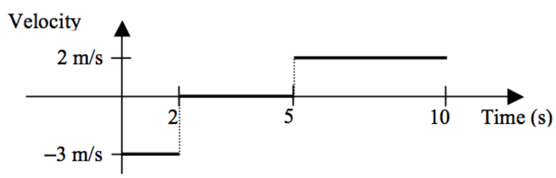
   a. Who is ahead at $t = 0.2$ s ? Explain.

b. Who is accelerating? Explain.
c. Who is going fastest at $t = 0.3$ s? Explain.
d. Which car has a constant velocity throughout? Explain.
e. Graph x vs. t and v vs. t. Put both cars on same graph; label which line is which car.
f. Which car is going faster at $t = 0.2$ s (Hint: Assume they travel the same distance between 0.1 and 0.2 seconds)?

6. In the picture below, a ball starting at rest rolls down a ramp, goes along at the bottom, and then back up a smaller ramp. Ignore friction and air resistance. Sketch the *vertical* position vs. time and *vertical* speed vs. time graphs that accurately describe this motion. Label your graphs with the times indicated in the picture.



7. Draw the position vs. time graph that corresponds to the velocity vs. time graph below. You may assume a starting position $x_0 = 0$. Label both axes of your graph with appropriate values.



8. Two cars are heading right towards each other, but are 12 km apart. One car is going 70 km/hr and the other is going 50 km/hr. How much time do they have before they collide head on?
9. The following data represent the first 30 seconds of actor Crispin Glover's drive to work.

TABLE 4.1:

Time (s)	Position (m)	Distance (m)
0	0	0
5	10	10
10	30	30
15	30	30
20	20	40
25	50	70
30	80	120

(a) Sketch the graphs of position vs. time and distance vs. time. Label your x and y axes appropriately.

(b) Why is there a discrepancy between the distance covered and the change in position during the time period between $t = 25$ s and $t = 30$ s?

(c) What do you think is going on between $t = 10$ s and $t = 15$ s?

(d) What is the displacement between $t = 10$ s and $t = 25$ s?

(e) What is the distance covered between $t = 10$ s and $t = 25$ s?

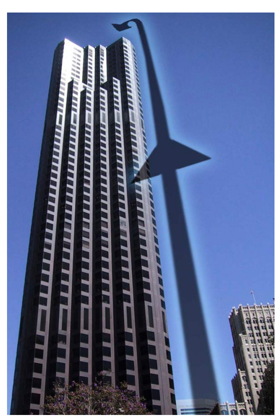
(f) What is the average velocity during the first 30 seconds of the trip?

(g) What is the average velocity between the times $t = 20$ s and $t = 30$ s?

(h) During which time interval(s) was the velocity negative?

(i) Sketch the velocity vs. time and speed vs. time graphs. Label your x and y axes appropriately.

10. Sketchy LeBaron, a used car salesman, claims his car is able to go from 0 to 60 mi/hr in 3.5 seconds.
    a. What is the average acceleration of this car? Give your answer in m/s^2. (Hint: you will have to perform a conversion.)
    b. How much distance does this car cover in these 3.5 seconds? Express your answer twice: in meters and in feet.
    c. What is the speed of the car in mi/hr after 2 seconds?
11. Michael Jordan had a vertical jump of about 48 inches.
    a. Convert this height into meters.
    b. Assuming no air resistance, at what speed did he leave the ground?
    c. What is his speed $3/4$ of the way up?
    d. What is his speed just before he hits the ground on the way down?
12. You are sitting on your bike at rest. Your brother comes running at you from behind at a speed of $2\ \text{m}/\text{s}$. At the exact moment he passes you, you start up on your bike with an acceleration of $2\ \text{m}/\text{s}^2$.
    a. Draw a picture of the situation, defining the starting positions, speeds, etc.
    b. At what time t do you have the same speed as your brother?
    c. At what time t do you pass your brother?
    d. Draw another picture of the exact moment you catch your brother. Label the drawing with the positions and speeds at that moment.
    e. Sketch a position vs. time graph for both you and your brother, labeling the important points (*i.e.,* starting point, when you catch him, etc.)
    f. Sketch a speed vs. time graph for both you and your brother, labeling the important points (*i.e.,* starting point, when you catch him, etc.)



13. You are standing at the foot of the Bank of America building in San Francisco, which is 52 floors (237 m) high. You launch a ball straight up in the air from the edge of the foot of the building. The initial vertical speed is $70\ \text{m}/\text{s}$. (For this problem, you may ignore your own height, which is very small compared to the height of the building.)
    a. How high up does the ball go?

b. How fast is the ball going right before it hits the top of the building?
c. For how many seconds total is the ball in the air?

14. Measure how high you can jump vertically on Earth. Then, figure out how high you would be able to jump on the Moon, where acceleration due to gravity is $1/6^{th}$ that of Earth. Assume you launch upwards with the same speed on the Moon as you do on the Earth.
15. A car is smashed into a wall during Weaverville's July 4^{th} Destruction Derby. The car is going 25 $\mathrm{m/s}$ just before it strikes the wall. It comes to a stop 0.8 seconds later. What is the average acceleration of the car during the collision?

16. A helicopter is traveling with a velocity of 12 $\mathrm{m/s}$ directly upward. Directly below the helicopter is a very large and very soft pillow. As it turns out, this is a good thing, because the helicopter is lifting a large man. When the man is 20 m above the pillow, he lets go of the rope.
    a. What is the speed of the man just before he lands on the pillow?
    b. How long is he in the air after he lets go?
    c. What is the greatest height reached by the man above the ground? (Hint: this should be greater than 20 m. Why?)
    d. What is the distance between the helicopter and the man three seconds after he lets go of the rope?
17. You are speeding towards a brick wall at a speed of 55 MPH. The brick wall is only 100 feet away.
    a. What is your speed in $\mathrm{m/s}$?
    b. What is the distance to the wall in meters?
    c. What is the minimum acceleration you should use to avoid hitting the wall?
18. What acceleration should you use to increase your speed from 10 $\mathrm{m/s}$ to 18 $\mathrm{m/s}$ over a distance of 55 m?
19. You drop a rock from the top of a cliff. The rock takes 3.5 seconds to reach the bottom.
    a. What is the initial speed of the rock?
    b. What is the magnitude (i.e., *numerical value*) of the acceleration of the rock at the moment it is dropped?
    c. What is the magnitude of the acceleration of the rock when it is half-way down the cliff?
    d. What is the height of the cliff?
20. An owl is flying along above your farm with positions and velocities given by the formulas

$x(t) = 5.0 + 0.5t + (1/2)(0.3)t^2;$ where t is in seconds and x is in meters from the barn;
$v(t) = 0.5 + (0.3)t$ where v is m/s

(a) What is the acceleration of the owl?

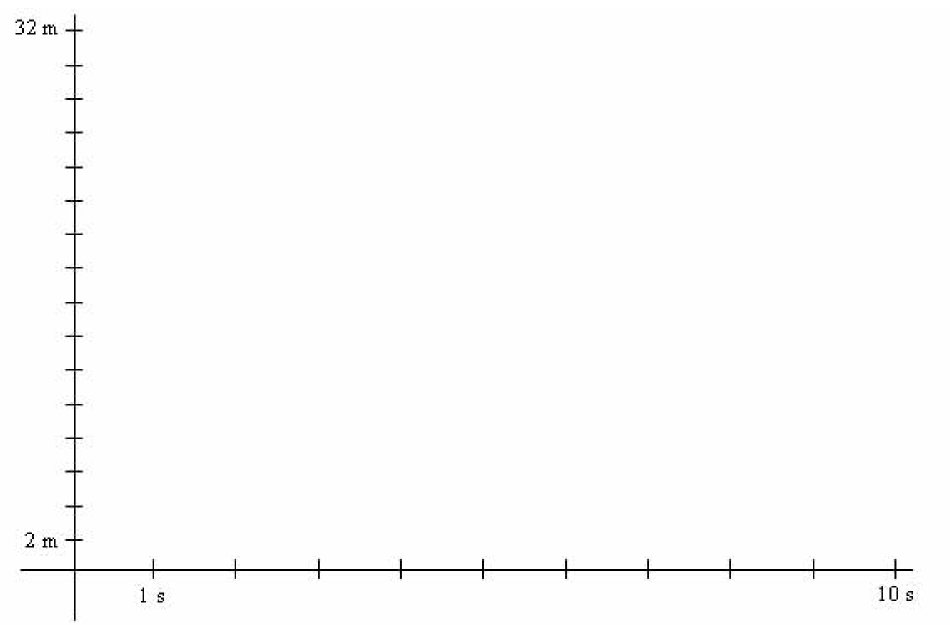
(b) What is the speed of the owl at $t = 0$?

(c) Fill in the missing elements of the table.

TABLE 4.2:

t	x	v
0.0 s	5 m	.5 m/s
1.0 s	5.65 m	.8 m/s
2.0 s	6.6 m	1.1 m/s
3.0 s		
4.0 s		
5.0 s		
6.0 s		
7.0 s		
8.0 s		
9.0 s		
10.0 s		

(d) Plot the x and t points on the following graph. Then, connect your points with a smoothly curving line. Be careful and neat and use pencil.



(e) Use the formula to calculate the speed of the owl in m/s at $t = 5$ seconds.

(f) Lightly draw in a tangent to your curve at the $t = 5$ s point. Then, measure the slope of this tangent by measuring the *rise* (in meters) and the *run* (in seconds). What is the slope in m/s?

(g) Were your answers to the last two parts the same? If so, why? If not, why not?

(h) Fill in the following table. This is going to be harder to do, because you are given x or v and are expected to find t. You may have to use the quadratic formula!

TABLE 4.3:

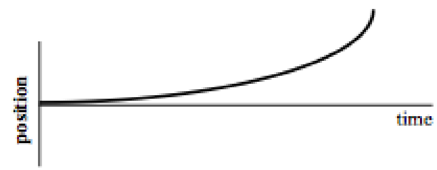
t	x	v
		2.6 m/s

TABLE 4.3: (continued)

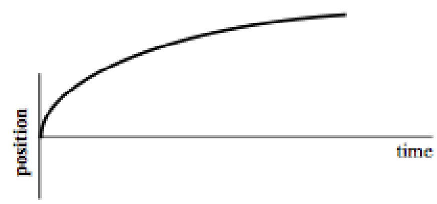
t	x	v
	17.1 m	
		3.14 m/s
	31.4 m	
		5.41 m/s

21. For each of the following graphs, write a few sentences about what kind of motions were made. Try to use the words we have defined in class (speed, velocity, position, acceleration) in your description.

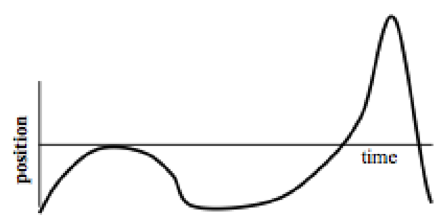
a.



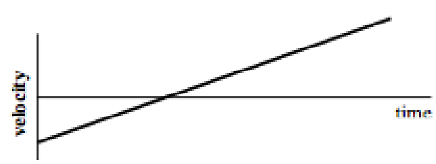
b.



c.



d.



Answers to Selected Problems

1. .
2. .

3. .
4. .
   1. Zyan
   2. Ashaan is accelerating because the distance he travels every 0.1 seconds is increasing, so the speed must be increasing
   3. Ashaan
   4. Zyan
   5. Ashaan
5. .
6. .
7. 6 minutes
8. (d) 20 meters (e) 40 meters (f) 2.67 m/s (g) 6 m/s (h) Between $t = 15$ s and $t = 20$ sec because your position goes from $x = 30$ m to $x = 20$m. (i) You made some sort of turn
   1. 7.7 m/s^2
   2. 47 m, 150 feet
   3. 34 m/s

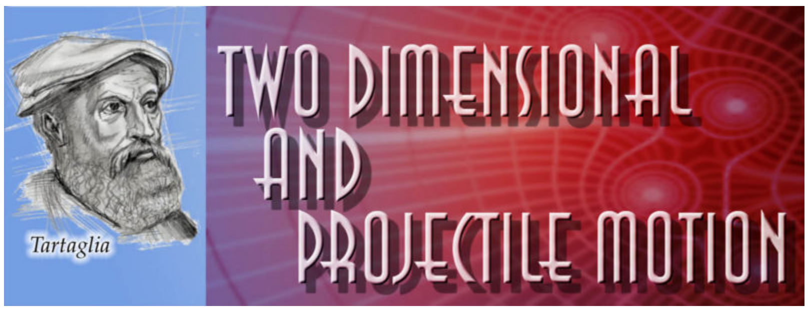
   1. 1.22 m
   2. 4.9 m/s
   3. 2.46 m/s
   4. -4.9 m/s
9. (b) 1 second (c) at 2 seconds (d) 4m
   1. 250 m
   2. 13 m/s, -13 m/s
   3. 14 s for round trip
10. Let's say we can jump 20 feet (6.1 m) in the air. ? Then, on the moon, we can jump 36.5 m straight up.
11. -31m/s^2
    1. 23 m/s
    2. 3.6 seconds
    3. 28 m
    4. 45m

    1. 25 m/s
    2. 30 m
    3. 2.5 m/s^2
12. 2 m/s^2
    1. $v_0 = 0$
    2. 10 m/s^2
    3. -10 m/s^2
    4. 60 m

    1. 0.3 m/s^2
    2. 0.5 m/s

Chapter 5 Two Dimensional and Projectile Motion Version 2

Chapter Outline



5.1 The Big Idea

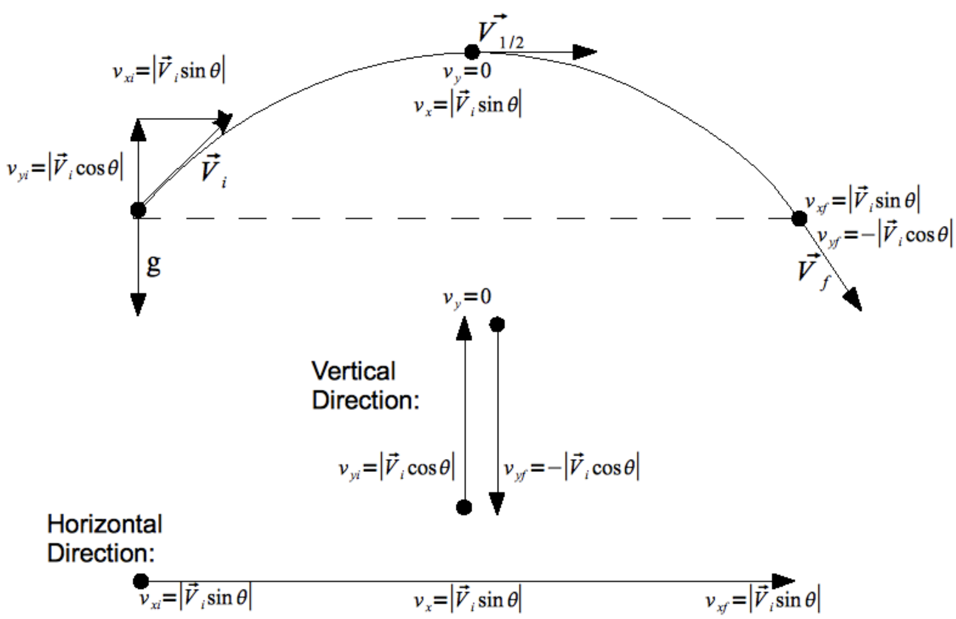
In this chapter, we explore the motion of projectiles under the influence of gravity — fired cannonballs, thrown basketballs, and other objects that have no way of propelling themselves and do not experience significant air resistance. From chapter 1, we know that vectors can be separated into components; if they are separated into perpendicular components the motion along each component can be treated independently (figure 1).

This is the insight that allows us to solve two dimensional projectile motion problems: we break any initial velocity vector into a component parallel to the ground and a component perpendicular to it. The force of gravity — which will be explained in more detail later — accelerates any object near the surface of the earth toward its center at a rate of $g = 9.8\text{m}/\text{s}^2$. This acceleration is in the direction perpendicular to the surface of the earth, conventionally labeled y.

Since in projectile motion under the sole influence of gravity any acceleration the object experiences is in the y direction, its horizontal, or x, velocity remains constant throughout its flight (at least in the absence of air resistance, which we ignore for the time being). To solve two dimensional motion problems, we apply the kinematics equations of one-dimensional motion to each of the two directions. In the y direction, we can use the uniform acceleration equations to solve for time in flight. Using this time, we can find how far the object traveled in the x direction also.

5.2 Solving Two Dimensional Motion Problems

Break the Initial Velocity into its Components



Apply the Kinematics Equations

Vertical Direction	**Horizontal Direction**
$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$	$x(t) = x_i + v_{ix}t$
$v_y(t) = v_{iy} - gt$	$v_x(t) = v_{ix}$
${v_y}^2 = {v_{0y}}^2 - 2g(\Delta y)$	
$a_y = -g = -9.8\text{m/s}^2 \approx -10\text{m/s}^2$	$a_x = 0$

5.3 Key Concepts

- In projectile motion, the horizontal displacement of an object from its starting point is called its *range.*
- Vertical (y) speed is zero only at the highest point of a thrown object's flight.
- To work these problems, separate the "Big Three" equations into two sets: one for the vertical direction, and one for the horizontal. Keep them separate.
- The only variable that can go into both sets of equations is time. You use time to communicate between the two directions.
- Since in the absence of air resistance there is no acceleration in the horizontal direction, this component of velocity does not change over time. This is a counter-intuitive notion for many. (Air resistance will cause velocity to decrease slightly or significantly depending on the object. But this factor is ignored for the time being.)
- Motion in the vertical direction must include the acceleration due to gravity, and therefore the velocity in the vertical direction changes over time.
- The shape of the path of an object undergoing projectile motion in two dimensions is a parabola.

5.4 Two Dimensional Example

Example 1

Question: A ball of mass m is moving horizontally with a speed of v_i off a cliff of height h. How much time does it take the ball to travel from the edge of the cliff to the ground? Express your answer in terms of g (acceleration due to gravity) and h (height of the cliff).

Solution: Since we are solving for the time, any motion in the x direction is not pertinent. We can just use the equation

$$f = \vec{v}$$

i + t
$\vec{v}$

f=_i + t

and solve for t. Notice though that v_{iy}, the ball's initial velocity in the y direction, is equal to zero when the ball rolls of the cliff. We can therefore disregard it; we have:

$$v_{\text{yf}} = a_y t$$

Therefore,

$$t = \frac{v_{\text{yf}}}{a}$$

Though we have solved for t, we have not solved for it in terms of the given quantities We can replace a with g because the only acceleration on the ball is due to gravity. We now need to replace v_{yf} with some combination of h and g. Using the equation

fv

^2=v_i^2+2ax,

we can solve for v

f^2 $inthetermswanted.Notethat$ x $heredenotesthedistancetraveledbytheobject,or$ h $.Itisn'tthehorizontal$ x $.Because$ v_yi^2=0 $,wecanonceagaindisregardit.Wenowreplace$ x $with$ h and a $with$ g $.ThisgivesusBecausewehavesolvedfor$ v_f^2 $wewillmaketheotherequation---theonesolvedfor$ t $---alsocontain$ v_f^2 $.Specifically,Nowwecansubstitute$ 2gh for v_f^2 $.Wecanthensolvefortheanswer:$

5.5 Two Dimensional Motion Problem Set

Draw detailed pictures for each problem (putting in all the data, such as initial velocity, time, etc.), and write down your questions when you get stuck.

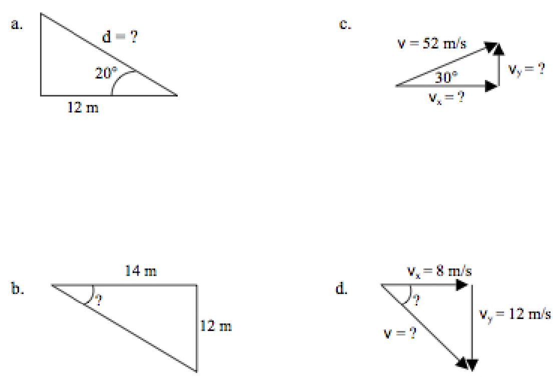
1. Determine which of the following is in projectile motion. Remember that "projectile motion" means that gravity is the only means of acceleration for the object.
   a. A jet airplane during takeoff.
   b. A baseball during a Barry Bonds home run.
   c. A spacecraft just after all the rockets turn off in Earth orbit.
   d. A basketball thrown towards a basket.
   e. A bullet shot out of a gun.
   f. An inter-continental ballistic missile.
   g. A package dropped out of an airplane as it ascends upward with constant speed.
2. Decide if each of the statements below is True or False. Then, explain your reasoning.
   a. At a projectile's highest point, its velocity is zero.
   b. At a projectile's highest point, its acceleration is zero.
   c. The rate of change of the x position is changing with time along the projectile path.
   d. The rate of change of the y position is changing with time along the projectile path.
   e. Suppose that after 2 s, an object has traveled 2 m in the horizontal direction. If the object is in projectile motion, it must travel 2 m in the vertical direction as well.
   f. Suppose a hunter fires his gun. Suppose as well that as the bullet flies out horizontally and undergoes projectile motion, the shell for the bullet falls directly downward. Then, the shell hits the ground before the bullet.
3. Imagine the path of a soccer ball in projectile motion. Which of the following is true at the highest point in its flight?
   a. $v_x = 0, v_y = 0, a_x = 0, a_y = 0$.
   b. $v_x > 0, v_y = 0, a_x = 0, a_y = 0$.
   c. $v_x = 0, v_y = 0, a_x = 0, a_y = -9.8\ \text{m/s}^2$.
   d. $v_x > 0, v_y = 0, a_x = 0, a_y = -9.8\ \text{m/s}^2$.
   e. $v_x = 0, v_y = 0, a_x = 0, a_y = -9.8\ \text{m/s}^2$.
4. A hunter with an air blaster gun is preparing to shoot at a monkey hanging from a tree. He is pointing his gun directly at the monkey. The monkey's got to think quickly! What is the monkey's best chance to avoid being smacked by the rubber ball?
   a. The monkey should stay right where he is: the bullet will pass beneath him due to gravity.
   b. The monkey should let go when the hunter fires. Since the gun is pointing right at him, he can avoid getting hit by falling to the ground.
   c. The monkey should stay right where he is: the bullet will sail above him since its vertical velocity increases by 9.8 m/s every second of flight.
   d. The monkey should let go when the hunter fires. He will fall faster than the bullet due to his greater mass, and it will fly over his head.
5. You are riding your bike in a straight line with a speed of 10 m/s. You accidentally drop your calculator out of your backpack from a height of 2.0 m above the ground. When it hits the ground, where is the calculator in relation to the position of your backpack? (Neglect air resistance.)
   a. You and your backpack are 6.3 m ahead of the calculator.

b. You and your backpack are directly above the calculator.
c. You and your backpack are 6.3 m behind the calculator.
d. None of the above.

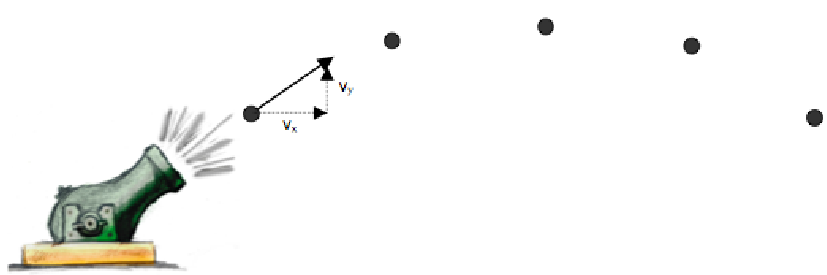
6. A ball of mass m is moving horizontally with speed v_0 off a cliff of height h, as shown. How much time does it take the rock to travel from the edge of the cliff to the ground?

a. $\sqrt{hv_0}$.
b. $\frac{h}{v_0}$.
c. $\frac{hv_0}{g}$.
d. $\frac{2h}{g}$.
e. $\sqrt{\frac{2h}{g}}$.

7. Find the missing legs or angles of the triangles shown.

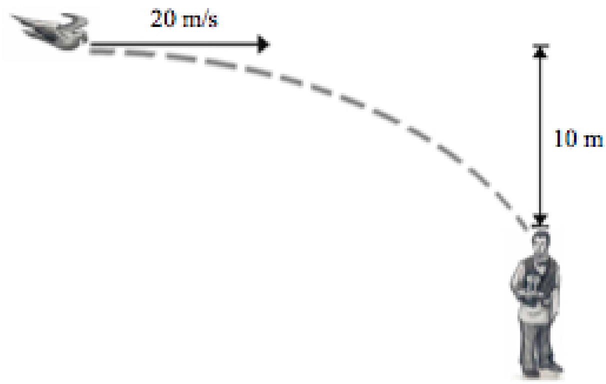


8. Draw in the $x-$ and $y-$velocity components for each dot along the path of the cannonball. The first one is done for you.



9. A stone is thrown horizontally at a speed of 8.0 m/s from the edge of a cliff 80 m in height. How far from the base of the cliff will the stone strike the ground?
10. A toy truck moves off the edge of a table that is 1.25 m high and lands 0.40 m from the base of the table.
    a. How much time passed between the moment the car left the table and the moment it hit the floor?
    b. What was the horizontal velocity of the car when it hit the ground?
11. A hawk in level flight 135 m above the ground drops the fish it caught. If the hawk's horizontal speed is 20.0 m/s, how far ahead of the drop point will the fish land?

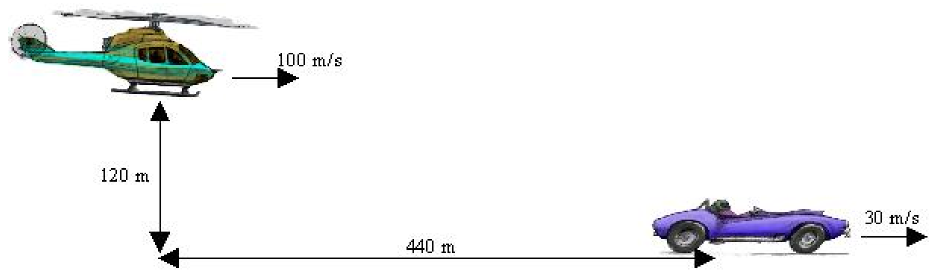
12. A pistol is fired horizontally toward a target 120 m away, but at the same height. The bullet's velocity is 200 m/s. How long does it take the bullet to get to the target? How far below the target does the bullet hit?
13. A bird, traveling at 20 m/s, wants to hit a waiter 10 m below with his dropping (see image). In order to hit the waiter, the bird must release his dropping some distance before he is directly overhead. What is this distance?



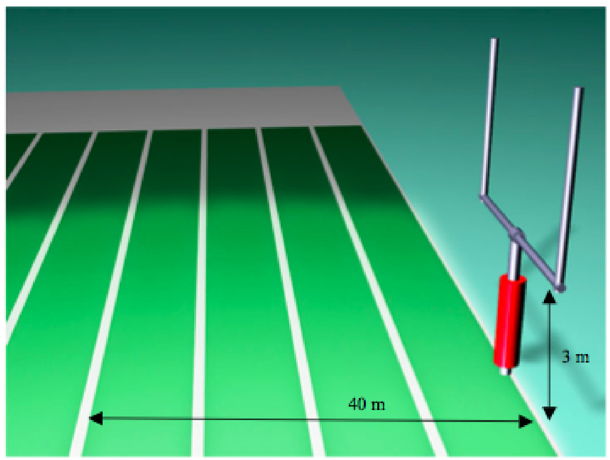
14. Joe Nedney of the *San Francisco 49ers* kicked a field goal with an initial velocity of 20 m/s at an angle of 60°.

    a. How long is the ball in the air? *Hint:* you may assume that the ball lands at same height as it starts at.
    b. What are the range and maximum height of the ball?

15. A racquetball thrown from the ground at an angle of 45° and with a speed of 22.5 m/s lands exactly 2.5 s later on the top of a nearby building. Calculate the horizontal distance it traveled and the height of the building.
16. Donovan McNabb throws a football. He throws it with an initial velocity of 30 m/s at an angle of 25°. How much time passes until the ball travels 35 m horizontally? What is the height of the ball after 0.5 seconds? (Assume that, when thrown, the ball is 2 m above the ground.)
17. Pablo Sandoval throws a baseball with a horizontal component of velocity of 25 m/s. After 2 seconds, the ball is 40 m above the release point. Calculate the horizontal distance it has traveled by this time, its initial vertical component of velocity, and its initial angle of projection. Also, is the ball on the way up or the way down at this moment in time?
18. Barry Bonds hits a 125 m(450′) home run that lands in the stands at an altitude 30 m above its starting altitude. Assuming that the ball left the bat at an angle of 45° from the horizontal, calculate how long the ball was in the air.
19. A golfer can drive a ball with an initial speed of 40.0 m/s. If the tee and the green are separated by 100 m, but are on the same level, at what angle should the ball be driven? (*Hint:* you should use $2\cos(x)\sin(x) = \sin(2x)$ at some point.)
20. How long will it take a bullet fired from a cliff at an initial velocity of 700 m/s, at an angle 30° below the horizontal, to reach the ground 200 m below?
21. A diver in Hawaii is jumping off a cliff 45 m high, but she notices that there is an outcropping of rocks 7 m out at the base. So, she must clear a horizontal distance of 7 m during the dive in order to survive. Assuming the diver jumps horizontally, what is his/her minimum push-off speed?
22. If Monte Ellis can jump 1.0 m high on Earth, how high can he jump on the moon assuming same initial velocity that he had on Earth (where gravity is 1/6 that of Earth's gravity)?
23. James Bond is trying to jump from a helicopter into a speeding Corvette to capture the bad guy. The car is going 30.0 m/s and the helicopter is flying completely horizontally at 100 m/s. The helicopter is 120 m above the car and 440 m behind the car. How long must James Bond wait to jump in order to safely make it into the car?



24. A field goal kicker lines up to kick a 44 yard (40 m) field goal. He kicks it with an initial velocity of 22 m/s at an angle of 55°. The field goal posts are 3 meters high.



a. Does he make the field goal?
b. What is the ball's velocity and direction of motion just as it reaches the field goal post (*i.e.,* after it has traveled 40 m in the horizontal direction)?

25. In a football game a punter kicks the ball a horizontal distance of 43 yards (39 m). On TV, they track the hang time, which reads 3.9 seconds. From this information, calculate the angle and speed at which the ball was kicked. *(Note for non-football watchers: the projectile starts and lands at the same height. It goes* 43 *yards horizontally in a time of* 3.9 *seconds)*

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. .
6. .
7. a. 13 m b. 41 degrees c. $v_y = 26$ m/s; $v_x = 45$ m/s d. 56 degrees, 14 m/s
8. .
9. 32 m
10. a. 0.5 s b. 0.8 m/s
11. 104 m
12. $t = 0.60$ s, 1.8 m below target
13. 28 m.

14. a. 3.5 s. b. 35 m; 15 m
15. 40 m; 8.5 m
16. 1.3 seconds, 7.1 meters
17. 50 m; $v_{0y} = 30$ m/s; 50^0; on the way up
18. 4.4 s
19. 19°
20. 0.5 s
21. 2.3 m/s
22. 6 m
23. 1.4 seconds
24. a. yes b. 14 m/s @ 23 degrees from horizontal
25. 22 m/s @ 62 degrees

CHAPTER 6 Newtons Laws Version 2

Chapter Outline

6.1 The Big Idea

In the previous chapters, we studied the behavior of accelerating objects in one and two dimensions. We did not, however, address the issue of *where* the acceleration comes from: in other words, why, in certain situations, do the velocities of objects change? It might make sense that a cart moves if I push it, but what about a dropped object: is it accelerating for a different reason, or for the same one? Is there something common to *all* accelerating objects?

Building on the insights of scientists before him, Isaac Newton created a mathematical analysis of moving and accelerating objects; the rules he discovered are now known as Newton's Laws of Motion. Newton is a legendary figure to physicists, and it's hard to underestimate his influence on the field. Actually, the substance of his Laws had been summarized by scientists before him. Still, the mathematical framework for their interpretation that Newton created was a revolutionary achievement, since it unified the existing knowledge of mechanics in a consistent system and cemented math as the accepted method of interpreting physical phenomena.

Here are Newton's Laws, in modern English:

Newton's First Law
: *Every body continues in its state of rest, or of uniform motion in a right (straight) line, unless it is compelled to change that state by forces impressed upon it.*

Newton's Second Law
: *The change of motion is proportional to the motive force impressed; and is made in the direction of the right (straight) line in which that force is impressed.*

Newton's Third Law
: *To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*

(Taken from the Principia in modern English, Isaac Newton, University of California Press, 1934).

6.2 Newton's Laws Explained

The **First Law is about inertia; objects at rest stay at rest unless acted upon and objects in motion continue that motion in a straight line unless acted upon**. Prior to Newton and Galileo, the prevailing view on motion was still Aristotle's. According to his theory the natural state of things is at rest; force is required to keep something moving at a constant rate. This made sense to people throughout history because on earth, friction and air resistance slow moving objects. When there is no air resistance (or other sources of friction), a situation approximated in space, Newton's first law is much more evident.

The "motion" Newton mentions in the Second Law is, in his language, the product of the mass and velocity of an object — we call this quantity momentum — so **the Second Law is actually the** famous equation**:**

$$\vec{F} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = m\vec{a} \qquad [1]$$

That is, the acceleration experienced by an object will be proportional to the applied force and inversely proportional to its mass. If there are multiple forces, they can be added as vectors and it is the *net* force that matters:

$$m\vec{a} = \vec{F}_{\text{net}} = \sum_i \vec{F}_i \qquad \text{Net force is the vector sum of all the forces}$$

$$ma_x = F_{\text{net},\,x} = \sum_i F_{ix} \qquad \text{Horizontal components add}$$

$$ma_y = F_{\text{net},\,y} = \sum_i F_{iy} \qquad \text{As do vertical ones}$$

When the net force on an object is zero, it is said to be in **translational equilibrium**:

$$\sum_i \vec{F}_i = 0 \qquad \text{Translational Equilibrium Condition [2]}$$

Finally, the **Third Law states that you can't push someone or something without being pushed back**. This law is somewhat confusing: if to each applied force there is an equal and opposite force, why does anything ever accelerate? The key is that the 'equal and opposite' forces act on different objects. If I push a cart, the cart is in turn pushing on me. However, I'm also pushing (and being pushed by) *the earth*, through my feet. Therefore, in the end, the cart and I move in the same direction and the earth moves opposite us. The cart-person system experienced a net force in one direction, while the earth experienced an equal and opposite force. According to Newton's *second* law, the acceleration objects experience due to applied forces is inversely proportional to their mass; clearly, the earth — with its gigantic mass — doesn't move very far compared to the cart and person.

Newton's Laws Example

Question: Tom and Mary are standing on identical skateboards. Tom and Mary push off of each other and travel in opposite directions.

a) If Tom (M) and Mary (m) have identical masses, who travels farther?

b) If Tom has a bigger mass than Mary, who goes farther?

c) If Tom and Mary have identical masses and Tom pushes twice as hard as Mary, who goes farther?

Solution

a) Neither. Both Tom and Mary will travel the same distance. The forced applied to each person is the same (Newton's Third Law). So

$$\cancel{M}a = \cancel{m}a$$

which cancels to

$$a = a$$

Therefore both people will travel the same distance because the acceleration controls how far someone will travel and Tom and Mary have equal acceleration.

b) Mary will go farther. Again, the same force is applied to both Mary and Tom so

$$Ma = ma$$

Since Tom has the larger mass, his acceleration must be smaller (acceleration and mass are inversely proportional). Finally, because Mary's acceleration is greater, she will travel farther.

c) Neither. Newton's Third Law states that for every action there is an equal and opposite reaction. Therefore if Tom pushes twice as hard as Mary, Mary will essentially be pushing back with the same strength. They will therefore travel the same distance.

6.3 What are Forces?

In other words, things tend to stay in the their current state of motion unless some "forces" are "impressed" on them. But where do such forces come from? *What* are they? **Force isn't a real *object*, but rather a *concept* used to describe actions**. We can think of it as the cause of any kind of "pushing" or "pulling" that an object experiences. As long as we can measure them consistently, forces can be treated like any other physical vector quantity. One way to state a major goal of physics is to find a method for consistently predicting the forces an object will experience under any circumstances, based on the circumstances. At this point, physicists have identified four basic forces that govern the universe:

- **The strong force**: The most powerful of the four forces, it holds nuclei together in atoms — but has a very short range. It has to overcome the massive electromagnetic repulsion between protons in a nucleus.
- **The electromagnetic force**: Responsible for the behavior of charged particles. Has infinite range.
- **The weak force**: Another nuclear force, responsible for much of the structure of stars and the universe in general. Its range is longer than that of the strong force, but still smaller than an atom.
- **Gravity**: Responsible for the attraction of all masses in the universe. Has infinite range.

All others — friction, air resistance, and other contact forces; buoyancy; the spring force — can be reduced to these fundamental forces. The fundamental forces are covered in more detail in later chapters.

Note that this classification does not tell us anything about *where* these forces come from, or how they are able to act seemingly at a distance, with "no strings attached". Newton himself said:

I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses.

In other words, we are interested in describing and predicting nature, rather than explaining its root causes. Forces acting a distance may seem strange in light of our experience with forces that *we* can apply to things (like pushing on rocks, etc), but to Newton asking about the nature of gravity was like asking about the nature of mass: it's just there, we can measure it, and that's it.

6.4 Common Forces

Universal Gravity

In previous chapters we learned that gravity — near the surface of planets, at least — is a force that accelerates objects at a constant rate. At this point we can extend this description using the framework of Newton's Laws.

Newton's Laws apply to all forces; but when he developed them only one was known: gravity. Newton's major insight — and one of the greatest in the history of science — was that the same force that causes objects to fall when released is also responsible for keeping the planets in orbit. According to some sources, he realized this while taking a stroll through some gardens and witnessing a falling apple.

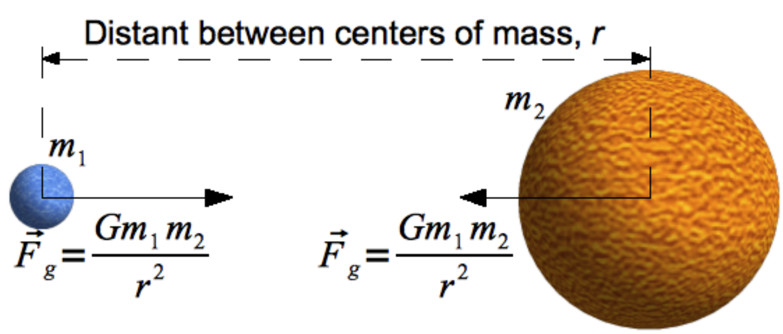
After considering the implications of this unification, Newton formulated the **Law of Universal Gravitation**: Any two objects in the universe, with masses m_1 and m_2 with their centers of mass at a distance r apart will experience a force of mutual attraction along the line joining their centers of mass equal to:

$$\vec{F}_G = \frac{Gm_1m_2}{r^2} \qquad \text{Universal Gravitation [3]},$$

where G is the Gravitational constant:

$$G = 6.67300 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

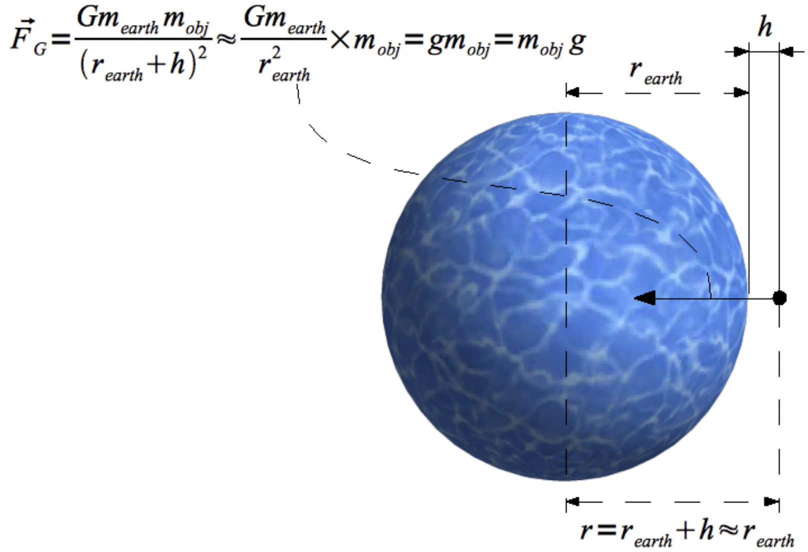
Here is an illustration of this law for two objects, for instance the earth and the sun:



Gravity on the Earth's Surface

In the chapter on energy, we saw that the gravitational potential energy formula for objects near earth, $U_g = mgh$, is a special case of a more general result. It so happens that the fact that gravity accelerates near earth objects at a constant rate is an almost identical result.

On the surface of a planet — such as earth — the r in formula [3] is very close to the radius of the planet, since a planet's center of mass is — usually — at its center. It also does not vary by much: for instance, the earth's radius is about 6,000 km, while the heights we consider for this book are on the order of at most a few kilometers — so we can say that for objects near the surface of the earth, the r in formula [3] is constant and equal to the earth's radius. This allows us to say that gravity is more or less constant on the surface of the earth. Here's an illustration:



For any object a height h above the surface of the earth, the force of gravity may be expressed as:

$$\vec{F}_G = \frac{Gm_{earth}m_{obj}}{(r_{earth}+h)^2} \text{ [4]}$$

Now we make the approximation that

$$r_{earth} + h \approx r_{earth}$$

then, we can rewrite [4] as

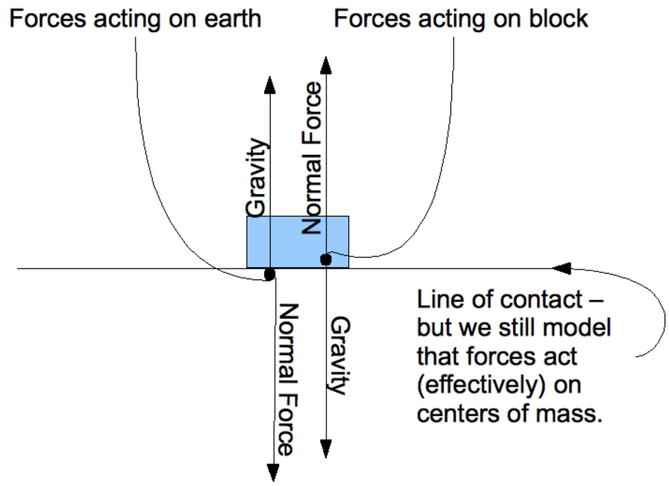
$$\vec{F}_G = \underbrace{\frac{Gm_{earth}}{r^2_{earth}}}_{\vec{g}_{earth}} \times m_{obj} = m_{obj} \times \vec{g} \qquad \text{[4] Gravity on Earth}$$

We can do this because the quantity in braces only has constants; we can combine them and call their product g. Remember, *this is an approximation that holds* ***only*** *when the r in formula [3] is more or less constant.*

We call the quantity mg an object's **weight**. Weight is different from mass — which is identical everywhere — since it depends on the gravitational force an object experiences. In fact, weight is the magnitude of that force. To find the weight of an object on another planet, star, or moon, use the appropriate values in formula [4].

Normal Force

Often, objects experience gravitational attraction but cannot move closer together because they are in contact. For instance, when you stand on the surface of the earth you are obviously not accelerating toward its center. According to Newton's Laws, there must be a force opposing gravity, so that the net force on both objects is zero. We call such a force the **Normal Force**. It is actually electromagnetic in nature (like other contact forces), and arises due to the repulsion of atoms in the two objects. Here is an illustration of the Normal force on a block sitting on earth:



Gravity and Normal Force Example

Question: A woman of mass 70.0 kg weighs herself in an elevator.

a) If she wants to weigh less, should she weigh herself when accelerating upward or downward? b) When the elevator is not accelerating, what does the scale read (i.e., what is the normal force that the scale exerts on the woman)? c) When the elevator is accelerating upward at 2.00 m/s^2, what does the scale read?

Answer a) If she wants to weigh less, she has to decrease her force (her weight is the force) on the scale. We will use the equation

$$F = ma$$

to determine in which situation she exerts less force on the scale.

If the elevator is accelerating upward then the acceleration would be greater. She would be pushed toward the floor of the elevator making her weight increase. Therefore, she should weigh herself when the elevator is going down.

b) When the elevator is not accelerating, the scale would read 70.0kg.

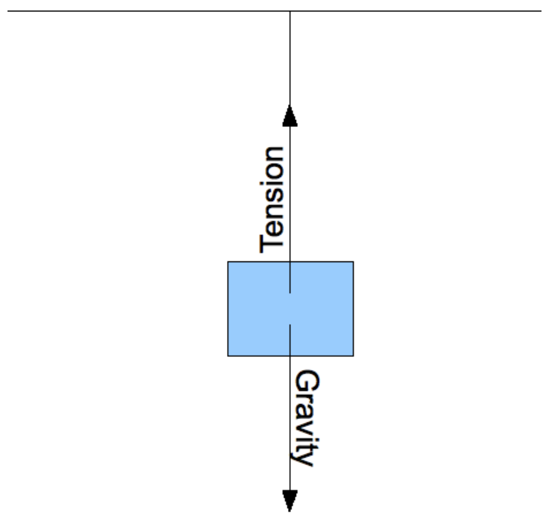
c) If the elevator was accelerating upward at a speed of 2.00m/s^2, then the scale would read

$$F = ma = 70\text{kg} \times (9.8\text{m/s}^2 + 2\text{m/s}^2) = 826\text{N}$$

which is 82.6kg.

Tension

Another force that often opposes gravity is known as **tension**. This force is provided by wires and strings when they hold objects above the earth. Like the Normal Force, it is electromagnetic in nature and arises due to the intermolecular bonds in the wire or string:



If the object is in equilibrium, tension must be equal in magnitude and opposite in direction to gravity. This force transfers the gravity acting on the object to whatever the wire or string is attached to; in the end it is usually a Normal Force — between the earth and whatever the wire is attached to — that ends up balancing out the force of gravity on the object.

Friction

Friction is a force that opposes motion. Any two objects in contact have what is called a mutual coefficient of friction. To find the force of friction between them, we multiply the normal force by this coefficient. Like the forces above, it arises due to electromagnetic interactions of atoms in two objects. There are actually two coefficients of friction: static and kinetic. Static friction will oppose *initial* motion of two objects relative to each other. Once the objects are moving, however, kinetic friction will oppose their continuing motion. Kinetic friction is lower than static friction, so it is easier to keep an object in motion than to set it in motion.

$$f_s \leq \mu_s |\vec{F}_N| \qquad [5] \text{ Static friction opposes potential motion of surfaces in contact}$$

$$f_k = \mu_k |\vec{F}_N| \qquad [6] \text{ Kinetic frictions opposes motion of surfaces in contact}$$

There are some things about friction that are not very intuitive:

- The magnitude of the friction force does not depend on the surface areas in contact.
- The magnitude of kinetic friction does not depend on the relative velocity or acceleration of the two objects.
- Friction always points in the direction opposing motion. If the net force (not counting friction) on an object is lower than the maximum possible value of static friction, friction will be equal to the net force in magnitude and opposite in direction.

Spring Force

Finally, the last force we will cover is that exerted by a stretched spring. Any spring has some equilibrium length, and if stretched in either direction it will push or pull with a force equal to:

$$\vec{F_{sp}} = -k\vec{\Delta x}$$ [7] Force of spring $\vec{\Delta x}$ from equilibrium

Spring Example

Question: A spring with a spring constant of $k = 400\text{N/m}$ has an uncompressed length of .23m and a fully compressed length of .15m. What is the force required to fully compress the spring?

Solution: We will use the equation

$$F = kx$$

to solve this. We simply have to plug in the known value for the spring and the distance to solve for the force.

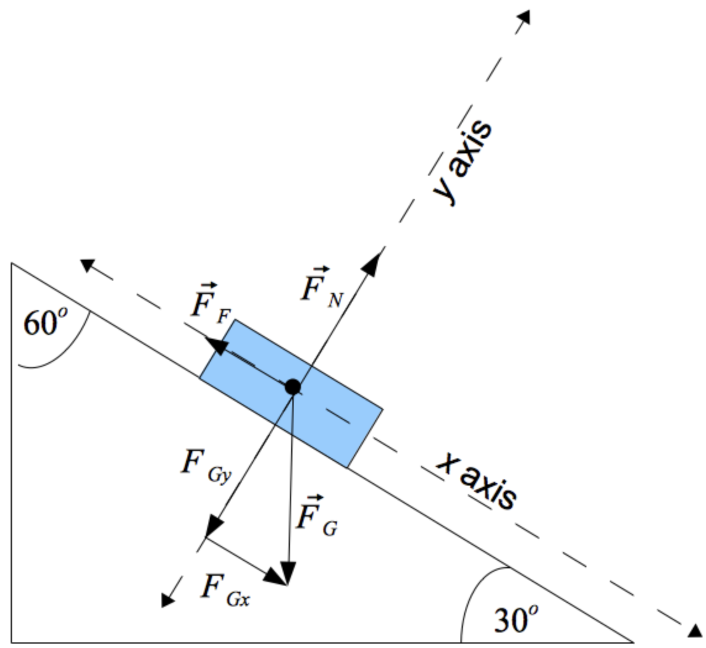
$$F = kx = (400\text{N/m})(.23\text{m} - .15\text{m}) = 32\text{N}$$

6.5 Short Summary

- An object will not change its state of motion (i.e., accelerate) unless an unbalanced force acts on it. Equal and oppositely directed forces on the same object do not produce acceleration.
- The force of gravity is called weight. Near the surface of a planet, it has magnitude mg and is directed perpendicular to its surface. This g is different from the Gravitational Constant, and differs from planet to planet.
- Your mass does not change when you move to other planets — although your weight does — because mass is a measure of how much *matter* your body contains, and not how much gravitational force you feel.
- To calculate the net force on an object, you need to calculate all the individual forces acting on the object and then add them as vectors.
- Newton's Third Law states for every force there is an equal but opposite reaction force. To distinguish a third law pair from merely oppositely directed pairs is difficult, but very important. Third law pairs must obey three rules: (1) Third law force pairs must be of the same type of force. (2) Third law force pairs are exerted on two different objects. (3) Third law force pairs are equal in magnitude and oppositely directed. *Example:* A block sits on a table. The Earth's gravity on the block and the force of the table on the block are equal and opposite. **But these are not third law pairs,** because they are both on the same object and the forces are of different types. The proper third law pairs are: (1) earth's gravity on block/block's gravity on earth and (2) table pushes on block/ block pushes on table.

6.6 Free-Body Diagram Example

Question: Using the diagram below, find the net force on the block. The block weighs 3kg and the inclined plane has a coefficient of friction of .6.



Answer:

The first step to solving a Newton's Laws problem is to identify the object in question. In our case, the block on the slope is the object of interest.

Next, we need to draw a free-body diagram. To do this, we need to identify all of the forces acting on the block and their direction. The forces are friction, which acts in the negative x direction, the normal force, which acts in the positive y direction, and gravity, which acts in a combination of the negative y direction and the positive x direction. Notice that we have rotated the picture so that the majority of the forces acting on the block are along the y or x axis. This does not change the answer to the problem because the direction of the forces is still the same relative to each other. When we have determined our answer, we can simply rotate it back to the original position.

Now we need to break down gravity (the only force not along one of the axises) into its component vectors (which do follow the axises).

$$\text{The x component of gravity} : 9.8\text{m/s}^2 \times cos60 = 4.9\text{m/s}^2$$
$$\text{The y component of gravity} : 9.8\text{m/s}^2 \times sin60 = 8.5\text{m/s}^2$$

Yet these are only the acceleration of gravity so we need to multiply them by the weight of the block to get the force.

$$F = ma = 3\text{kg} \times 4.9\text{m/s}^2 = 14.7\text{N}F = ma = 3\text{kg} \times 8.5\text{m/s}^2 = 25.5\text{N}$$

Now that we have solved for the force of the y-component of gravity we know the normal force (they are equal). Therefore the normal force is 25.5N. Now that we have the normal force and the coefficient of static friction, we can find the force of friction.

$$F_s = \mu_s F_N = .6 \times 25.5\text{N} = 15.3\text{N}$$

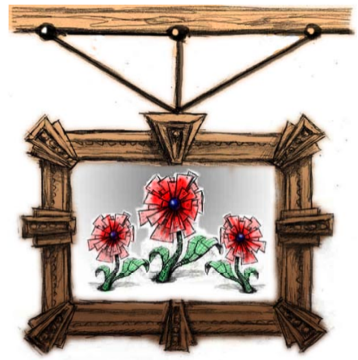
The force of static friction is greater than the component of gravity that is forcing the block down the inclined plane. Therefore the force of friction will match the force of the x-component of gravity. So the net force on the block is

$$\begin{aligned} \text{net force in the x} - \text{direction}: \quad & \overbrace{14.7\text{N}}^{x-component\ of\ gravity} - \overbrace{14.7\text{N}}^{force\ of\ friction} = 0\text{N} \\ \text{net force in the y} - \text{direction}: \quad & \underbrace{25.5\text{N}}_{Normal\ Force} - \underbrace{25.5\text{N}}_{y-component\ of\ gravity} = 0\text{N} \end{aligned}$$

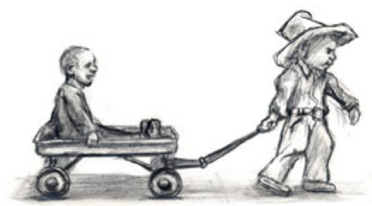
Therefore the net force on the block is 0N.

6.7 Newton's Laws Problem Set

1. A VW Bug hits a huge truck head-on. Each vehicle was initially going 50 MPH.
    a. Which vehicle experiences the greater force?
    b. Which experiences the greater acceleration?
2. Is it possible for me to wave my hand and keep the rest of my body perfectly still? Why or why not?
3. How does a rocket accelerate in space, where there is nothing to 'push off' against?
4. Is there a net force on a hammer when you hold it steady above the ground? If you let the hammer drop, what's the net force on the hammer while it is falling to the ground?
5. If an object is moving at constant velocity or at rest, what is the minimum number of forces acting on it (other than zero)?
6. If an object is accelerating, what is the minimum number of forces acting on it?
7. You are standing on a bathroom scale. Can you reduce your weight by pulling up on your shoes? (Try it.)
8. When pulling a paper towel from a paper towel roll, why is a quick jerk more effective than a slow pull?
9. You and your friend are standing on identical skateboards with an industrial-strength compressed spring in between you. After the spring is released, it falls straight to the ground and the two of you fly apart.
    a. If you have identical masses, who travels farther?
    b. If your friend has a bigger mass who goes farther?
    c. If your friend has a bigger mass who feels the larger force?
    d. If you guys have identical masses, even if you push on the spring, why isn't it possible to go further than your friend?
10. Explain the normal force in terms of the microscopic forces between molecules in a surface.
11. A stone with a mass of 10 kg is sitting on the ground, not moving.
    a. What is the weight of the stone?
    b. What is the normal force acting on the stone?
12. The stone from the last question is now being pulled horizontally along the ground at constant speed in the positive x direction. Is there a net force on the stone?
13. A spring with spring constant $k = 400\ \text{N/m}$ has an uncompressed length of 0.23 m. When fully compressed, it has a length of 0.15 m. What force is required to fully compress the spring?
14. Measuring velocity is hard: for instance, can you tell how fast you're going around the Sun right now? Measuring acceleration is comparatively easy — you can *feel* accelerations. Here's a clever way to determine your acceleration. As you accelerate your car on a flat stretch, you notice that the fuzzy dice hanging from your rearview mirror are no longer hanging straight up and down. In fact, they are making a 30° angle with respect to the vertical. What is your acceleration? (Hint: Draw a FBD. Consider both x and y equations.)
15. Draw free body diagrams (FBDs) for all of the following objects involved (in **bold**) and label all the forces appropriately. Make sure the lengths of the vectors in your FBDs are proportional to the strength of the force: smaller forces get shorter arrows!
    a. A **man** stands in an elevator that is accelerating upward at $2\ \text{m/s}^2$.
    b. A boy is dragging a **sled** at a constant speed. The boy is pulling the sled with a rope at a 30° angle.
    c. Your **foot** presses against the ground as you walk.
    d. The **picture** shown here is attached to the ceiling by three wires.

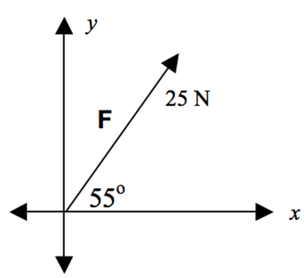


16. Analyze the situation shown here with a big kid pulling a little kid in a wagon. You'll notice that there are a lot of different forces acting on the system. Let's think about what happens the moment the sled begins to move.

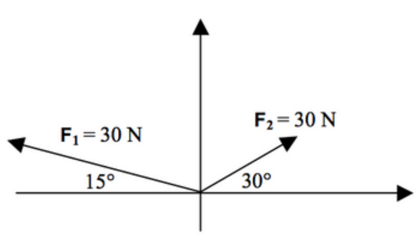


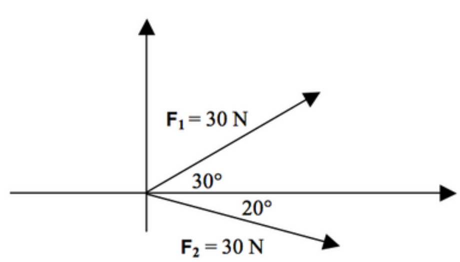
a. First, draw the free body diagram of the big kid. Include all the forces you can think of, including friction. Then do the same for the little kid.
b. Identify all third law pairs. Decide which forces act on the two body system and which are extraneous.
c. Explain what conditions would make it possible for the two-body system to move forward.

17. Break the force vector F on the right into its x and y components, F_x and F_y.

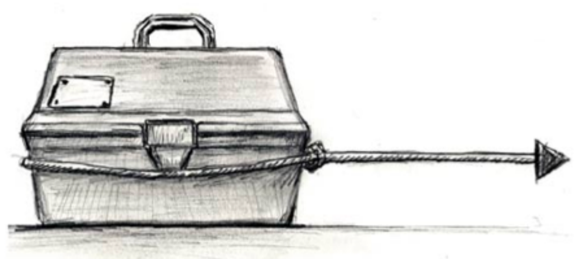


18. For both figures below, find the net force and its direction (*i.e.,* the magnitude of $F = F_1 + F_2$ and the angle it makes with the $x-$axis). Draw in F.

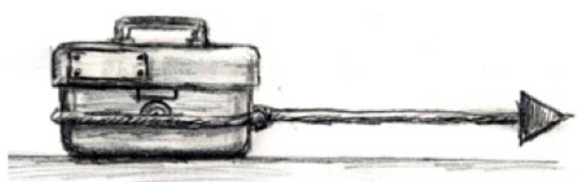




19. Andreas and Kaya are pulling a wagon. Andreas is pulling with a force of 50 N towards the northeast. Kaya is pulling with a force of 50 N towards the southeast. The wagon has a mass of 23 kg. What is the acceleration and direction of motion of the wagon?
20. Laura and Alan are pulling a wagon. Laura is pulling with a force of 50 N towards the northeast. Alan is pulling with a force of 50 N directly east. The wagon has a mass of 23 kg. What is the acceleration and direction of motion of the wagon?
21. When the 20 kg box shown below is pulled with a force of 100 N, it just starts to move (i.e., the maximum value of static friction is overcome with a force of 100 N). What is the value of the coefficient of static friction, μ_S?



22. A different box, this time 5 kg in mass, is being pulled with a force of 20 N and is sliding with an acceleration of 2 m/s^2. Find the coefficient of kinetic friction, μ_K.



23. The man is hanging from a rope wrapped around a pulley and attached to both of his shoulders. The pulley is fixed to the wall. The rope is designed to hold 500 N of weight; at higher tension, it will break. Let's say he has a mass of 80 kg. Draw a free body diagram and explain (using Newton's Laws) whether or not the rope will break

Problem 23

Problem 24

24. Now the man ties one end of the rope to the ground and is held up by the other. Does the rope break in this situation? What precisely is the difference between this problem and the one before?
25. For a boy who weighs 500 N on Earth what are his mass and weight on the moon (where $g = 1.6\ \text{m}/\text{s}^2$)?
26. A woman of mass 70.0 kg weighs herself in an elevator.

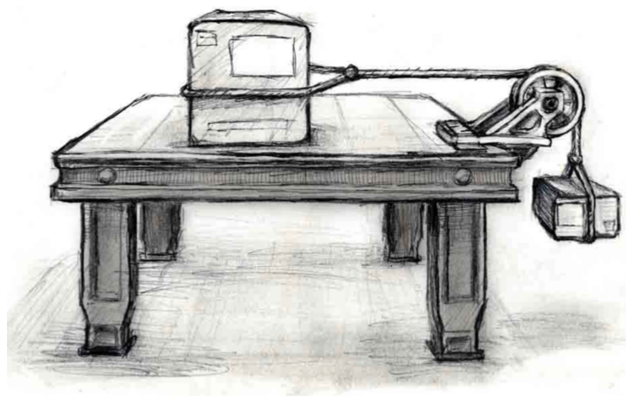
   a. If she wants to weigh less, should she weigh herself when accelerating upward or downward?

b. When the elevator is not accelerating, what does the scale read (i.e., what is the normal force that the scale exerts on the woman)?
c. When the elevator is accelerating upward at $2.00\ \mathrm{m/s^2}$, what does the scale read?

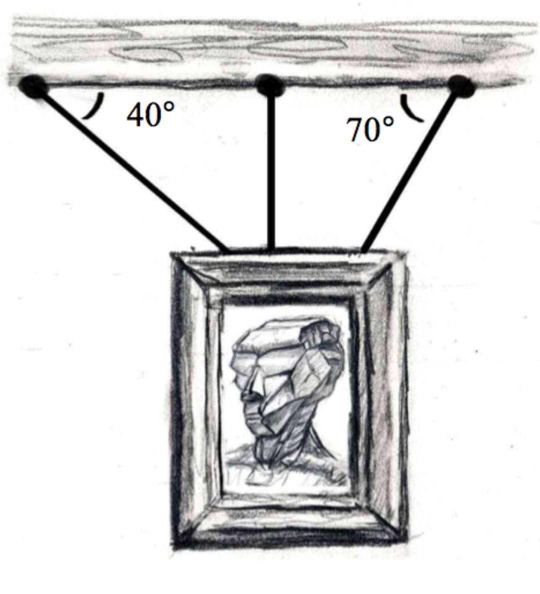
27. A crane is lowering a box of mass 50 kg with an acceleration of $2.0\ \mathrm{m/s^2}$.

a. Find the tension F_T in the cable.
b. If the crane lowers the box at a constant speed, what is the tension F_T in the cable?

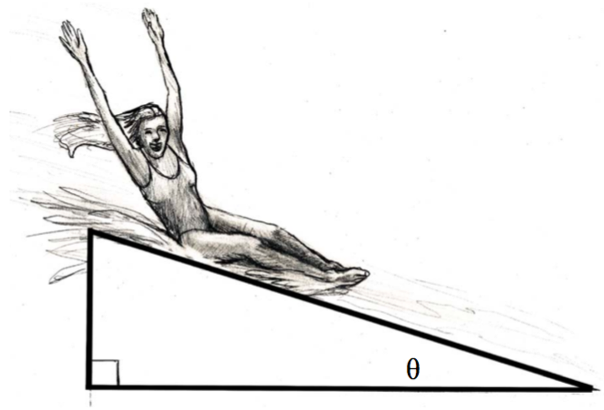
28. The large box on the table is 30 kg and is connected via a rope and pulley to a smaller 10 kg box, which is hanging. The 10 kg mass is the highest mass you can hang without moving the box on the table. Find the coefficient of static friction μ_S.



29. Find the mass of the painting. The tension in the leftmost rope is 7.2 N, in the middle rope it is 16 N, and in the rightmost rope it is 16 N.



30. Find Brittany's acceleration down the frictionless waterslide in terms of her mass m, the angle θ of the incline, and the acceleration of gravity g.



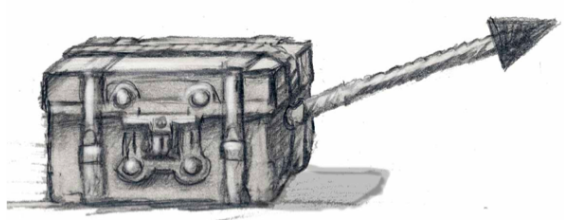
31. The physics professor holds an eraser up against a wall by pushing it directly against the wall with a completely horizontal force of 20 N. The eraser has a mass of 0.5 kg. The wall has coefficients of friction $\mu_S = 0.8$ and $\mu_K = 0.6$.

a. Draw a free body diagram for the eraser.
b. What is the normal force F_N acting on the eraser?
c. What is the frictional force F_S equal to?
d. What is the maximum mass m the eraser could have and still not fall down?
e. What would happen if the wall and eraser were both frictionless?

32. A tractor of mass 580 kg accelerates up a 10° incline from rest to a speed of 10 m/s in 4 s. For all of answers below, provide a magnitude and a direction.

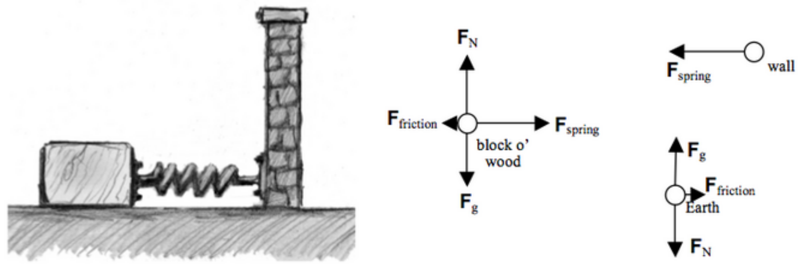
a. What net force F_{net} has been applied to the tractor?
b. What is the normal force, F_N on the tractor?
c. What is the force of gravity F_g on the tractor?
d. What force has been applied to the tractor so that it moves uphill?
e. What is the source of this force?

33. A heavy box (mass 25 kg) is dragged along the floor by a kid at a 30° angle to the horizontal with a force of 80 N (which is the maximum force the kid can apply).

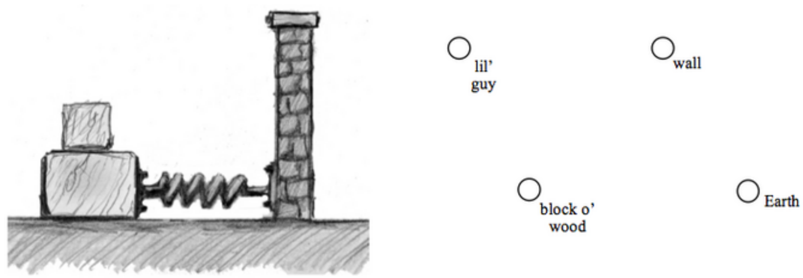


a. Draw the free body diagram.
b. What is the normal force F_N?
c. Does the normal force decrease or increase as the angle of pull increases? Explain.
d. Assuming no friction, what is the acceleration of the box?
e. Assuming it begins at rest, what is its speed after ten seconds?
f. Is it possible for the kid to lift the box by pulling straight up on the rope?
g. In the absence of friction, what is the net force in the $x-$direction if the kid pulls at a 30°○ angle?
h. In the absence of friction, what is the net force in the $x-$direction if the kid pulls at a 45° angle?
i. In the absence of friction, what is the net force in the $x-$direction if the kid pulls at a60° angle?
j. The kid pulls the box at constant velocity at an angle of 30°. What is the coefficient of kinetic friction μ_K between the box and the floor?
k. The kid pulls the box at an angle of 30°○, producing an acceleration of 2 m/s^2. What is the coefficient of kinetic friction μ_K between the box and the floor?

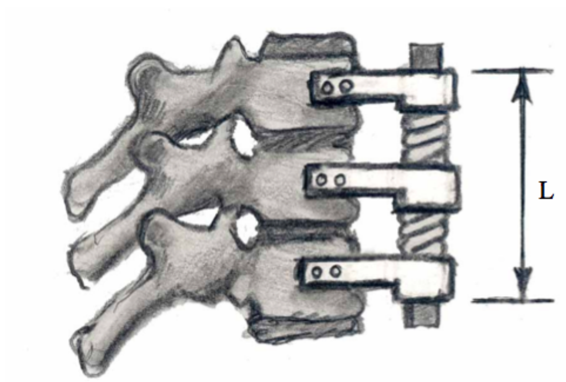
34. For the following situation, identify the 3^{rd} law force pairs on the associated free body diagrams. Label each member of one pair "A,″ each member of the next pair "B,″ and so on. The spring is stretched so that it is pulling the block of wood to the right.



Draw free body diagrams for the situation below. Notice that we are pulling the bottom block *out from beneath* the top block. There is friction between the blocks! After you have drawn your FBDs, identify the 3^{rd} law force pairs, as above.



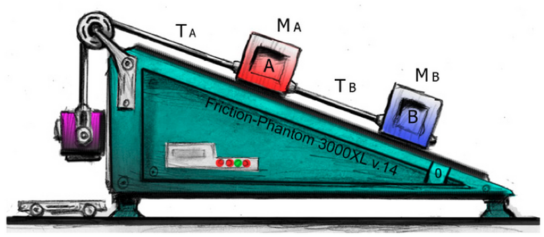
35. Spinal implant problem — this is a real life bio-med engineering problem!



Here's the situation: both springs are compressed by an amount x_o. The rod of length L is fixed to both the top plate and the bottom plate. The two springs, each with spring constant k, are wrapped around the rod on both sides of the middle plate, but are free to move because they are not attached to the rod or the plates. The middle plate has negligible mass, and is constrained in its motion by the compression forces of the top and bottom springs. The medical implementation of this device is to screw the top plate to one vertebrae and the middle plate to the vertebrae directly below. The bottom plate is suspended in space. Instead of fusing broken vertebrates together, this implant allows movement somewhat analogous to the natural movement of functioning vertebrae. Below you will do the exact calculations that an engineer did to get this device patented and available for use at hospitals.

a. Find the force, F, on the middle plate for the region of its movement $\triangle x \leq x_o$. Give your answer in terms of the constants given. (*Hint: In this region both springs are providing opposite compression forces*.)
b. Find the force, F, on the middle plate for the region of its movement $\triangle x \geq x_o$. Give your answer in terms of the constants given. (*Hint: In this region, only one spring is in contact with the middle plate.*)
c. Graph F vs. x. Label the values for force for the transition region in terms of the constants given.

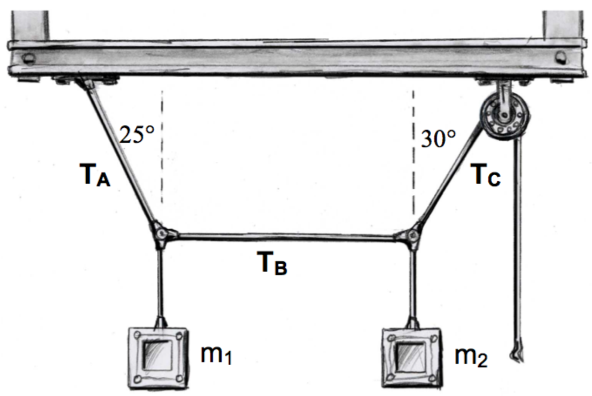
36. You design a mechanism for lifting boxes up an inclined plane by using a vertically hanging mass to pull them, as shown in the figure below.



The pulley at the top of the incline is massless and frictionless. The larger mass, M, is accelerating downward with a measured acceleration a. The smaller masses are m_A and m_B ; the angle of the incline is θ, and the coefficient of kinetic friction between each of the masses and the incline has been measured and determined to be μ_K.

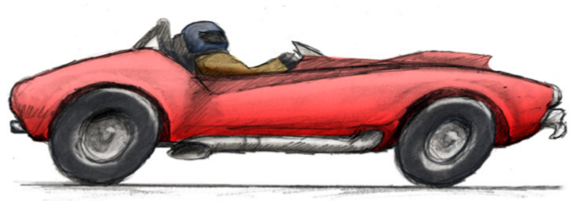
a. Draw free body diagrams for each of the three masses.
b. Calculate the magnitude of the frictional force on each of the smaller masses in terms of the given quantities.
c. Calculate the net force on the hanging mass in terms of the given quantities.
d. Calculate the magnitudes of the two tension forces T_A and T_B in terms of the given quantities.
e. Design and state a strategy for solving for how long it will take the larger mass to hit the ground, assuming at this moment it is at a height h above the ground. Do not attempt to solve this: simply state the strategy for solving it.

37. You build a device for lifting objects, as shown below. A rope is attached to the ceiling and two masses are allowed to hang from it. The end of the rope passes around a pulley (right) where you can pull it downward to lift the two objects upward. The angles of the ropes, measured with respect to the vertical, are shown. Assume the bodies are at rest initially.



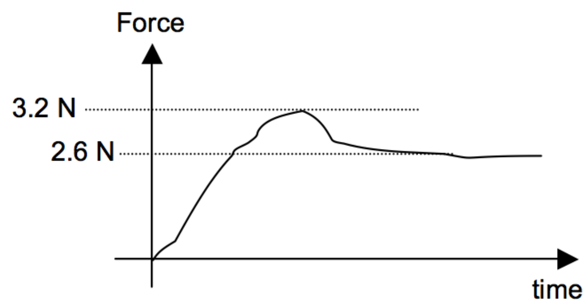
a. Suppose you are able to measure the masses m_1 and m_2 of the two hanging objects as well as the tension T_C. Do you then have enough information to determine the other two tensions, T_A and T_B? Explain your reasoning.
b. If you only knew the tensions T_A and T_C, would you have enough information to determine the masses m_1 and m_2? If so, write m_1 and m_2 in terms of T_A and T_C. If not, what further information would you require?

38. A stunt driver is approaching a cliff at very high speed. Sensors in his car have measured the acceleration and velocity of the car, as well as all forces acting on it, for various times. The driver's motion can be broken down into the following steps: Step 1: The driver, beginning at rest, accelerates his car on a horizontal road for ten seconds. Sensors show that there is a force in the direction of motion of 6000 N, but additional forces acting in the opposite direction with magnitude 1000 N. The mass of the car is 1250 kg. Step 2: Approaching the cliff, the driver takes his foot off of the gas pedal (There is no further force in the direction of motion.) and brakes, increasing the force opposing motion from 1000 N to 2500 N. This continues for five seconds until he reaches the cliff. Step 3: The driver flies off the cliff, which is 44.1 m high and begins projectile motion.



(a) Ignoring air resistance, how long is the stunt driver in the air?
(b) For Step 1:
   i. Draw a free body diagram, naming all the forces on the car.
   ii. Calculate the magnitude of the net force.
   iii. Find the change in velocity over the stated time period.
   iv. Make a graph of velocity in the $x-$direction vs. time over the stated time period.
   v. Calculate the distance the driver covered in the stated time period. Do this by finding the area under the curve in your graph of (iv). Then, check your result by using the equations for kinematics.
(c) Repeat (b) for Step 2.
(d) Calculate the distance that the stunt driver should land from the bottom of the cliff.

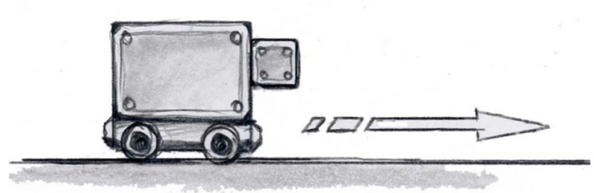
39. You are pulling open a stuck drawer, but since you're a physics geek you're pulling it open with an electronic device that measures force! You measure the following behavior. The drawer has a weight of 7 N.



Draw a graph of friction force vs. time.

40. Draw arrows representing the **forces** acting on the cannonball as it flies through the air. Assume that air resistance is small compared to gravity, but not negligible.

41. A tug of war erupts between you and your sweetie. Assume your mass is 60 kg and the coefficient of friction between your feet and the ground is 0.5 (good shoes). Your sweetie's mass is 85 kg and the coefficient of friction between his/her feet and the ground is 0.35 (socks). Who is going to win? Explain, making use of a calculation.
42. A block has a little block hanging out to its side, as shown:



As you know, if the situation is left like this, the little block will just fall. But if we accelerate the leftmost block to the right, this will create a normal force between the little block and the big block, and if there is a coefficient of friction between them, then the little block won't slide down! Clever, eh?

a. The mass of the little block is 0.15 kg. What frictional force is required to keep it from falling? (State a magnitude and direction.)
b. If both blocks are accelerating to the right with an acceleration $a = 14.0\ \mathrm{m/s^2}$, what is the normal force on the little block provided by the big block?
c. What is the minimum coefficient of static friction required?

Answers to Selected Problems

1. .
2. .
3. .
4. Zero; weight of the hammer minus the air resistance.
5. 2 forces
6. 1 force
7. No
8. The towel's inertia resists the acceleration
9. a. Same distance b. You go farther c. Same amount of force
10. .
11. a. 98 N b. 98 N
12. .
13. 32 N
14. $5.7\ \mathrm{m/s^2}$
15. .
16. .
17. $F_x = 14\ \mathrm{N}, F_y = 20\ \mathrm{N}$
18. Left picture: $F = 23\mathrm{N}\ 98^\circ$, right picture:$F = 54\ \mathrm{N}\ 5^\circ$
19. $3\ \mathrm{m/s^2}$ east
20. $4\ \mathrm{m/s^2}; 22.5^\circ$ NE
21. 0.51
22. 0.2
23. The rope will not break because his weight of 784 N is distributed between the two ropes.
24. Yes, because his weight of 784 N is greater than what the rope can hold.
25. Mass is 51 kg and weight is 82 N
26. a. While accelerating down b. 686 N c. 826 N
27. a. 390 N b. 490 N
28. 0.33
29. 3.6 kg
30. $g \sin\theta$
31. b.20 N c. 4.9 N d. 1.63 kg e. Eraser would slip down the wall
32. a. 1450 N b. 5600 N c. 5700 N d. Friction between the tires and the ground e. Fuel, engine, or equal and opposite reaction
33. b. 210 N c. no, the box is flat so the normal force doesn't change d. $2.8\ \mathrm{m/s^2}$ e.28 m/s f. no g. 69 N h. 57 N i. 40 N j. 0.33 k. 0.09
34. .
35. a. zero b. $-kx0$
36. b. $f_1 = \mu_k m_1 g\cos\theta; f_2 = \mu_k m_2 g\cos\theta$ c. Ma d. $T_A = (m_1 + m_2)(a + \mu\cos\theta)$ and $T_B = m_2 a + \mu m_2\cos\theta$ e. Solve

by using $d = 1/2at^2$ and substituting h for d

37. a. Yes, because it is static and you know the angle and m_1 b. Yes, T_A and the angle gives you m_1 and the angle and T_C gives you $m_2, m_1 = T_A \cos 25/g$ and $m_2 = T_C \cos 30/g$
38. a. 3 seconds d. 90 m
39. .
40. .
41. .
42. a. 1.5 N; 2.1 N; 0.71

CHAPTER 7 Centripetal Forces Version 2

Chapter Outline

7.1 Forces so Far

In the absence of a net applied force, moving objects travel in a straight lines; this is Newton's First Law. If their velocity changes, *even only in direction*, there must be an applied force. When something experiences a net applied force, there are several possibilities:

1. The force is constant in magnitude and direction and points along the line of motion, or the object is at rest. In this case, the object will accelerate or decelerate in the direction of the force. The object's position and velocity can be found using the so called 'big three' equations.
2. The force is constant in magnitude and direction and acting at some non-right angle to the direction of motion. In this case, the object's velocity vector can be broken down into perpendicular components in such a way that on is parallel to the force. Then the problem is reduced to two one dimensional problems — along the force and perpendicular to it — which can be solved according to case 1 above. An example of this is parabolic motion under the influence of gravity.
3. The force is constant in magnitude, the object is in motion, and the force is *always perpendicular to the velocity vector*. In this case, the object will move in a circle. This is a kind of 'opposite' of case 1: then, the object's speed changed — whether increased or decreased — by the largest possible amount for a given force, while now the object's speed *does not change at all*.
4. The force is not constant
    a. The force is due to a compressed or extended spring. We will cover this later.
    b. All other cases: beyond the scope of this book.

7.2 Centripetal Forces

Summary

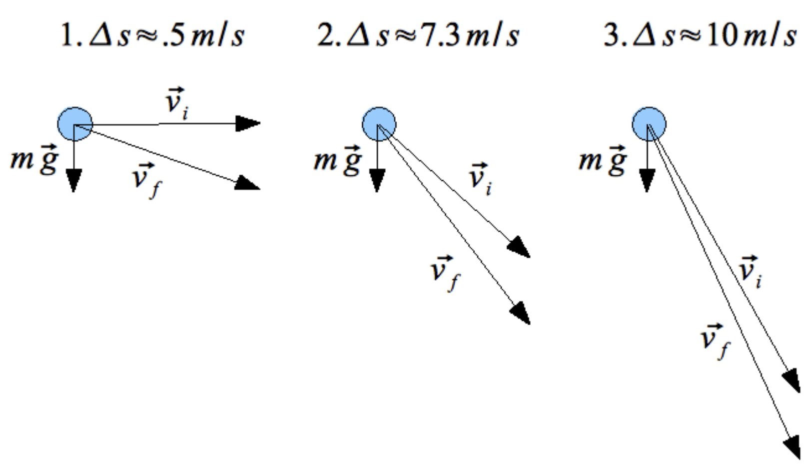
Forces that cause objects to follow circular paths — case 3 above — are known as **centripetal**, or 'center seeking', forces. Such forces must *continuously change direction* to stay perpendicular to the velocity vector. We saw in the chapters on vectors and kinematics that vectors cannot impact motion in directions perpendicular to them. This is why the horizontal velocity of projectiles on earth does not change (as in two dimensional motion).

Speed vs. Direction

Think of a ball rolling horizontally off a cliff. At first, its velocity is perpendicular to the force of gravity. As it falls, its velocity in the x direction stays constant, but it accelerates downward due to gravity. This ball will not travel in circle, though, because gravity is *only* perpendicular to its velocity at the instant it leaves the cliff. Eventually, the ball's velocity components will be equal. After some time, if the cliff is tall enough, the ball's vertical velocity will dwarf its horizontal velocity.

Let's now compare how gravity affects the ball's *speed* at different 1 second intervals during its flight.

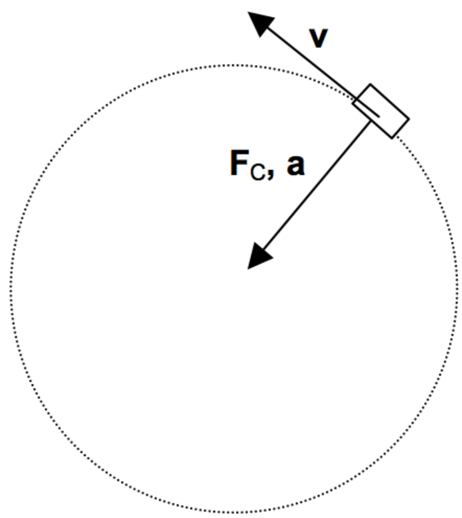
1. Let's say this ball initially had a horizontal velocity — and therefore also speed — of 100 m/s, and a vertical speed of 0. After the first second, its vertical velocity will about 10 m/s (assume $g = 10\text{m/s}$. Using the pythagorean theorem, we find that the speed is now $\sqrt{100^2 + 10^2} \approx 100.5\text{m/s}$, a change of less than .5 m/s.
2. Now consider the ball after 10 seconds, when its velocity components are equal. Between the 10th and 11th second, its speed goes from $\sqrt{100^2 + 100^2} \approx 141.4\text{m/s}$ to $\sqrt{100^2 + (100 + 10)^2} \approx 148.7\text{m/s}$, a much bigger increase in the same time.
3. Finally, for the sake of argument, let's say the cliff is mount Everest and the ball keeps falling for 100 seconds. Now, in one second, its speed goes from $\sqrt{100^2 + (1000)^2} \approx 1005\text{m/s}$ to $\sqrt{100^2 + (1000 + 10)^2} \approx 1014.9\text{m/s}$.



In other words, *it is much easier for a force to change the speed of an object when it points along its velocity vector*. Forces are capable of changing an object's velocity — speed and direction: the more parallel to motion the force, the more it changes speed; the more perpendicular, direction (if we had analyzed the effect on the angle of the velocity vector above instead of speed, the results would have been reversed).

Circular Motion

Knowing this, we can understand why, when force is *always* perpendicular to the velocity vector, an object's speed *never* changes, while its direction changes continuously. If the force is constant and magnitude, the direction of the object's velocity must change at a constant rate — otherwise the situation would be asymmetrical. In other words, the object will travel in a circle, with instantaneous velocity tangent to it and instantaneous force pointing toward the center. At any given time, the relationship between force, acceleration, and velocity is illustrated here:



Circular motion is kind of a limiting case of the 'first second' scenario above — if the force had always been perpendicular to the ball's velocity, it wouldn't have accelerated downward for an entire second. No matter how weak a centripetal force, it will in principle always cause a moving object to travel in a circle. This may seem counterintuitive, but is actually a direct result of the arguments above.

7.3 Characterizing The Force and Motion

If a mass m is traveling with velocity $\vec{v}$ and experiences a centripetal —always perpendicular — force $\vec{F}_c$, it will travel in a circle of radius

$$r = \frac{mv^2}{|\vec{F}|} \text{ [1]}$$

Alternatively, to keep this mass moving at this velocity in a circle of this radius, one needs to apply a centripetal force of

$$\vec{F}_c = \frac{mv^2}{r} \text{ [2]}$$

By Newton's Second Law, this is equivalent to a centripetal acceleration of:

$$\vec{F}_c = \cancel{m}\vec{a}_c = \cancel{m}\frac{v^2}{r} \text{ [3]}$$

7.4 Gravity as a Centripetal Force

When can Gravity Act as a Centripetal Force?

We saw last chapter that the force of Gravity causes an attraction between two objects of mass m_1 and m_2 at a distance r of

$$\vec{F}_G = \frac{Gm_1m_2}{r^2}. \text{ [4]}$$

By Newton's Third Law, both objects experience the force: equal in magnitude and opposite in direction, and both will move as an effect of it. If one of the objects is much lighter than the other (like the earth is to the sun, or a satellite is to earth) we can approximate the situation by saying that the heavier mass (the sun) does not move, since its acceleration will be far smaller due to its large mass. Then, if the lighter mass remains at a relatively constant absolute distance from the heavier one (remember, centripetal force needs to be constant in magnitude), we can say that the lighter mass experiences an *effectively* centripetal force.

Math of Centripetal Gravity

Gravity is not always a centripetal force. This is a really important point. It only acts as a centripetal force when conditions approximate those listed above — very much like it isn't constant near the surface of the earth, but very close to it.

If gravity provides centripetal force and acceleration, we can set [2] equal to [4]. It's important to remember that in [2] m refers to the lighter mass, since that is the one traveling. Then,

$$\frac{G\cancel{m_1}m_2}{\cancel{r^2}_{\,r}} = \frac{\cancel{m_1}v^2}{\cancel{r}}$$

So, the relationship between velocity and radius for a circular orbit of a light object around an heavy mass (note the mass of the lighter object cancels) is:

$$Gm_2 = {v_{\text{orb}}}^2 r_{\text{orb}}$$

7.5 Key Concepts

- For something in orbit, an orbital period, T, is the time it takes to make one complete rotation.
- If a particle travels a full circle or orbit — a distance of $2\pi r$ — in an amount of time T, then its speed is distance over time or $\frac{2\pi r}{T}$.
- An object moving in a circle has an instantaneous velocity vector *tangential* to the circle of its path. The force and acceleration vectors point to the center of the circle.
- Net force and acceleration *always* have the same direction.
- Centripetal acceleration is just the acceleration provided by centripetal forces.
- A **geosynchronous** orbit occurs when a satellite completes one orbit of the Earth every 24 hours. Since it revolves at the same rate as the earth, it will stay above the same location.

7.6 Key Applications

- To find the maximum speed that a car can take a corner on a flat road without skidding out, set the force of friction equal to the centripetal force.
- To find the tension in the rope of a swinging pendulum, remember that it is the *sum* of the tension and gravity that produces a net upward centripetal force. A common mistake is just setting the centripetal force equal to the tension.
- To find the speed of a planet or satellite in an orbit, follow the example above.

7.7 Examples

Example 1

Question: You buy new tires for your car in order to take turns a little faster (uh, not advised—always drive slowly). The new tires double your coefficient of friction with the road. With the old tires you could take a particular turn at a speed of v_i. What is the maximum speed you can now take the turn without skidding out.

Solution: To find the maximum speed a car can take a corner on a flat road without skidding out, set the force of friction equal to the centripetal force. This is because the centripetal force pushes the car off the road and the frictional force keeps the car on the road. Therefore, if the centripetal force and the frictional force were equal, the car would be going the maximum speed it could go on that turn without sliding off the road.

$$F_c = \frac{m{v_i}^2}{r} = F_k$$

Then we solve for v.

$$\frac{m{v_i}^2}{r} = F_k \Rightarrow {v_i}^2 = \frac{F_k r}{m} \Rightarrow v_i = \sqrt{\frac{F_k r}{m}}$$

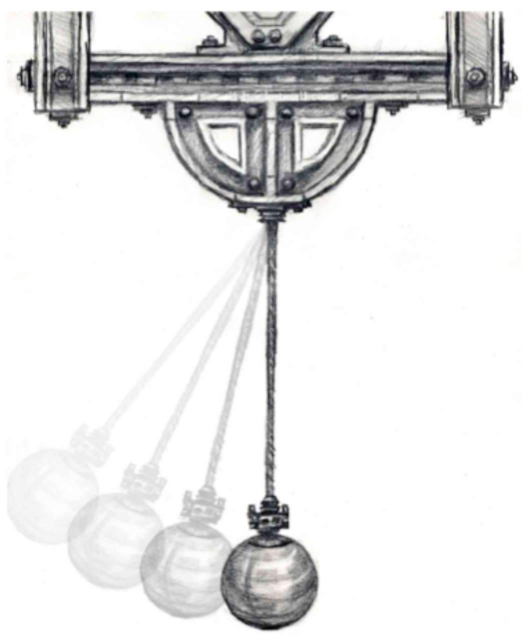
If we replace the frictional force with twice the frictional force we get the new speed.

$$v_f = \sqrt{\frac{2F_k r}{m}}$$

The new speed is $\sqrt{2}$ greater than the original speed.

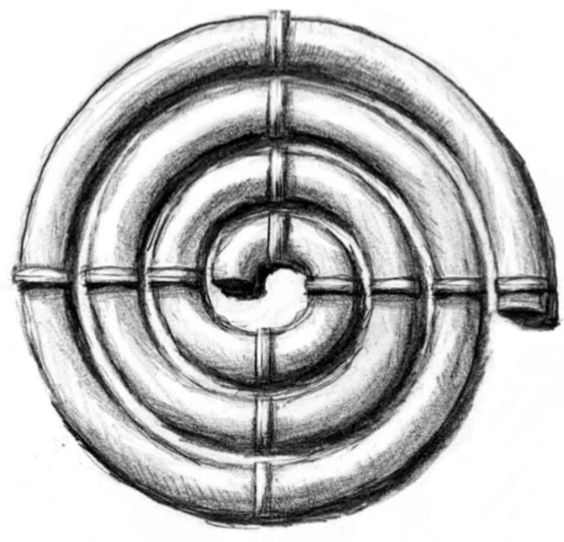
7.8 Centripetal Forces Problem Set

1. When you make a right turn at constant speed in your car what is the force that causes *you* (not the car) to change the direction of *your* velocity? Choose the best possible answer.
   a. Friction between your butt and the seat
   b. Inertia
   c. Air resistance
   d. Tension
   e. All of the above
   f. None of the above
2. You buy new tires for your car in order to take turns a little faster (uh, not advised — always drive slowly). The new tires double your coefficient of friction with the road. With the old tires you could take a particular turn at a speed v_o. What is the maximum speed you can now take the turn without skidding out?
   a. $4v_o$
   b. $2v_o$
   c. v_o
   d. $\sqrt{2}v_o$
   e. Not enough information given



3. A pendulum consisting of a rope with a ball attached at the end is swinging back and forth. As it swings downward to the right the ball is released at its lowest point. Decide which way the ball attached at the end of the string will go at the moment it is released.
   a. Straight upwards
   b. Straight downwards

c. Directly right
d. Directly left
e. It will stop

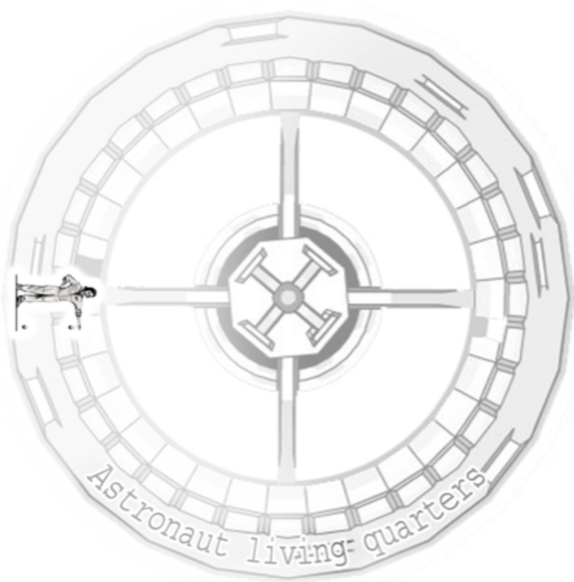


4. A ball is spiraling outward in the tube shown to the above. Which way will the ball go after it leaves the tube?
   a. Towards the top of the page
   b. Towards the bottom of the page
   c. Continue spiraling outward in the clockwise direction
   d. Continue in a circle with the radius equal to that of the spiral as it leaves the tube
   e. None of the above
5. An object of mass 10 kg is in a circular orbit of radius 10 m at a velocity of 10 m/s.
   a. Calculate the centripetal force (in *N*) required to maintain this orbit.
   b. What is the acceleration of this object?
6. Suppose you are spinning a child around in a circle by her arms. The radius of her orbit around you is 1 meter. Her speed is 1 m/s. Her mass is 25 kg.
   a. What is the tension in your arms?
   b. In her arms?
7. A racecar is traveling at a speed of 80.0 m/s on a circular racetrack of radius 450 m.
   a. What is its centripetal acceleration in $\mathrm{m/s^2}$?
   b. What is the centripetal force on the racecar if its mass is 500 kg?
   c. What provides the necessary centripetal force in this case?
8. The radius of the Earth is 6380 km. Calculate the velocity of a person standing at the equator due to the Earth's 24 hour rotation. Calculate the centripetal acceleration of this person and express it as a fraction of the acceleration g due to gravity. Is there any danger of "flying off"?
9. Neutron stars are the corpses of stars left over after supernova explosions. They are the size of a small city, but can spin several times per second. (Try to imagine this in your head.) Consider a neutron star of radius 10 km that spins with a period of 0.8 seconds. Imagine a person is standing at the equator of this neutron star.
   a. Calculate the centripetal acceleration of this person and express it as a multiple of the acceleration *g* due to gravity (on Earth).
   b. Now, find the minimum acceleration due to gravity that the neutron star must have in order to keep the person from flying off.

10. Calculate the force of gravity between the Sun and the Earth. (The relevant data are included in Appendix *B*.)
11. Calculate the force of gravity between two human beings, assuming that each has a mass of 80 kg and that they are standing 1 m apart. Is this a large force?
12. Prove g is *approximately* 10 m/s^2 on Earth by following these steps:
    a. Calculate the force of gravity between a falling object (for example an apple) and that of Earth. Use the symbol m_o to represent the mass of the falling object.
    b. Now divide that force by the object's mass to find the acceleration g of the object.
13. Our Milky Way galaxy is orbited by a few hundred "globular" clusters of stars, some of the most ancient objects in the universe. Globular cluster M13 is orbiting at a distance of 26,000 light-years (one light-year is 9.46×10^{15} m) and has an orbital period of 220 million years. The mass of the cluster is 10^6 times the mass of the Sun.
    a. What is the amount of centripetal force required to keep this cluster in orbit?
    b. What is the source of this force?# Based on this information, what is the mass of our galaxy? If you assume that the galaxy contains nothing, but Solar-mass stars (each with an approximate mass of 2×10^{30} kg), how many stars are in our galaxy?
14. Calculate the centripetal acceleration of the Earth around the Sun.
15. You are speeding around a turn of radius 30.0 m at a constant speed of 15.0 m/s.

    a. What is the minimum coefficient of friction μ between your car tires and the road necessary for you to retain control?
    b. Even if the road is terribly icy, you will still move in a circle because you are slamming into the walls. What centripetal forces must the walls exert on you if you do not lose speed? Assume $m = 650$ kg.
16. Calculate the gravitational force that your pencil or pen pulls on you. Use the center of your chest as the center of mass (and thus the mark for the distance measurement) and estimate all masses and distances.
    a. If there were no other forces present, what would your acceleration be towards your pencil? Is this a large or small acceleration?
    b. Why, in fact, doesn't your pencil accelerate towards you?
17. A digital TV satellite is placed in geosynchronous orbit around Earth, so it is always in the same spot in the sky.
    a. Using the fact that the satellite will have the same period of revolution as Earth, calculate the radius of its orbit.
    b. What is the ratio of the radius of this orbit to the radius of the Earth?
    c. Draw a sketch, to scale, of the Earth and the orbit of this digital TV satellite.
    d. If the mass of the satellite were to double, would the radius of the satellite's orbit be larger, smaller, or the same? Why?
18. A top secret spy satellite is designed to orbit the Earth twice each day (*i.e.,* twice as fast as the Earth's rotation). What is the height of this orbit above the Earth's surface?

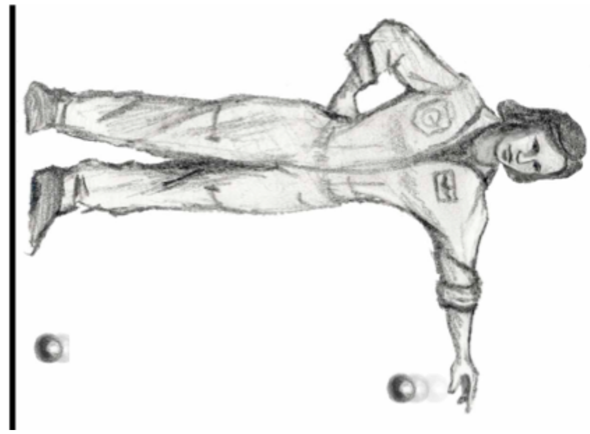
19. Two stars with masses 3.00×10^{31} kg and 7.00×10^{30} kg are orbiting each other under the influence of each other's gravity. We want to send a satellite in between them to study their behavior. However, the satellite needs to be at a point where the gravitational forces from the two stars are equal. The distance between the two stars is 2.0×10^{10} m. Find the distance from the more massive star to where the satellite should be placed. (*Hint:* Distance from the satellite to one of the stars is the variable.)
20. Calculate the mass of the Earth using *only: (i)* Newton's Universal Law of Gravity; *(ii)* the Moon-Earth distance (Appendix *B*); and *(iii)* the fact that it takes the Moon 27 days to orbit the Earth.
21. A student comes up to you and says, "I can visualize the force of tension, the force of friction, and the other forces, but I can't visualize *centripetal* force." As you know, a centripetal force must be *provided* by tension, friction, or some other "familiar" force. Write a two or three sentence explanation, in your own words, to help the confused student.



22. A space station was established far from the gravitational field of Earth. Extended stays in zero gravity are not healthy for human beings. Thus, for the comfort of the astronauts, the station is rotated so that the astronauts *feel* there is an internal gravity. The rotation speed is such that the *apparent* acceleration of gravity is 9.8 m/s^2. The direction of rotation is counter-clockwise.

    a. If the radius of the station is 80 m, what is its rotational speed, v?
    b. Draw vectors representing the astronaut's velocity and acceleration.
    c. Draw a free body diagram for the astronaut.
    d. Is the astronaut exerting a force on the space station? If so, calculate its magnitude. Her mass $m = 65$ kg.
    e. The astronaut drops a ball, which *appears* to accelerate to the 'floor,' (see picture) at 9.8 m/s^2.

        a. Draw the velocity and acceleration vectors for the ball while it is in the air.
        b. What force(s) are acting on the ball while it is in the air?
        c. Draw the acceleration and velocity vectors after the ball hits the floor and comes to rest.
        d. What force(s) act on the ball after it hits the ground?



Answers to Selected Problems

1. .
2. .
3. .
4. .
5. a. 100 N b. 10 m/s^2
6. a. 25 N towards her b. 25 N towards you
7. a. 14.2 m/s^2 b. 7.1×10^3 N c. friction between the tires and the road
8. .0034g
9. a. 6.2×10^5 m/s^2 b. The same as a.
10. 3.56×10^{22}N
11. 4.2×10^{-7} N; very small force
12. $g = 9.8$ m/s^2; you'll get close to this number but not exactly due to some other small effects
13. a. 4×10^{26} N b. gravity c. 2×10^{41} kg
14. .006 m/s^2
15. a. .765 b. 4880 N
16. a. $\sim 10^{-8}$ N very small force b. Your pencil does not accelerate toward you because the frictional force on your pencil is much greater than this force.
17. a. 4.23×10^7m b. 6.6 R_e d. The same, the radius is independent of mass
18. 1.9×10^7m
19. You get two answers for r, one is outside of the two stars one is between them, that's the one you want, 1.32×10^{10}m from the larger star.
20. .
21. .
22. a. $v = 28$ m/s b. v—down, a—right c. f—right d. Yes, 640N

CHAPTER 8 Momentum Conservation Version 2

Chapter Outline

8.1 The Big Idea

The universe has many remarkable qualities, among them a rather beautiful symmetry: the total amount of motion in the universe is constant. This law only makes sense if we measure "motion" in a specific way: as the product of mass and velocity. This product, called *momentum,* can be transferred from one object to another in a collision. The rapidity with which momentum is exchanged over time is determined by the forces involved in the collision. This is the second of the five fundamental conservation laws in physics. The other four are conservation of energy, angular momentum, charge and CPT. (See Feynman's Diagrams for an explanation of CPT.)

8.2 Key Equations and Definitions

We start with a definition of momentum. Since mass is a scalar and velocity is a vector, momentum — their product — is a vector. The momentum of an object with mass m traveling at a velocity v is:

$$\vec{p} = m\vec{v} \text{ [1]}$$

It points in the direction of an object's velocity, and has a magnitude equal to the object's mass times its speed. For a system of many objects, the momentum of the system is equal to the sum of the individual momentum vectors:

$$\vec{p_{sys}} = \sum \vec{p}_i \text{ [2]}$$

Newton referred to momentum in his Second Law; in his terms, if an object of m experiences a net force F_{net} in a period Δt, the following relationship holds:

$$\vec{F_{net}} = \frac{\Delta \vec{p}}{\Delta t} = m\frac{\Delta \vec{v}}{\Delta t} = m\vec{a} \text{ [3]}$$

In other words, *an unbalanced force changes an object's momentum*, with a change equal to

$$\Delta \vec{p} = \vec{F_{net}} \Delta t \text{ [4]}$$

Again, momentum is such an important quantity that Newton defined his Second Law in terms of it.

Finally, according to the law of conservation of momentum, the final momentum of a closed system — like its energy — is equal to its initial value. Using [2], we can write:

$$\sum \vec{p_{\text{initial}}} = \sum \vec{p}$$

final [5]

An important point is that since momentum is a vector, both its magnitude *and* direction are conserved. The (vector) sum of the initial momentum vectors will be equal to the sum of the final vectors.

Much like forces cannot affect motion in direction perpendicular to them—think horizontal velocity of projectiles in 2-D, perpendicular components of the momentum vector are independent. This means that [3] implies that any mutually perpendicular components of momentum would have to be conserved as well, in particular:

$$\sum \vec{p_{\text{yi}}} = \sum \vec{p_{\text{yf}}} \text{ [6]}$$
$$\sum \vec{p_{\text{xi}}} = \sum \vec{p_{\text{xf}}} \text{ [7]}$$

This fact is useful in two dimensional problems, where you can set up equations for each component.

8.3 Key Concepts

- The total momentum of the universe is always the same and is equal to zero. The total momentum of an isolated system never changes.
- Momentum can be transferred from one body to another. In an isolated system in which momentum is transferred internally, the total initial momentum is the same as the total final momentum.
- Momentum conservation is especially important in collisions, where the total momentum just before the collision is the same as the total momentum after the collision.
- The force imparted on an object is equal to the change in momentum divided by the time interval over which the objects are in contact.
- **Internal forces** are forces for which both Newton's Third Law force pairs are contained within the system. For example, consider a two-car head-on collision. Define the *system* as just the two cars. In this case, internal forces include that of the fenders pushing on each other, the contact forces between the bolts, washers, and nuts in the engines, etc.
- **External forces** are forces that act on the system from outside. In our previous example, external forces include the force of gravity acting on both cars (because the other part of the force pair, the pull of gravity the Earth experiences coming from the cars, is not included in the system) and the forces of friction between the tires and the road.
- If there are no external forces acting on a system of objects, the initial momentum of the system will be the same as the final momentum of the system. Otherwise, the final momentum will change by $\Delta\vec{p} = \vec{F}\Delta t$. We call such a change in momentum $\Delta\vec{p}$ an **impulse**.

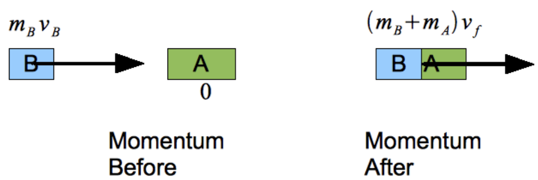
8.4 Key Applications

- Two cars collide head-on—two subatomic particles collide in an accelerator—a bird slams horizontally into a glass office building: all of these are examples of **one-dimensional (straight line) collisions**. For these, pay extra attention to direction: define one direction as positive and the other as negative, and be consistent with signs. Remember, in one dimension vectors are just numbers with signs.
- A firecracker in mid-air explodes—two children push off each other on roller skates—an atomic nucleus breaks apart during a radioactive decay: all of these are examples of ***disintegration*** **problems**. The initial momentum beforehand is zero, so the final momentum afterwards must also be zero. Momenta along any set of perpendicular vectors (like $(\vec{x}, \vec{y}, \vec{z})$) must also be 0.
- A spacecraft burns off momentum by colliding with air molecules as it descends—hail stones pummel the top of your car—a wet rag is thrown at and sticks to the wall: all of these are examples of ***impulse*** **problems**, where the change in momentum of one object and the reaction to the applied force are considered. What is important here is the rate: you need to come up with an average time Δt that the collision(s) last so that you can figure out the force $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$, according to [4].
- Remember as well that if a particle has momentum $\vec{p}$, and it experiences an impulse that turns it around completely, with new momentum $-\vec{p}$, then the total change in momentum has magnitude $2p$. It is harder to reflect something than to stop it.
- Momentum vectors add just like any other vectors. Refer to the addition of vectors material in Chapter 1.

8.5 Examples

Example 1

Question: Two blocks collide on a frictionless surface. Afterwards, they have a combined mass of 10kg and a speed of 2.5m/s. Before the collision, block A, which has a mass of 8.0kg, was at rest. What was the mass and initial speed of block B?



Solution: To find mass of block B we have a simple subtraction problem. We know that the combined mass is 10kg and the mass of block A is 8.0kg.

$$10\text{kg} - 8.0\text{kg} = 2.0\text{kg}$$

Now that we know the mass of both blocks we can find the speed of block B. We will use conservation of momentum. This was a completely inelastic collision. We know this because the blocks stuck together after the collision. This problem is one dimensional, because all motion happens along the same line. Thus we will use the equation

$$(m_A + m_B)v_f = m_A \times v_A + m_B \times v_B$$

and solve for the velocity of block B.

$$(m_A + m_B)v_f = m_A \times v_A + m_B v_B \Rightarrow \frac{(m_A + m_B)(v_f) - (m_A)(v_A)}{m_B} = v_B$$

Now we simply plug in what we know to solve for the velocity.

$$\frac{(2.0\text{kg} + 8.0\text{kg})(2.5\text{m/s}) - (8.0\text{kg})(0\text{m/s})}{2.0\text{kg}} = 12.5\text{m/s}$$

Example 2

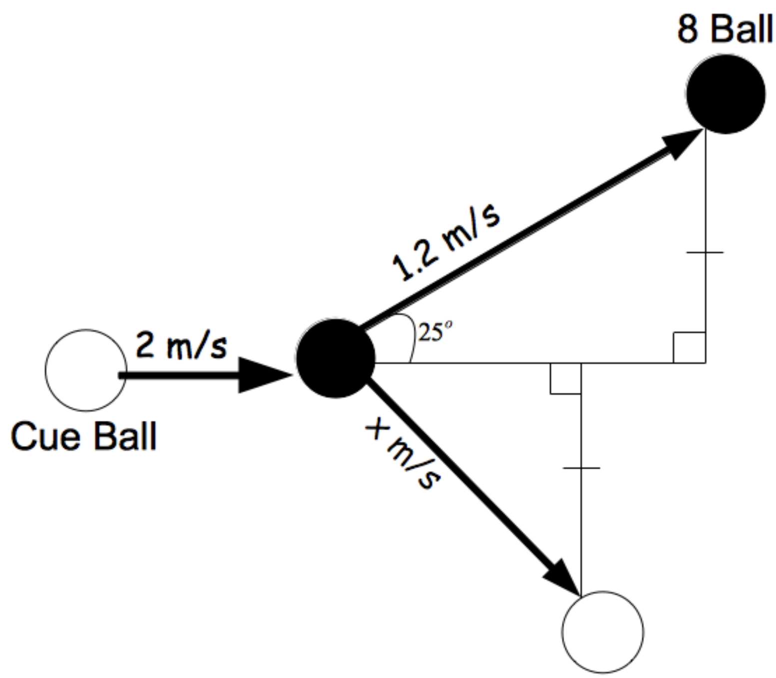
Question: Chris and Ashley are playing pool on a frictionless table. Ashley hits the cue ball into the 8 ball with a velocity of 1.2m/s. The cue ball (c) and the 8 ball (e) react as shown in the diagram. The 8 ball and the cue ball both have a mass of .17kg. What is the velocity of the cue ball? What is the direction (the angle) of the cue ball?

Answer: We know the equation for conservation of momentum, along with the masses of the objects in question as well two of the three velocities. Therefore all we need to do is manipulate the conservation of momentum equation so that it is solved for the velocity of the cue ball after the collision and then plug in the known values to get the velocity of the cue ball.

$$m_c v_{ic} + m_e v_{ie} = m_c v_{fc} + m_e v_{fe}$$

$$v_{fc} = \frac{m_c v_{ic} + m_e v_{ie} - m_e v_{fe}}{m_c} = \frac{.17\text{kg} \times 2.0\text{m/s} + .17\text{kg} \times 0\text{m/s} - .17\text{kg} \times 1.2\text{m/s}}{.17\text{kg}} = .80\text{m/s}$$

Now we want to find the direction of the cue ball. To do this we will use the diagram below.



We know that the momentum in the y direction of the two balls is equal. Therefore we can say that the velocity in the y direction is also equal because the masses of the two balls are equal.

$$m_c v_c y = m_e v_e y \to v_c y = v_e y$$

Given this and the diagram, we can find the direction of the cue ball. After 1 second, the 8 ball will have traveled 1.2m. Therefore we can find the distance it has traveled in the y direction.

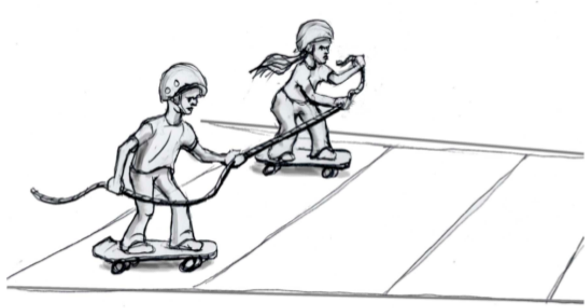
$$sin25^o = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{1.2\text{m}} \to x = sin25 \times 1.2\text{m} = .51\text{m}$$

Therefore, in one second the cue ball will have traveled .51m in the y direction as well. We also know how far in total the cue ball travels in one second (.80m). Thus we can find the direction of the cue ball.

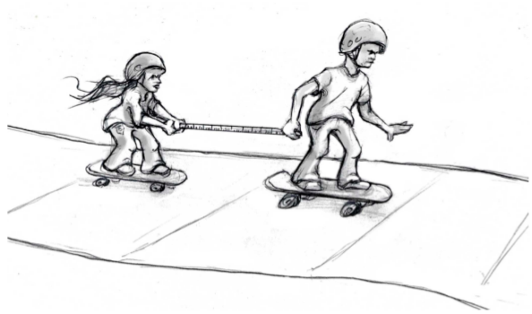
$$sin^{-1} \frac{\text{opposite}}{\text{hypotenuse}} = sin^{-1} \frac{.51\text{m}}{.80\text{m}} = 40^o$$

8.6 Momentum Conservation Problem Set

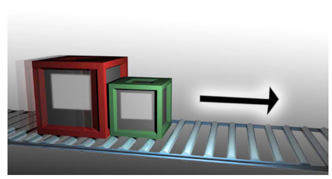
1. You find yourself in the middle of a frozen lake. There is no friction between your feet and the ice of the lake. You need to get home for dinner. Which strategy will work best?
    a. Press down harder with your shoes as you walk to shore.
    b. Take off your jacket. Then, throw it in the direction opposite to the shore.
    c. Wiggle your butt until you start to move in the direction of the shore.
    d. Call for help from the great Greek god Poseidon.
2. You jump off of the top of your house and hope to land on a wooden deck below. Consider the following possible outcomes:
    a. You hit the deck, but it isn't wood! A camouflaged trampoline slows you down over a time period of 0.2 seconds and sends you flying back up into the air.
    b. You hit the deck with your knees locked in a straight-legged position. The collision time is 0.01 seconds.
    c. You hit the deck and bend your legs, lengthening the collision time to 0.2 seconds.
    d. You hit the deck, but it isn't wood! It is simply a piece of paper painted to look like a deck. Below is an infinite void and you continue to fall, forever.
        a. Which method will involve the greatest force acting on you?
        b. Which method will involve the least force acting on you?
        c. Which method will land you on the deck in the least pain?
        d. Which method involves the least impulse delivered to you?
        e. Which method involves the greatest impulse delivered to you?



3. You and your sister are riding skateboards side by side at the same speed. You are holding one end of a rope and she is holding the other. Assume there is no friction between the wheels and the ground. If your sister lets go of the rope, how does your speed change?
    a. It stays the same.
    b. It doubles.
    c. It reduces by half.

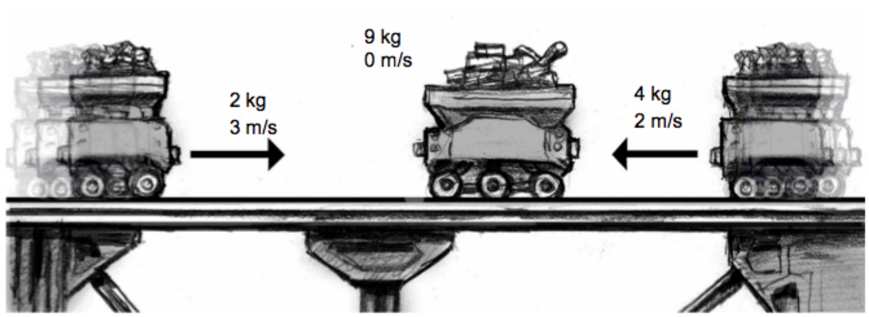


4. You and your sister are riding skateboards (see Problem 3), but now she is riding behind you. You are holding one end of a meter stick and she is holding the other. At an agreed time, you push back on the stick hard enough to get her to stop. What happens to your speed? Choose one. (For the purposes of this problem pretend you and your sister weigh the same amount.)
   a. It stays the same.
   b. It doubles.
   c. It reduces by half.
5. You punch the wall with your fist. Clearly your fist has momentum before it hits the wall. It is equally clear that after hitting the wall, your fist has no momentum. But momentum is always conserved! Explain.
6. An astronaut is using a drill to fix the gyroscopes on the Hubble telescope. Suddenly, she loses her footing and floats away from the telescope. What should she do to save herself?
7. You look up one morning and see that a 30 kg chunk of asbestos from your ceiling is falling on you! Would you be better off if the chunk hit you and stuck to your forehead, or if it hit you and bounced upward? Explain your answer.
8. A 5.00 kg firecracker explodes into two parts: one part has a mass of 3.00 kg and moves at a velocity of 25.0 m/s towards the west. The other part has a mass of 2.00 kg. What is the velocity of the second piece as a result of the explosion?
9. A firecracker lying on the ground explodes, breaking into two pieces. One piece has twice the mass of the other. What is the ratio of their speeds?
10. You throw your 6.0 kg skateboard down the street, giving it a speed of 4.0 m/s. Your friend, the Frog, jumps on your skateboard from rest as it passes by. Frog has a mass of 60 kg.
    a. What is the momentum of the skateboard before Frog jumps on it?
    b. Find Frog's speed after he jumps on the skateboard.
    c. What impulse did Frog deliver to the skateboard?
    d. If the impulse was delivered over 0.2 seconds, what was the average force imparted to the skateboard?
    e. What was the average force imparted to the Frog? Explain.



11. Two blocks collide on a frictionless surface, as shown. Afterwards, they have a combined mass of 10 kg and a speed of 2.5 m/s. Before the collision, one of the blocks was at rest. This block had a mass of 8.0 kg. What was the mass and initial speed of the second block?
12. While driving in your pickup truck down Highway 280 between San Francisco and Palo Alto, an asteroid lands in your truck bed! Despite its 220 kg mass, the asteroid does not destroy your 1200 kg truck. In fact, it landed perfectly vertically. Before the asteroid hit, you were going 25 m/s. After it hit, how fast were you going?
13. A baseball player faces a 80.0 m/s pitch. In a matter of .020 seconds he swings the bat, hitting a 50.0 m/s line drive back at the pitcher. Calculate the force on the bat while in contact with the ball.

14. An astronaut is 100 m away from her spaceship doing repairs with a 10.0 kg wrench. The astronaut's total mass is 90.0 kg and the ship has a mass of 1.00×10^4 kg. If she throws the wrench in the opposite direction of the spaceship at 10.0 m/s how long would it take for her to reach the ship?
15. A place kicker applies an average force of 2400 N to a football of .040 kg. The force is applied at an angle of 20.0 degrees from the horizontal. Contact time is .010 sec.
    a. Find the velocity of the ball upon leaving the foot.
    b. Assuming no air resistance find the time to reach the goal posts 40.0 m away.
    c. The posts are 4.00 m high. Is the kick good? By how much?



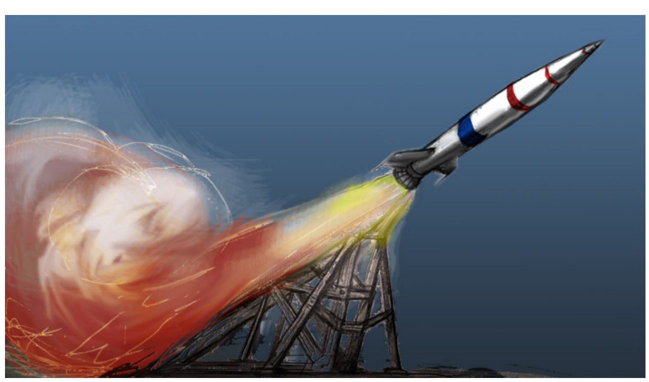
16. In the above picture, the carts are moving on a level, frictionless track. After the collision all three carts stick together. Determine the direction and speed of the combined carts after the collision. (Assume 3−significant digit accuracy.)

17. Your author's Italian cousin crashed into a tree. He was originally going 36 km/hr. Assume it took 0.40 seconds for the tree to bring him to a stop. The mass of the cousin and the car is 450 kg.
    a. What average force did he experience? Include a direction in your answer.
    b. What average force did the tree experience? Include a direction in your answer.
    c. Express this force in pounds.
    d. How many g's of acceleration did he experience?
18. The train engine and its four boxcars are coasting at 40 m/s. The engine train has mass of 5,500 kg and the boxcars have masses, from left to right, of 1,000 kg, 1,500 kg, 2,000 kg, and 3,000 kg. (For this problem, you may neglect the small external forces of friction and air resistance.)

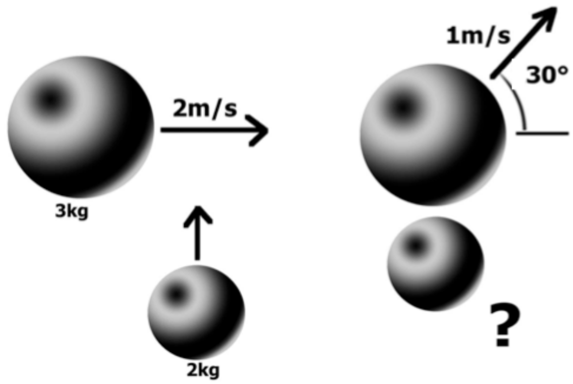
a. What happens to the speed of the train when it releases the last boxcar? (*Hint: Think before you blindly calculate.*)
b. If the train can shoot boxcars backwards at 30 $\mathrm{m/s}$ *relative to the train's speed,* how many boxcars does the train need to shoot out in order to obtain a speed of 58.75 $\mathrm{m/s}$?

19. Serena Williams volleys a tennis ball hit to her at 30 $\mathrm{m/s}$. She blasts it back to the other court at 50 $\mathrm{m/s}$. A standard tennis ball has mass of 0.057 kg. If Serena applied an average force of 500 N to the ball while it was in contact with the racket, how long did the contact last?

20. Zoran's spacecraft, with mass 12,000 kg, is traveling to space. The structure and capsule of the craft have a mass of 2,000 kg; the rest is fuel. The rocket shoots out 0.10 $\mathrm{kg/s}$ of fuel particles with a velocity of 700 $\mathrm{m/s}$ with respect to the craft.
   a. What is the acceleration of the rocket in the first second?
   b. What is the average acceleration of the rocket after the first ten minutes have passed?



21. In Sacramento a 4000 kg SUV is traveling 30 m/s south on Truxel crashes into an empty school bus, 7000 kg traveling east on San Juan. The collision is perfectly inelastic.
    a. Find the velocity of the wreck just after collision
    b. Find the direction in which the wreck initially moves
22. A 3 kg ball is moving 2 m/s in the positive $x-$direction when it is struck dead center by a 2 kg ball moving in the positive $y-$direction. After collision the 3 kg ball moves at 1 m/s30 degrees from the positive $x-$axis.



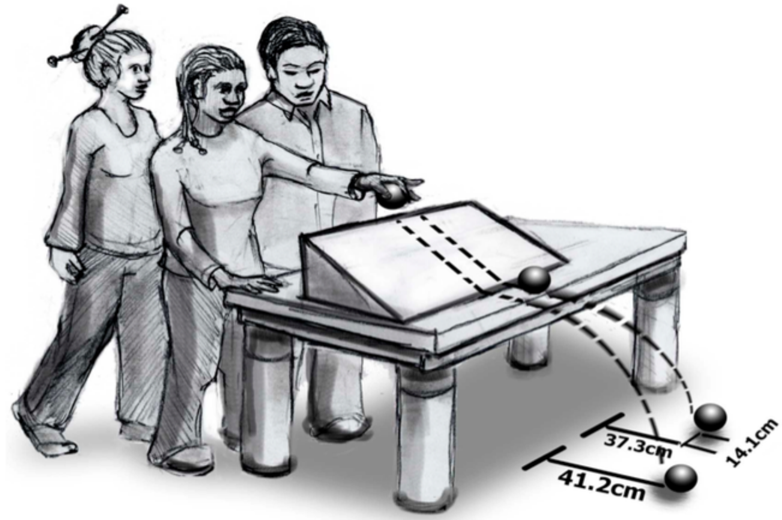
(a) To $2-$significant digit accuracy fill out the following table:

TABLE 8.1:

	3 kg ball p_x	3 kg ball p_y	2 kg ball p_x	2 kg ball p_y
Momentum before				
Momentum after collision				

23. (b) Find the velocity and direction of the 2 kg ball.
    (c) Use the table to prove momentum is conserved.
    (d) Prove that kinetic energy is not conserved.
24. Students are doing an experiment on the lab table. A steel ball is rolled down a small ramp and allowed to hit the floor. Its impact point is carefully marked. Next a second ball of the same mass is put upon a set screw and a collision takes place such that both balls go off at an angle and hit the floor. All measurements are taken with a meter stick on the floor with a co-ordinate system such that just below the impact point is the origin. The following data is collected:

    (a) no collision: 41.2 cm
    (b) target ball: 37.3 cm in the direction of motion and 14.1 cm perpendicular to the direction of motion



i. From this data predict the impact position of the other ball.
ii. One of the lab groups declares that the data on the floor alone demonstrate to a 2% accuracy that the collision was elastic. Show their reasoning.
iii. Another lab group says they can't make that determination without knowing the velocity the balls have on impact. They ask for a timer. The instructor says you don't need one; use your meter stick. Explain.
iv. Design an experiment to prove momentum conservation with balls of different masses, giving apparatus, procedure and design. Give some sample numbers.

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. 37.5 m/s
9. $v_1 = 2v_2$
10. a. $24\frac{kg-m}{5}$ b. 0.364 m/s c. $22\frac{kg-m}{5}$ d. 109 N e. 109 N due to Newton's third law
11. 2.0 kg, 125 m/s
12. 21 m/s to the left
13. 3250 N
14. a. 90 sec b. 1.7×10^5 sec
15. a. 60 m/s b. .700 sec c. yes, 8.16 m
16. 0.13 m/s to the left
17. a. 11000 N to the left b. tree experienced same average force of 11000 N but to the right c. 2500 lb. d. about 2.5 "g"s of acceleration
18. a. no change b. the last two cars
19. a. 0.00912 s

20. a. 0.0058 m/s^2 b. 3.5 m/s^2
21. a.15 m/s b. 49° S of E
22. b. 4.6 m/s 68°

CHAPTER 9 Energy and Force Version 2

Chapter Outline

9.1 The Big Idea

The law of *conservation of momentum* states that in any closed system (including the universe) the total quantity of momentum is constant. Momentum can be transferred from one body to another, but none is lost or gained. If a system has its momentum changed from the outside it is caused by an *impulse,* which transfers momentum from one body to another.

When any two bodies in the universe interact, they can exchange energy, momentum, or both. We saw in an earlier chapter that the law of *conservation of energy* states that in any closed system (including the universe) the total quantity of energy remains fixed. Energy is transferred from one form to another, but not lost or gained. If energy is put into a system from the outside or vice versa it is often in the form of *work,* which is a transfer of energy between bodies.

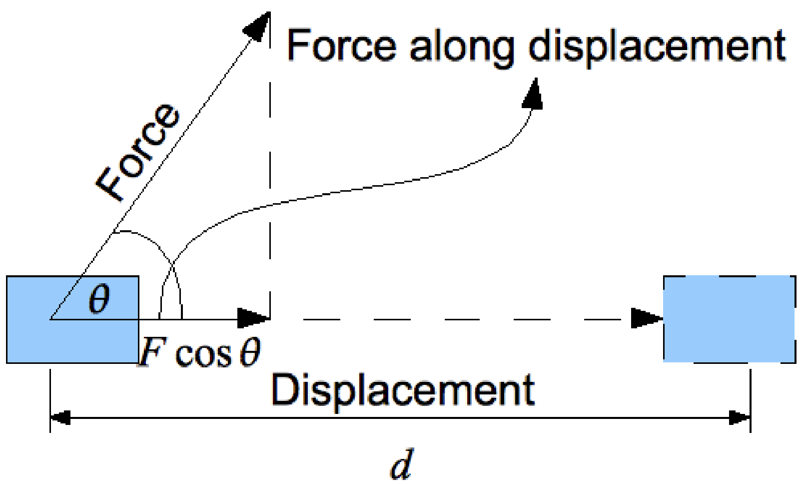
At this point, we have an opportunity to explore the relationship between force, energy and work. These are really important concepts in physics, so we should take our time to understand them. At this point the situation can be summarized like this: energy is the capacity to create motion or change, force is what creates the change, and work is a book keeping tool to keep track of forces.

9.2 Math of Force, Energy, and Work

When an object moves in the direction of an applied force, we say that the force does **work** on the object. Note that the force may be slowing the object down, speeding it up, maintaining its velocity — any number of things. In all cases, the net work done is given by this formula:

$$W = \vec{F} \cdot \vec{d} = \vec{F} \cdot \Delta \vec{x} \qquad \text{[1] Work is the dot product of force and displacement.}$$

In other words, if an object has traveled a distance d under force $\vec{F}$, the work done on it will equal to d multiplied by the component of $\vec{F}$ along the object's path. Consider the following example of a block moving horizontally with a force applied at some angle:



Here the net work done on the object by the force will be $Fd\cos\theta$.

9.3 Work-Energy Principle

The reason the concept of work is so useful is because of a theorem, called the **work-energy principle**, which states that *the change in an object's kinetic energy is equal to the net work done on it*:

$$\Delta K_e = W_{net} \text{ [2]}$$

Although we cannot derive this principle in general, we can do it for the case that interests us most: constant acceleration. In the following derivation, we assume that the force is along motion. This doesn't reduce the generality of the result, but makes the derivation more tractable because we don't need to worry about vectors or angles.

Recall that an object's kinetic energy is given by the formula:

$$K_e = \frac{1}{2}mv^2 \text{ [3]}$$

Consider an object of mass m accelerated from a velocity v_i to v_f under a constant force. The change in kinetic energy, according to [2], is equal to:

$$\Delta K_e = K_{ei} - K_{ef} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) \text{ [4]}$$

Now let's see how much work this took. To find this, we need to find the distance such an object will travel under these conditions. We can do this by using the third of our 'Big three' equations, namely:

$${v_f}^2 = {v_i}^2 + 2a\Delta x \text{ [5]}$$

alternatively,

$$\Delta x = \frac{{v_f}^2 - {v_i}^2}{2a} \text{ [6]}$$

Plugging in [6] and Newton's Third Law, $F = ma$, into [2], we find:

$$W = F\Delta x = ma \times \frac{{v_f}^2 - {v_i}^2}{2a} = \frac{1}{2}m(v_f^2 - v_i^2) \text{ [7],}$$

which was our result in [4].

Using the Work-Energy Principle

The Work-Energy Principle can be used to derive a variety of useful results. Consider, for instance, an object dropped a height Δh under the influence of gravity. This object will experience constant acceleration. Therefore, we can again use equation [6], substituting gravity for acceleration and Δh for distance:

$$\Delta h = \frac{{v_f}^2 - {v_i}^2}{2g}$$

multiplying both sides by mg, we find:

$$mg\Delta h = m\not{g}\frac{{v_f}^2 - {v_i}^2}{2\not{g}} = \Delta K_e \text{ [8]}$$

In other words, the work performed on the object by gravity in this case is $mg\Delta h$. We refer to this quantity as gravitational potential energy; here, we have derived it as a function of height. For most forces (exceptions are friction, air resistance, and other forces that convert energy into heat), potential energy can be understood as the ability to perform work.

Spring Force

A spring with spring constant k a distance Δx from equilibrium experiences a restorative force equal to:

$$F_s = -k\Delta x \text{ [9]}$$

This is a force that can change an object's kinetic energy, and therefore do work. So, it has a potential energy associated with it as well. This quantity is given by:

$$E_{sp} = \frac{1}{2}k\Delta x^2 \qquad \text{[10] Spring Potential Energy}$$

The derivation of [10] is left to the reader. Hint: find the average force an object experiences while moving from $x = 0$ to $x = \Delta x$ while attached to a spring. The net work is then this force times the displacement. Since this quantity (work) must equal to the change in the object's kinetic energy, it is also equal to the potential energy of the spring. This derivation is very similar to the derivation of the kinematics equations — look those up.

9.4 Summary of Key Equations and Definitions

Here is a summary of important concepts from this and the past few chapters. The point of this chapter is to combine all of our knowledge so far to solve new kinds of problems.

$$\text{Transfers}\begin{cases} W = \mathbf{F}\cdot\mathbf{d} & \text{Work is the dot product of force and displacement} \\ P = \frac{\Delta E}{\Delta t} & \text{Power is the rate of change of energy of a system, in Watts (J/s)} \\ \mathbf{J} = \blacksquare\mathbf{p} = \mathbf{F}\Delta t & \text{Impulse is the change in a system's momentum} \end{cases}$$

$$\text{Conservation Laws}\begin{cases} \sum \mathbf{p}_{\text{initial}} = \sum \mathbf{p}_{\textbf{final}} & \text{Total momentum is constant in closed systems} \\ \sum E_{\text{initial}} = \sum E_{final} & \text{Total energy is constant in closed systems} \\ \sum K_{\text{initial}} = \sum K_{final} & \text{Kinetic energy conserved \textbf{only} in elastic collisions} \end{cases}$$

One important type of problem is called a **collision** problem. In cases where collisions are elastic, kinetic energy and momentum are conserved. In inelastic collisions, only momentum is conserved.

9.5 Key Concepts

- An **impulse** occurs when momentum is transferred from one system to another. You can always determine the impulse by finding the changes in momentum, which are done by forces acting over a period of time. If you graph force vs. time of impact, the area under the curve is the impulse.
- Work is simply how much energy was transferred from one system to another system. You can always find the work done *on* an object (or done *by* an object) by determining how much energy has been transferred into or out of the object through forces. If you graph force vs. distance, the area under the curve is work. (The semantics take some getting used to: if *you* do work on *me,* then you have lost energy, and I have gained energy.)

9.6 Key Applications

- When working a problem that asks for *height* or *speed,* energy conservation is almost always the easiest approach.
- Potential energy of gravity, U_g, is always measured with respect to some arbitrary 'zero' height defined to be where the gravitational potential energy is zero. You can set this height equal to zero at any altitude you like. Be consistent with your choice throughout the problem. Often it is easiest to set it to zero at the lowest point in the problem.
- Some problems require you to use both energy conservation *and* momentum conservation. Remember, in every collision, momentum is conserved. Kinetic energy, on the other hand, is not always conserved, since some kinetic energy may be lost to heat.
- If a system involves no energy losses due to heat or sound, no change in potential energy and no work is done by anybody to anybody else, then kinetic energy is conserved. Collisions where this occurs are called *elastic.* In elastic collisions, both kinetic energy and momentum are conserved. In *inelastic* collisions kinetic energy is not conserved; only momentum is conserved.
- Sometimes energy is "lost" when crushing an object. For instance, if you throw silly putty against a wall, much of the energy goes into flattening the silly putty (changing intermolecular bonds). Treat this as lost energy, similar to sound, chemical changes, or heat. In an inelastic collision, things stick, energy is lost, and so kinetic energy is not conserved.
- When calculating work, use the component of the force that is in the same direction as the motion. Components of force perpendicular to the direction of the motion don't do work. (Note that centripetal forces never do work, since they are always perpendicular to the direction of motion.)
- When calculating impulse the time to use is when the force is in contact with the body.

9.7 Work and Energy Examples

Example 1

Question: A pile driver lifts a 500 kg mass a vertical distance of 20 m in 1.1 sec. It uses 225 kW of supplied power to do this.

a) How much work was done by the pile driver?

b) How much power was used in actually lifting the mass?

c) What is the efficiency of the machine? (This is the ratio of power used to power supplied.)

d) The mass is dropped on a pile and falls 20 m. If it loses 40,000 J on the way down to the ground due to air resistance, what is its speed when it hits the pile?

Answer:

a) We will use the equation for work and plug in the known values to get the amount of work done by the pile driver.

$$W = Fd = mad = 500\text{kg} \times 9.8\text{m/s}^2 \times 20\text{m} = 9.8 \times 10^4\text{J}$$

b) We will use the power equation and plug in the known values to the power used.

$$P = \frac{W}{\delta t} = \frac{9.8 \times 10^4\text{J}}{1.1\text{s}} = 89000\text{W} \times \frac{1\text{kW}}{1000\text{W}} = 89\text{kW}$$

c) This is simply a division problem.

$$\frac{\text{power used}}{\text{power supplied}} = \frac{89\text{kW}}{225\text{kW}} = .40$$

d) We have already solved for the amount of energy the mass has after the pile driver performs work on it (it has 9.8×10^4J). If on the way down it loses 40000J due to air resistance, then it effectively has

$$98000\text{J} - 40000\text{J} = 58000\text{J}$$

of energy. So we will set the kinetic energy equation equal to the total energy and solve for v. This will give us the velocity of the mass when it hits the ground because right before the mass hits the ground, all of the potential energy will have been converted into kinetic energy.

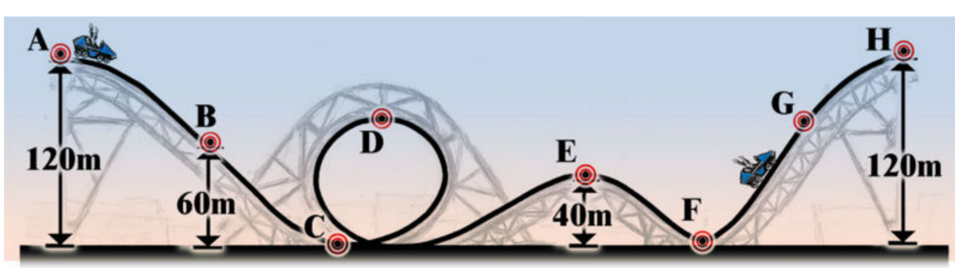
$$58000\text{J} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{58000\text{J} \times 2}{m}} = \sqrt{\frac{58000\text{J} \times 2}{500\text{kg}}} = 15.2\text{m/s}$$

9.8 Energy and Force Problem Set

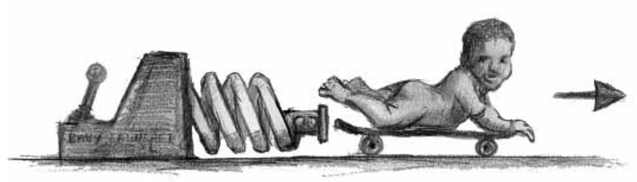
1. At 8:00 AM, a bomb exploded in mid-air. Which of the following is true of the pieces of the bomb after explosion? (Select all that apply.)
   a. The vector sum of the momenta of all the pieces is zero.
   b. The total kinetic energy of all the pieces is zero.
   c. The chemical potential energy of the bomb has been converted entirely to the kinetic energy of the pieces.
   d. Energy is lost from the system to sound, heat, and a pressure wave.
2. A rock with mass m is dropped from a cliff of height h. What is its speed when it gets to the bottom of the cliff?
   a. $\sqrt{mg}$
   b. $2gh$
   c. $\sqrt{2gh}$
   d. gh
   e. None of the above
3. Two cats, Felix and Meekwad, collide. Felix has a mass of 2 kg and an initial velocity of 10 m/s to the west. Meekwad has a mass of 1 kg and is initially at rest. After the collision, Felix has a velocity of 4 m/s to the west and Meekwad has a velocity of 12 m/s to the west. Verify that momentum was conserved. Then, determine the kinetic energies of the system before and after the collision. What happened?! (All numbers are exact.)
4. You are at rest on your bicycle at the top of a hill that is 20 m tall. You start rolling down the hill. At the bottom of the hill you have a speed of 22 m/s. Your mass is 80 kg. Assuming no energy is gained by or lost to any other source, which of the following must be true?
   a. The wind must be doing work on you.
   b. You must be doing work on the wind.
   c. No work has been done on either you or the wind.
   d. Not enough information to choose from the first three.
5. A snowboarder, starting at rest at the top of a mountain, flies down the slope, goes off a jump and crashes through a second-story window at the ski lodge. Retell this story, but describe it using the language of energy. Be sure to describe both how and when the skier gained and lost energy during her journey.
6. An airplane with mass 200,000 kg is traveling with a speed of 268 m/s.
   (a) What is the kinetic energy of the plane at this speed?

   A wind picks up, which causes the plane to lose 1.20×10^8 J per second.

   (b) How fast is the plane going after 25.0 seconds?
7. A roller coaster begins at rest 120 m above the ground, as shown. Assume no friction from the wheels and air, and that no energy is lost to heat, sound, and so on. The radius of the loop is 40 m.

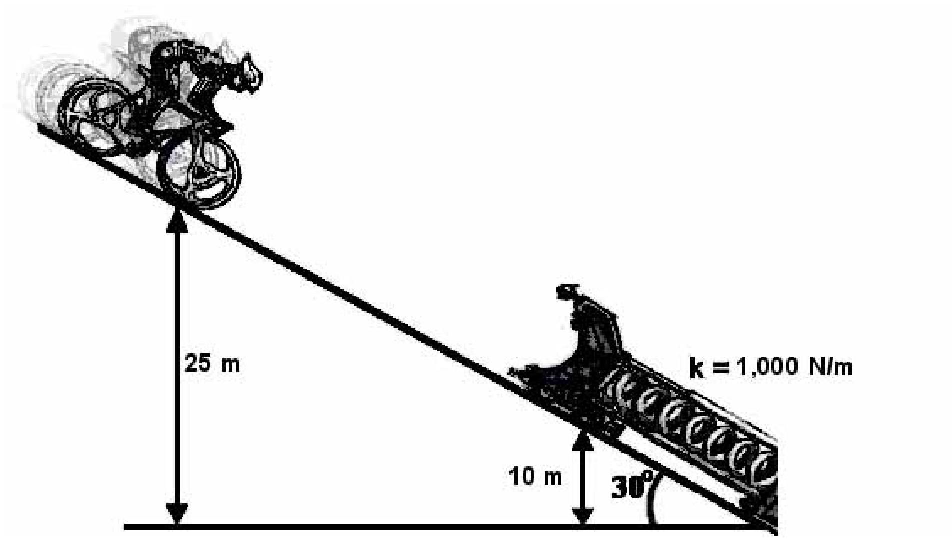


a. Find the speed of the roller coaster at points B, C, D, E, F, and H.
b. Assume that 25% of the initial potential energy of the coaster is lost due to heat, sound, and air resistance along its route. How far short of point H will the coaster stop?
c. Does the coaster actually make it through the loop without falling? (Hint: You might review the material from Chapter 6 to answer this part.)



8. In the picture above, a 9.0 kg baby on a skateboard is about to be launched horizontally. The spring constant is 300 N/m and the spring is compressed 0.4 m. For the following questions, ignore the small energy loss due to the friction in the wheels of the skateboard and the rotational energy used up to make the wheels spin.

   a. What is the speed of the baby after the spring has reached its uncompressed length?
   b. After being launched, the baby encounters a hill 7 m high. Will the baby make it to the top? If so, what is his speed at the top? If not, how high does he make it?
   c. Are you finally convinced that your authors have lost their minds? Look at that picture!



9. When the biker is at the top of the ramp shown above, he has a speed of 10 m/s and is at a height of 25 m. The bike and person have a total mass of 100 kg. He speeds into the contraption at the end of the ramp, which slows him to a stop.

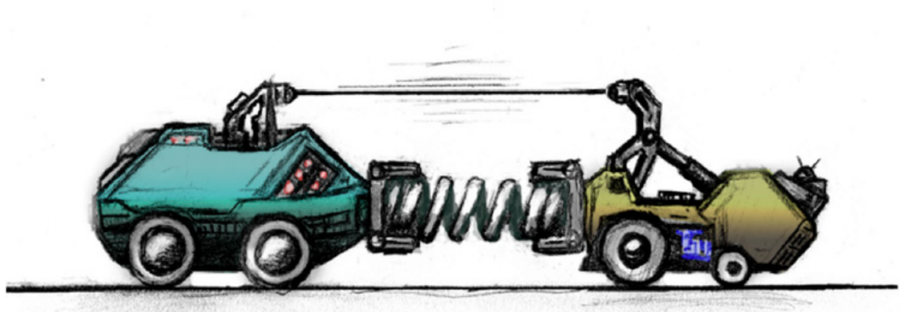
   a. What is his initial total energy? (Hint: Set $U_g = 0$ at the very bottom of the ramp.)
   b. What is the length of the spring when it is maximally compressed by the biker? (Hint: The spring does *not* compress all the way to the ground so there is still some gravitational potential energy. It will help to draw some triangles.)

10. An elevator in an old apartment building in Switzerland has four huge springs at the bottom of the shaft to cushion its fall in case the cable breaks. The springs have an uncompressed height of about 1 meter. Estimate the spring constant necessary to stop this elevator, following these steps:

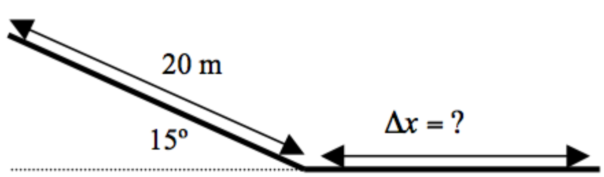
    a. First, guesstimate the mass of the elevator with a few passengers inside.
    b. Now, estimate the height of a five-story building.
    c. Lastly, use conservation of energy to estimate the spring constant.

11. You are driving your buddy to class in a car of mass 900 kg at a speed of 50 m/s. You and your passenger each have 80 kg of mass. Suddenly, a deer runs out in front of your car. The coefficient of friction between the tires and the freeway cement is $\mu_k = 0.9$. In addition there is an average force of friction of 6,000 N exerted by air resistance, friction of the wheels and axles, etc. in the time it takes the car to stop.
    a. What is your stopping distance if you skid to a stop?
    b. What is your stopping distance if you roll to a stop (*i.e.*, if the brakes don't lock)?
12. You are skiing down a hill. You start at rest at a height 120 m above the bottom. The slope has a 10.0° grade. Assume the total mass of skier and equipment is 75.0 kg.

    a. Ignore all energy losses due to friction. What is your speed at the bottom?
    b. If, however, you just make it to the bottom with zero speed what would be the average force of friction, including air resistance?
13. Two horrific contraptions on frictionless wheels are compressing a spring ($k = 400$ N/m) by 0.5 m compared to its uncompressed (equilibrium) length. Each of the 500 kg vehicles is stationary and they are connected by a string. The string is cut! Find the speeds of the masses once they lose contact with the spring.



14. You slide down a hill on top of a big ice block as shown in the diagram. Your speed at the top of the hill is zero. The coefficient of kinetic friction on the slide down the hill is zero ($\mu_k = 0$). The coefficient of kinetic friction on the level part just beneath the hill is 0.1($\mu_k = 0.1$).



    a. What is your speed just as you reach the bottom of the hill?

b. How far will you slide before you come to a stop?

15. A 70 kg woman falls from a height of 2.0 m and lands on a springy mattress.

a. If the springs compress by 0.5 m, what is the spring constant of the mattress?
b. f no energy is lost from the system, what height will she bounce back up to?

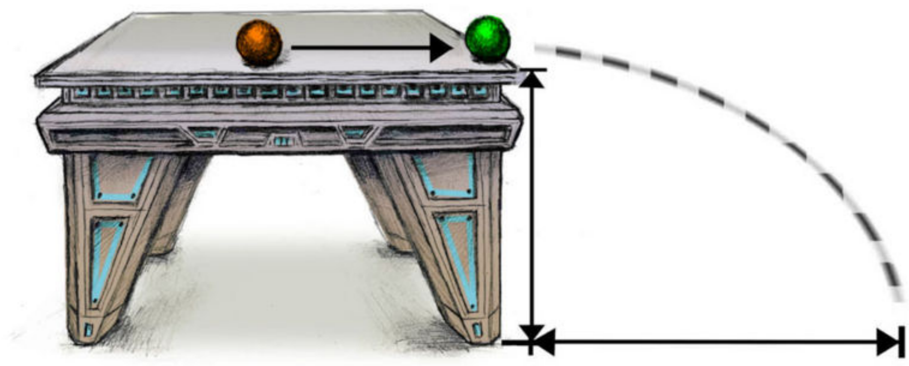
16. Marciel is at rest on his skateboard (total mass 50 kg) until he catches a ball traveling with a speed of 50 m/s. The baseball has a mass of 2 kg. What percent of the original kinetic energy is transferred into heat, sound, deformation of the baseball, and other non-mechanical forms when the collision occurs?

17. You throw a 0.5 kg lump of clay with a speed of 5 m/s at a 15 kg bowling ball hanging from a vertical rope. The bowling ball swings up to a height of 0.01 m compared to its initial height. Was this an elastic collision? Justify your answer.

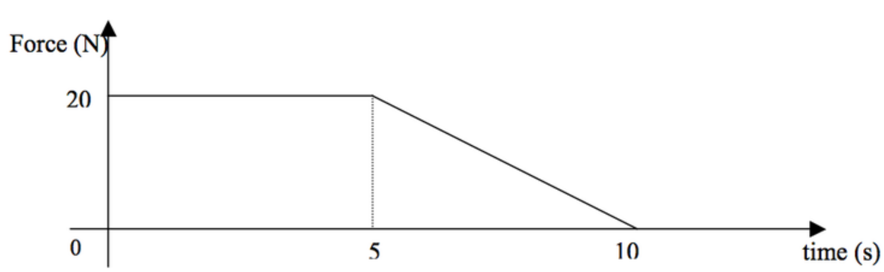
18. The 20 g bullet shown below is traveling to the right with a speed of 20 m/s. A 1.0 kg block is hanging from the ceiling from a rope 2.0 m in length.

    a. What is the maximum height that the bullet-block system will reach, if the bullet embeds itself in the block?
    b. What is the maximum angle the rope makes with the vertical after the collision?

19. You are playing pool and you hit the cue ball with a speed of 2 m/s at the $8-$ball (which is stationary). Assume an elastic collision and that both balls are the same mass. Find the speed and direction of both balls after the collision, assuming neither flies off at any angle.
20. A 0.045 kg golf ball with a speed of 42.0 m/s collides elastically head-on with a 0.17 kg pool ball at rest. Find the speed and direction of both balls after the collision.
21. Ball A is traveling along a flat table with a speed of 5.0 m/s, as shown below. Ball B, which has the same mass, is initially at rest, but is knocked off the 1.5m high table in an elastic collision with Ball A. Find the horizontal distance that Ball B travels before hitting the floor.



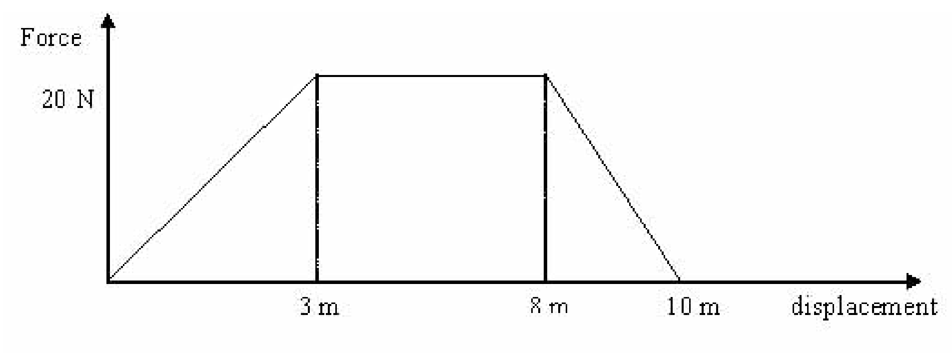
22. Manrico (80.0 kg) and Leonora (60.0 kg) are figure skaters. They are moving toward each other. Manrico's speed is 2.00 m/s; Leonora's speed is 4.00 m/s. When they meet, Leonora flies into Manrico's arms.
    a. With what speed does the entwined couple move?
    b. In which direction are they moving?
    c. How much kinetic energy is lost in the collision?
23. Aida slides down a 20.0 m high hill on a frictionless sled (combined mass 40.0 kg). At the bottom of the hill, she collides with Radames on his sled (combined mass 50.0 kg). The two children cling together and move along a horizontal plane that has a coefficient of kinetic friction of 0.10.
    a. What was Aida's speed before the collision?
    b. What was the combined speed immediately after collision?
    c. How far along the level plane do they move before stopping?
24. A pile driver lifts a 500 kg mass a vertical distance of 20 m in 1.1 sec. It uses 225 kW of supplied power to do this.
    a. How much power was used in actually lifting the mass?
    b. What is the efficiency of the machine? (This is the ratio of power used to power supplied.)
    c. The mass is dropped on a pile and falls 20 m. If it loses 40,000 J on the way down to the ground due to air resistance, what is its speed when it hits the pile?
25. Investigating a traffic collision, you determine that a fast-moving car (mass 600 kg) hit and stuck to a second car (mass 800 kg), which was initially at rest. The two cars slid a distance of 30.0 m over rough pavement with a coefficient of friction of 0.60 before coming to a halt. What was the speed of the first car? Was the driver above the posted 60 MPH speed limit?
26. Force is applied in the direction of motion to a 15.0 kg cart on a frictionless surface. The motion is along a straight line and when $t = 0$, then $x = 0$ and $v = 0$. (The displacement and velocity of the cart are initially zero.) Look at the following graph:



    a. What is the change in momentum during the first 5 sec?

b. What is the change in velocity during the first 10 sec?
c. What is the acceleration at 4 sec?
d. What is the total work done on the cart by the force from 0 – 10 sec?
e. What is the displacement after 5 sec?

27. Force is applied in the direction of motion to a 4.00 kg cart on a frictionless surface. The motion is along a straight line and when $t = 0, v = 0$ and $x = 0$. look at the following graph:



a. What is the acceleration of the cart when the displacement is 4 m?
b. What work was done on the cart between $x = 3$ m and $x = 8$ m?
c. What is the total work done on the cart between 0 – 10 m?
d. What is the speed of the cart at $x = 10$ m?
e. What is the impulse given the cart by the force from 1 – 10 m?
f. What is the speed at $x = 8$ m?
g. How much time elapsed from when the cart was at $x = 8$ to when it got to $x = 10$ m?

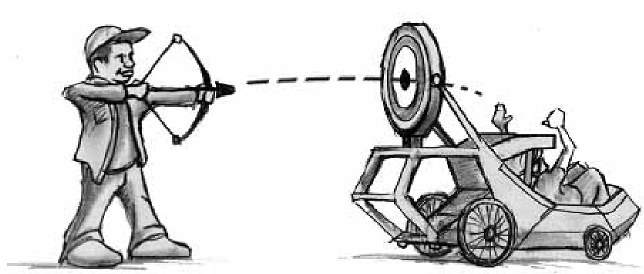
28. You are to design an experiment to measure the average force an archer exerts on the bow as she pulls it back prior to releasing the arrow. The mass of the arrow is known. The only lab equipment you can use is a meter stick.

a. Give the procedure of the experiment and include a diagram with the quantities to be measured shown.
b. Give sample calculations using realistic numbers.
c. What is the single most important inherent error in the experiment?
d. Explain if this error would tend to make the force that it measured greater or lesser than the actual force and why.

29. Molly eats a 500 kcal$(2.09 \times 10^6$ J) power bar before the big pole vault. The bar's energy content comes from changing chemical bonds from a high to a low state and expelling gases. However, 25.0% of the bar's energy is lost expelling gases and 60.0% is needed by the body for various biological functions.

a. How much energy is available to Molly for the run?
b. Energy losses due to air resistance and friction on the run are 200,000 J, Molly's increased heart rate and blood pressure use 55,000 J of the available energy during the run. What top speed can the 50.0 kg Molly expect to attain?
c. The kinetic energy is transferred to the pole, which is "compressed" like a spring of $k = 2720$ N/m; air resistance energy loss on the way up is 300 J, and as she crosses the bar she has a horizontal speed of 2.00 m/s. If Molly rises to a height equal to the expansion of the pole what is that height she reaches?
d. On the way down she encounters another 300 J of air resistance. How much heat in the end is given up when she hits the dirt and comes to a stop?

30. A new fun foam target on wheels for archery students has been invented. The arrow of mass, m, and speed, v_0, goes through the target and emerges at the other end with reduced speed, $v_0/2$. The mass of the target is 7 m. Ignore friction and air resistance.



a. What is the final speed of the target?
b. What is the kinetic energy of the arrow after it leaves the target?
c. What is the final kinetic energy of the target?
d. What percent of the initial energy of the arrow was lost in the shooting?

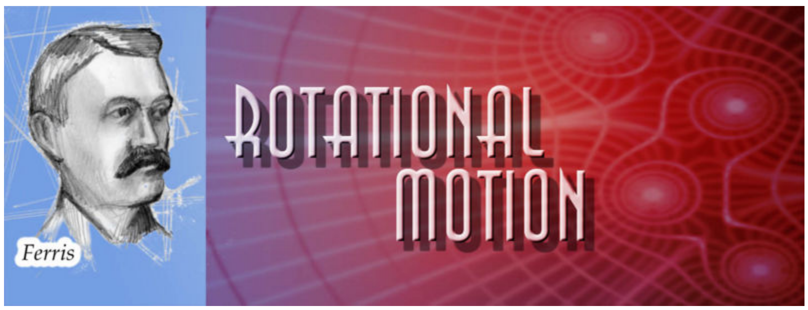
Answers to Selected Problems

1. .

2. .
3. .
4. .
5. .
6. a. 7.18×10^9 J b. 204 m/s
7. a. 34 m/s @ B; 28 m/s @ D; 40 m/s @ E; 49 m/s @ C and F; 0 m/s @ H b. 30 m c. Yes, it makes the loop
8. a. 2.3 m/s c. No, the baby will not clear the hill.
9. a. 29,500 J b. Spring has compressed length of 13 m
10. .
11. a. 86 m b. 220 m
12. a. 48.5 m/s b. 128 N
13. 0.32 m/s each
14. a. 10 m/s b. 52 m
15. a. 1.1×10^4 N/m b. 2 m above the spring
16. 96%
17. .
18. a. .008 m b. 5.12°
19. 8 m/s same direction as the cue ball and 0 m/s
20. $v_{golf} = -24.5$ m/s; vpool = 17.6 m/s
21. 2.8 m
22. a. 0.57 m/s b. Leonora's c. 617 J
23. a. 19.8 m/s b. 8.8 m/s c. 39.5 m
24. a. 89 kW b. 0.4 c. 15.1 m/s
25. 43.8 m/s
26. .
27. .
28. .
29. a. 3.15×10^5 J b. 18.0 m/s c. 2.41 m d. 7900 J
30. a. $v_0/14$ b. $m{v_0}^2/8$ c. $7m{v_0}^2/392$ d. 71%

CHAPTER 10 Rotational Motion Version 2

Chapter Outline



10.1 The Big Idea

In the chapter on centripetal forces, we learned that in some situations, objects move in circles. The purpose of this chapter is to describe and formalize such motion. The fundamental physics behind it is based on the *conservation of angular momentum.* This vector quantity is the product of rotational velocity and rotational inertia. In any closed system (including the universe) the quantity of angular momentum is fixed. Angular momentum can be transferred from one body to another, but cannot be lost or gained. If a system has its angular momentum changed from the outside it is caused by a torque. Torque is a force applied at a distance from the center of rotation.

Rotational motion has many analogies to linear motion. By studying it in this framework, we can make use of many of our previous results. In fact, most of rotational motion can be understood by looking at the following figure and applying results from previous chapters.

10.2 Formalizing Rotational Motion

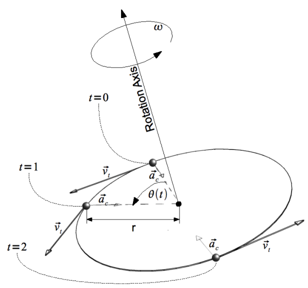


FIGURE 10.1

Illustration of Rotational Motion

Key Concepts

- To determine the *rotation axis,* wrap your right hand's fingers in the direction of rotation and your thumb will point along the axis (see figure).
- When something rotates in a circle, it moves through a *position angle* θ that runs from 0 to 2π radians and starts over again at 0. The physical distance it moves is called the *path length.* If the radius of the circle is larger, the path length traveled is longer. According to the arc length formula $s = r\theta$, the path length Δs traveled by something at radius r through an angle θ is:

$$\Delta s = r\Delta\theta \; [1]$$

- Just like the linear velocity is the rate of change of distance, angular velocity, usually called ω, is the rate of change of θ. The direction of angular velocity is either clockwise or counterclockwise. Analogously, the rate of change of ω is the angular acceleration α.
- The linear velocity and linear acceleration of rotating object also depend on the radius of rotation, which is called the *moment arm* (See figure) If something is rotating at a constant angular velocity, it moves more quickly if it is farther from the center of rotation. For instance, people at the Earth's equator are moving faster than people at northern latitudes, even though their day is still 24 hours long – this is because they have a greater circumference to travel in the same amount of time. According to [1],

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{rt} = \frac{v}{r} \text{ or } v = \omega r \; [2]$$

- Alternatively, we could derive [2] by setting the time to travel a path length equal to the circumference, $2\pi r$ at speed v equal to the time it takes to travel one full angular revolution, 2π at angular velocity ω.
- In exactly the same fashion we can derive the fact that angular acceleration α is related to linear acceleration a in the following way:

$$a = \alpha r \; [3]$$

Note: The above two relations hold for the situations where it is a single object (like the Earth, merry go round, etc.) or if a rolling object is not slipping with respect to the ground or if a pulley is not slipping with respect to the rope.

- The angular acceleration is not the same as centripetal acceleration, which always points toward the center. Angular acceleration is always in the direction or against the direction of angular velocity. The linear acceleration associated with it points along instantaneous velocity.
- Since the mathematics is identical, under constant angular acceleration we can have the big three equations for circular motion.
- Just as linear accelerations are caused by forces, angular accelerations are caused by *torques.*
- **Torques** produce angular accelerations, but just as masses resist acceleration (due to inertia), there is an inertia that opposes angular acceleration. The measure of this inertial resistance depends on the mass, but more importantly on the distribution of the mass in a given object. **The *moment of inertia, I*, is the rotational version of mass.** Values for the moment of inertia of common objects are given in problem 2. Torques have only two directions: those that produce clockwise (CW) and those that produce counterclockwise (CCW) rotations. The angular acceleration or change in would be in the direction of the torque.
- Imagine spinning a fairly heavy disk. To make it spin, you don't push *towards* the disk center– that will just move it in a straight line. To spin it, you need to push along the side, much like when you spin a basketball. Thus, the *torque* you exert on a disk to make it accelerate depends only on the component of the force perpendicular to the radius of rotation:
- Many separate torques can be applied to an object. The angular acceleration produced is $\alpha_{net} = \frac{\tau_{net}}{I}$
- When an object is rolling without slipping this means that $v = r\omega$ and $a = r\alpha$. This is also true in the situation of a rope on a pulley that is rotating the pulley without slipping. Using this correspondence between linear and angular speed and acceleration is very useful for solving problems, but is only true if there is no slipping.
- The *angular momentum* of a spinning object is $L = I\omega$. Torques produce a change in angular momentum with time: $\tau = \frac{\Delta L}{\Delta t}$
- Spinning objects have a kinetic energy, given by $K_{rot} = \frac{1}{2}I\omega^2$.

10.3 Analogies Between Linear and Rotational Motion

$$\text{Linear}\begin{cases}\text{Quantity} & \text{Units}\\ \vec{x} & \text{m}\\ \vec{v} & \text{m/s}\\ m & \text{kg}\\ \vec{F}=\frac{\Delta\vec{p}}{\Delta t} & \text{N}\\ \vec{a}=\frac{\vec{F_{Net}}}{m} & \text{m/s}^2\\ \vec{p}=m\vec{v} & \text{kg m/s}\\ K=\frac{1}{2}mv^2 & \text{J}\end{cases}\quad \text{Rotational}\begin{cases}\text{Quantity} & \text{Units}\\ \vec{\theta} & \text{Radians}\\ \omega & \text{Radians/s}\\ \vec{I} & \text{kg m}^2\\ \tau=\frac{\Delta\vec{L}}{\Delta t} & \text{N m}\\ \alpha=\frac{\vec{\tau_{Net}}}{I} & \text{Radians/s}^2\\ \vec{L}=\vec{I}\omega & \text{kg m}^2\text{/s}\\ K=\frac{1}{2}I\omega^2 & \text{J}\end{cases}$$

In addition to [1], [2], and [3], there are other important relationships for rotational motion. These are summarized in table 1.1.

TABLE 10.1:

Equation	Explanation
$a_c = v^2/r = r\omega^2$	the centripetal acceleration of an object.
$\omega = 2\pi/T = 2\pi f$	Relationship between period and frequency.
$\theta(t) = \theta_0 + \omega_0 t + 1/2\alpha t^2$	The 'Big Three' equations work for rotational motion too!
$\omega(t) = \omega_0 + \alpha t$	Rotational equivalent of $v_f = v_i + at$
$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	Rotational equivalent of $v_f^2 = v_i^2 + 2ad$
$\alpha = \tau_{net}/I$	Angular accelerations are produced by net torques,with inertia opposing acceleration; this is the rotational analog of $a = F_{net}/m$
$\tau_{net} = \Sigma\tau_i = I\alpha$	The net torque is the vector sum of all the torques acting on the object. When adding torques it is necessary to subtract CW from CCW torques.
$\vec{\tau} = \vec{r} \times \vec{F} = r_\perp F = rF_\perp$	Individual torques are determined by multiplying the force applied by the *perpendicular* component of the moment arm
$L = I\omega$	Angular momentum is the product of moment of inertia and angular velocity.
$\tau = \triangle L/\triangle t$	Torques produce changes in angular momentum; this is the rotational analog of $F = \triangle p/\triangle t$
$K = 1/2I\omega^2$	Rotational motion contributes to kinetic energy as well!

10.4 Example 1

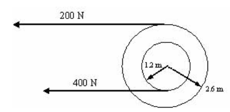
Question: A game of tug-of-war is played. . . but with a twist (ha!). Each team has its own rope attached to a merry-go-round. One team pulls counterclockwise with a force of 200N. The other team pulls clockwise with a force of 400N. But there is another twist. The counterclockwise team's rope is attached 2.6m from the center of the merry go round and the clockwise team's rope is attached 1.2m from the center of the circle.

a) Who wins?

b) By how much? That is, what is the net torque?

c) Assume that the merry-go-round is weighted down with a large pile of steel plates. It is so massive that it has a moment of inertia of $2000\text{kg} \times \text{m}^2$. What is the angular acceleration?

d) How long will it take the merry-go-round to complete one revolution?



Solution:

a) To find out who wins, we need to find which team is pulling with the greater torque. Therefore, we will use the equation

$$\tau = rF$$

counterclockwise team:

$$\tau = rF = 2.6\text{m} \times 200\text{N} = 520\text{N} \times \text{m}$$

clockwise team:

$$\tau = rF = 1.2\text{m} \times 400\text{N} = 480\text{N} \times \text{m}$$

So the counterclockwise team wins.

b) To figure out the net torque we simply subtract the two torques.

$$520\text{N} \times \text{m} - 480\text{N} \times \text{m} = 40\text{N} \times \text{m}$$

So we have a $40\text{N} \times \text{m}$ counterclockwise net torque.

c) To find the angular acceleration we use the equation

$$\alpha = \frac{\tau}{l}$$

Since we know both the net torque and the moment of inertia, all we have to do is plug these values in.

$$\alpha = \frac{\tau}{l} = \frac{40\text{N}}{2000\text{kg} \times \text{m}^2} = .02\text{r/s}^2$$

d) Finally, we want to know the time of one rotation. To do this we will use the equation

$$\theta = \theta_i + w_i t + \frac{1}{2}\alpha t^2$$

We are only concerned with $\frac{1}{2}\alpha t^2$ because θ_i and w_i both equal 0. All we need to do is solve for time.

$$\theta = \frac{1}{2}\alpha t^2 \Rightarrow 2\theta \times \alpha = t^2 \Rightarrow \sqrt{\frac{2\theta}{\alpha}} = t$$

Now we plug in the known values to get time.

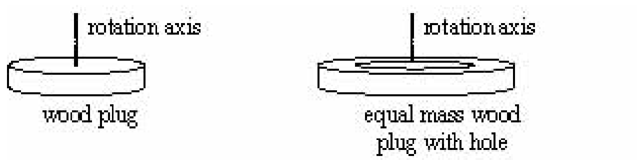
$$\sqrt{\frac{2 \times 2\pi}{.02\text{r/s}^2}} = 25\text{s}$$

10.5 Rotational Motion Problem Set

1. The wood plug, shown below, has a lower moment of inertia than the steel plug because it has a lower mass.

(a) Which of these plugs would be easier to spin on its axis? Explain.

Even though they have the same mass, the plug on the right has a higher moment of inertia (I), than the plug on the left, since the mass is distributed at greater radius.



(b) Which of the plugs would have a greater angular momentum if they were spinning with the same angular velocity? Explain.

2. Here is a table of some moments of inertia of commonly found objects:

Object	*Drawing*	*Moment of Inertia*
Disk (rotated about center)		$\frac{1}{2}MR^2$
Ring (rotated about center)		MR^2
Rod or plank (rotated about center)		$\frac{1}{12}ML^2$
Rod or plank (rotated about end)		$\frac{1}{3}ML^2$
Sphere		$\frac{2}{5}MR^2$
Satellite	R	MR^2

a. Calculate the moment of inertia of the Earth about its spin axis.
b. Calculate the moment of inertia of the Earth as it revolves around the Sun.
c. Calculate the moment of inertia of a hula hoop with mass 2 kg and radius 0.5 m.
d. Calculate the moment of inertia of a rod 0.75 m in length and mass 1.5 kg rotating about one end.
e. Repeat d., but calculate the moment of inertia about the center of the rod.

3. Imagine standing on the North Pole of the Earth as it spins. You would barely notice it, but you would turn all the way around over 24 hours, without covering any real distance. Compare this to people standing on the equator: they go all the way around the entire circumference of the Earth every 24 hours! Decide whether the following statements are TRUE or FALSE. Then, explain your thinking.

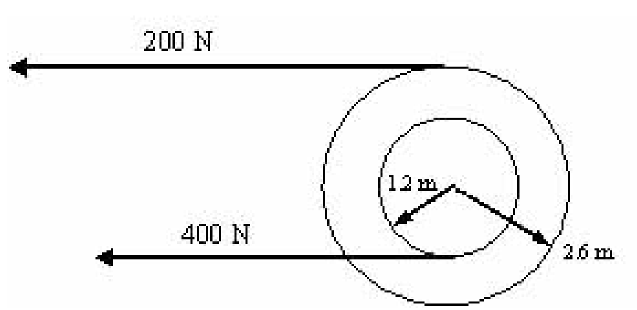
a. The person at the North Pole and the person at the equator rotate by 2π radians in 86,400 seconds.
b. The angular velocity of the person at the equator is $2\pi/86400$ radians per second.
c. Our angular velocity in San Francisco is $2\pi/86400$ radians per second.

d. Every point on the Earth travels the same distance every day.
e. Every point on the Earth rotates through the same angle every day.
f. The angular momentum of the Earth is the same each day.
g. The angular momentum of the Earth is $2/5MR^2\omega$.
h. The rotational kinetic energy of the Earth is $1/5MR^2\omega^2$.
i. The *orbital* kinetic energy of the Earth is $1/2MR^2\omega^2$, where *R* refers to the distance from the Earth to the Sun.

4. You spin up some pizza dough from rest with an angular acceleration of 5 rad/s^2.

a. How many radians has the pizza dough spun through in the first 10 seconds?
b. How many times has the pizza dough spun around in this time?
c. What is its angular velocity after 5 seconds?
d. What is providing the torque that allows the angular acceleration to occur?
e. Calculate the moment of inertia of a flat disk of pizza dough with mass 1.5 kg and radius 0.6 m.
f. Calculate the rotational kinetic energy of your pizza dough at $t = 5$ s and $t = 10$ s.

5. Your bike brakes went out! You put your feet on the wheel to slow it down. The rotational kinetic energy of the wheel begins to decrease. Where is this energy going?
6. Consider hitting someone with a Wiffle ball bat. Will it hurt them more if you grab the end or the middle of the bat when you swing it? Explain your thinking, but do so using the vocabulary of *moment of inertia* (treat the bat as a rod), *angular momentum* (imagine the bat swings down in a semi-circle), and *torque* (in this case, torques caused by the contact forces the other person's head and the bat are exerting on each other).
7. Why does the Earth keep going around the Sun? Shouldn't we be spiraling farther and farther downward towards the Sun, eventually falling into it? Why do low-Earth satellites eventually spiral down and burn up in the atmosphere, while the Moon never will?
8. If most of the mass of the Earth were concentrated at the core (say, in a ball of dense iron), would the moment of inertia of the Earth be higher or lower than it is now? (Assume the total mass stays the same.)
9. Two spheres of the same mass are spinning in your garage. The first is 10 cm in diameter and made of iron. The second is 20 cm in diameter but is a thin plastic sphere filled with air. Which is harder to slow down? Why? (And why are two spheres spinning in your garage?)
10. A game of tug-o-war is played ... but with a twist (ha!). Each team has its own rope attached to a merry-go-round. One team pulls clockwise, the other counterclockwise. Each pulls at a different point and with a different force, as shown.

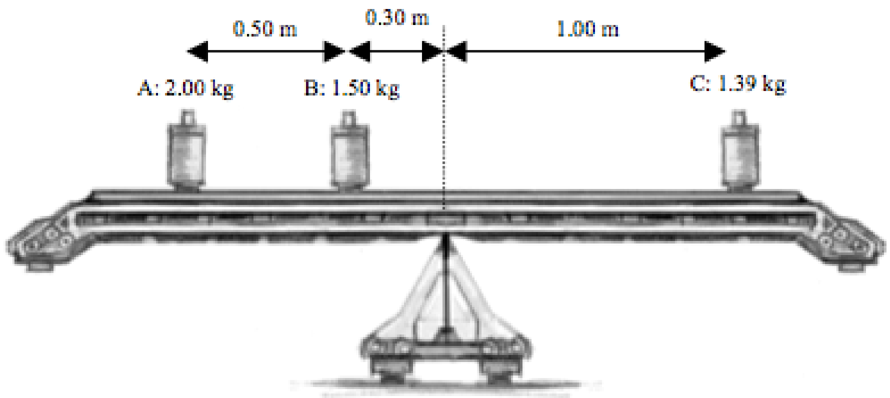


a. Who wins?
b. By how much? That is, what is the net torque?
c. Assume that the merry-go-round is weighted down with a large pile of steel plates. It is so massive that it has a moment of inertia of 2000 $\text{kg} \cdot m^2$. What is its angular acceleration?
d. How long will it take the merry-go-round to spin around once completely?

11. You have two coins; one is a standard U.S. quarter, and the other is a coin of equal mass and size, but with a hole cut out of the center.

a. Which coin has a higher moment of inertia?
b. Which coin would have the greater angular momentum if they are both spun at the same angular velocity?

12. A wooden plank is balanced on a pivot, as shown below. Weights are placed at various places on the plank.



Consider the torque on the plank caused by weight A.

a. What force, precisely, is responsible for this torque?
b. What is the magnitude (value) of this force, in Newtons?
c. What is the moment arm of the torque produced by weight A?
d. What is the magnitude of this torque, in $N \cdot m$?
e. Repeat parts (a – d) for weights B and C.
f. Calculate the net torque. Is the plank balanced? Explain.

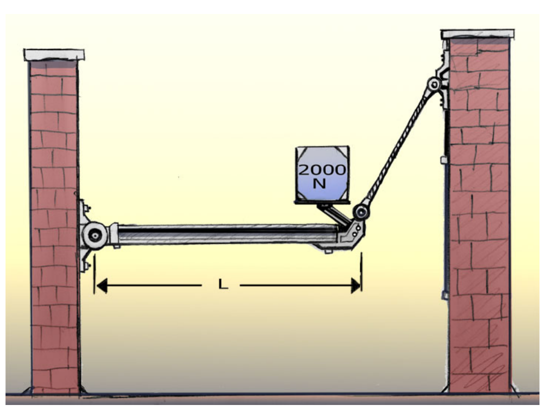
13. A star is rotating with a period of 10.0 days. It collapses with no loss in mass to a white dwarf with a radius of .001 of its original radius.

a. What is its initial angular velocity?
b. What is its angular velocity after collapse?

14. For a ball rolling without slipping with a radius of 0.10 m, a moment of inertia of 25.0 kg $- m^2$, and a linear velocity of 10.0 m/s calculate the following:

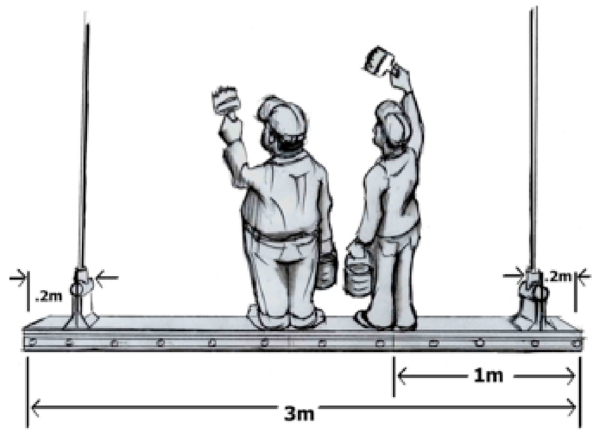
a. The angular velocity.
b. The rotational kinetic energy.
c. The angular momentum.
d. The torque needed to double its linear velocity in 0.2 sec.

15. A merry-go-round consists of a uniform solid disc of 225 kg and a radius of 6.0 m. A single 80 kg person stands on the edge when it is coasting at 0.20 revolutions /sec. How fast would the device be rotating after the person has walked 3.5 m toward the center. (The moments of inertia of compound objects add.)



16. In the figure we have a horizontal beam of length, L, pivoted on one end and supporting 2000 N on the other. Find the tension in the supporting cable, which is at the same point at the weight and is at an angle of 30 degrees to the vertical. Ignore the weight of the beam.

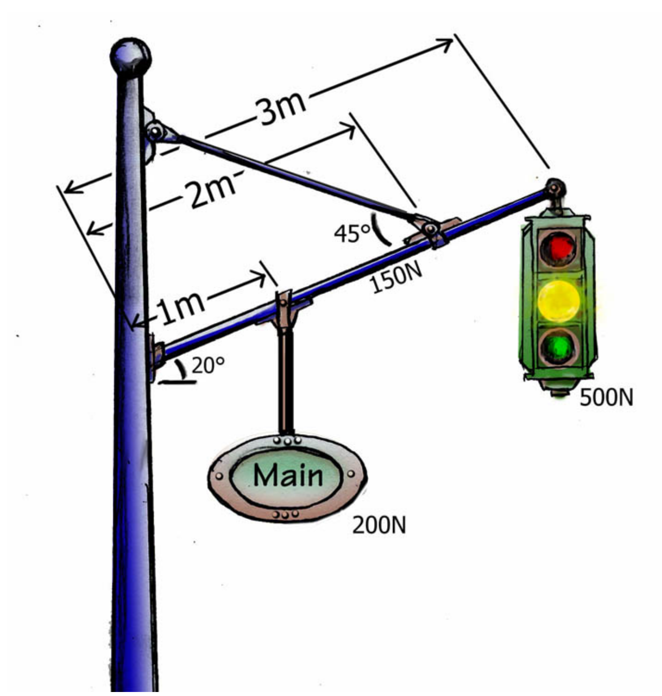
17. Two painters are on the fourth floor of a Victorian house on a scaffold, which weighs 400 N. The scaffold is 3.00 m long, supported by two ropes, each located 0.20 m from the end of the scaffold. The first painter of mass 75 kg is standing at the center; the second of mass, 65.0 kg, is standing 1.00 m from one end.



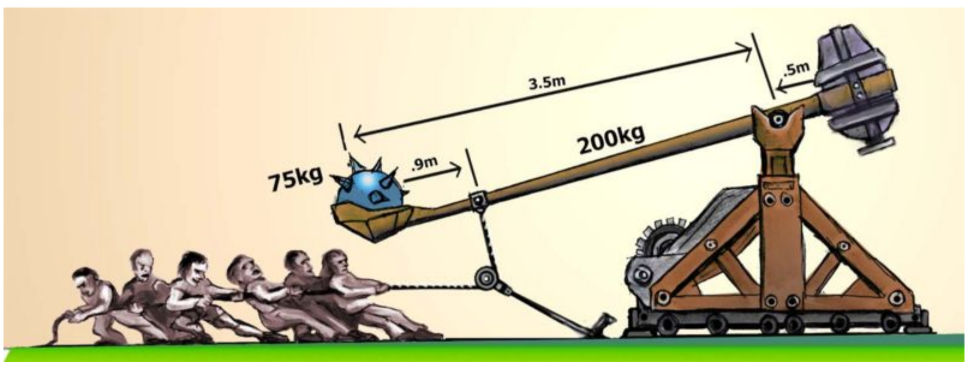
a. Draw a free body diagram, showing all forces and all torques. (Pick one of the ropes as a pivot point.)
b. Calculate the tension in the two ropes.
c. Calculate the moment of inertia for rotation around the pivot point, which is supported by the rope with the least tension. (This will be a compound moment of inertia made of three components.)
d. Calculate the instantaneous angular acceleration assuming the rope of greatest tension breaks.

18. A horizontal 60 N beam. 1.4 m in length has a 100 N weight on the end. It is supported by a cable, which is connected to the horizontal beam at an angle of 37 degrees at 1.0 m from the wall. Further support is provided by the wall hinge, which exerts a force of unknown direction, but which has a vertical (friction) component and a horizontal (normal) component.

a. Find the tension in the cable.
b. Find the two components of the force on the hinge (magnitude and direction).
c. Find the coefficient of friction of wall and hinge.



19. On a busy intersection a 3.00 m beam of 150 N is connected to a post at an angle upwards of 20.0 degrees to the horizontal. From the beam straight down hang a 200N sign 1.00 m from the post and a 500 N signal light at the end of the beam. The beam is supported by a cable, which connects to the beam 2.00 m from the post at an angle of 45.0 degrees measured from the beam; also by the hinge to the post, which has horizontal and vertical components of unknown direction.
    a. Find the tension in the cable.
    b. Find the magnitude and direction of the horizontal and vertical forces on the hinge.
    c. Find the total moment of inertia around the hinge as the axis.
    d. Find the instantaneous angular acceleration of the beam if the cable were to break.
20. There is a uniform rod of mass 2.0 kg of length 2.0 m. It has a mass of 2.6 kg at one end. It is attached to the ceiling .40 m from the end with the mass. The string comes in at a 53 degree angle to the rod.
    a. Calculate the total torque on the rod.
    b. Determine its direction of rotation.
    c. Explain, but don't calculate, what happens to the angular acceleration as it rotates toward a vertical position.



21. The medieval catapult consists of a 200 kg beam with a heavy ballast at one end and a projectile of 75.0 kg at the other end. The pivot is located 0.5 m from the ballast and a force with a downward component of 550 N is applied by prisoners to keep it steady until the commander gives the word to release it. The beam is 4.00 m long and the force is applied 0.900 m from the projectile end. Consider the situation when the beam is perfectly horizontal.
    a. Draw a free-body diagram labeling all torques.
    b. Find the mass of the ballast.
    c. Find the force on the horizontal support.
    d. How would the angular acceleration change as the beam moves from the horizontal to the vertical position. (Give a qualitative explanation.)
    e. In order to maximize range at what angle should the projectile be released?
    f. What additional information and/or calculation would have to be done to determine the range of the projectile?

Answers to Selected Problems

1. .
2. a. 9.74×10^{37} kg m2 b. 1.33×10^{47} kg m^2 c. 0.5 kg m^2 d. 0.28 kg m^2 e. 0.07 kg m^2
3. a. True, all rotate 2π for $86,400;$sec which is 24 hours, b. True, $\omega = 2\pi/\text{t}$ and $t = 86,400$ s f. True, L is the same g. $L = I\omega$ and $I = 2/5\ \text{mr}^2$ h. True, $K = \frac{1}{2}I\omega^2$ $I = 2/5\ \text{mr}^2$ sub $-$ in $K = 1/5\ \text{mr}^2\omega^2$ i. True, $K = \frac{1}{2}I\omega^2$ $I =\ \text{mr}^2$ sub $-$ in $K = \frac{1}{2}mr^2\omega^2$

4. a. 250 rad b. 40 rad c. 25 rad/s d. Force applied perpendicular to radius allows α e. 0.27 kg m^2, f. $K^5 = 84$ J and $K^1 0 = 340$ J
5. .
6. Moment of inertia at the end $1/3\,\mathrm{ML}^2$ at the center $1/12\,\mathrm{ML}^2$, angular momentum, $L = I\omega$ and torque, $\tau = I\alpha$ change the in the same way
7. .
8. Lower
9. Iron ball
10. a. 200 N team b. 40 N c. 0.02 $\mathrm{rad/s}^2$ d. 25 s
11. a. Coin with the hole b. Coin with the hole
12. a. weight b. 19.6 N c. plank's length (0.8m) left of the pivot d. 15.7 N m, e. Ba. weight, Bb. 14.7 N, Bc. plank's length (0.3m) left of the pivot, Bd. 4.4 N m, Ca. weight, Cb. 13.6 N, Cc. plank's length (1.00 m) right of the pivot, Cd. 13.6 N m, f) 6.5 N m CC, g) no, net torque doesn't equal zero
13. a. 7.27×10^{-6} Hz b. 7.27 Hz
14. a. 100 Hz b. 1.25×10^5 J c. $2500\ \mathrm{J} - \mathrm{s}$ d. $12,500\ \mathrm{m} - \mathrm{N}$
15. 28 rev/sec
16. 2300 N
17. b. 771 N, 1030 N c. 554 kgm^2 d. $4.81\mathrm{rad/sec}^2$
18. a. 300 N b. $240N, -22$ N c. .092
19. a. 2280 N b. 856 n toward beam, 106 N down c. 425 kgm^2 d. 3.39 $\mathrm{rad/sec}^2$
20. a. -1.28 Nm 20. CCW
21. a. 1411 kg c. 17410 N d. angular acc goes down as arm moves to vertical

10.6 References

1. Alex Zaliznyak. Rotational. Public Domain

CHAPTER 11 Simple Harmonic Motion Version 2

Chapter Outline

11.1 The Big Idea

The development of devices to measure time, like the pendulum, led to the analysis of *periodic motion*. Such motion repeats itself in equal intervals of time (called periods) and is also referred to as *harmonic motion.* When an object moves back and forth over the *same path* in harmonic motion it is said to be *oscillating.* If the distance such an object travels in one oscillation remains constant, it is called *simple* harmonic motion (SHM). A grandfather clock's pendulum and the quartz crystal in a modern watch are examples of SHM.

11.2 Key Concepts

- The oscillating object does not lose any energy in SHM. Friction is assumed to be zero.
- In harmonic motion there is always a *restorative force,* which attempts to *restore* the oscillating object to its equilibrium position. The restorative force changes during an oscillation and depends on the position of the object. In a spring the force is given by Hooke's Law: $\vec{F} = -k\vec{x}$; in a pendulum it is the component of gravity along the path.
- Objects in simple harmonic motion do not obey the "Big Three" equations of motion because the acceleration is not constant. As a spring compresses, the force (and hence the acceleration) increases. Similarly, as a pendulum swings, the tangential component of the force of gravity changes. The equations of motion for SHM are given in the Key Equations section.
- The period, T, is the amount of time needed for the harmonic motion to repeat itself, or for the object to go one full cycle. In SHM, T is the time it takes the object to return to its exact starting point and starting direction.
- The frequency, f is the number of cycles an object goes through in 1 second. Frequency is measured in Hertz (Hz). 1 Hz = 1 cycle per sec.
- The amplitude, A, is the distance from the *equilibrium* (or center) *point* of motion to either its lowest or highest point (*end points*). The amplitude, therefore, is half of the total distance covered by the oscillating object. The amplitude can vary in harmonic motion, but is constant in SHM.
- The kinetic energy and the speed are at a maximum at the equilibrium point, but the potential energy and restorative force is zero there.
- At the *end points* the potential energy is at a maximum, while the kinetic energy and speed are zero. However at the end points the restorative force and acceleration are at a maximum.
- In SHM since energy is conserved, often, the most fruitful method of calculating position and velocity is to set the total energy equal to the sum of kinetic and potential energies. Similarly force and acceleration are best calculated by using $\sum \vec{F} = m\vec{a}$.

11.3 Key Equations and Definitions

$$\text{Period Equations}\begin{cases} T = \frac{1}{f} & \text{Period is the inverse of frequency} \\ T_{\text{spring}} = 2\pi\sqrt{\dfrac{m}{k}} & \text{Period of mass } m \text{ on a spring with constant } k \\ T_{\text{pendulum}} = 2\pi\sqrt{\dfrac{L}{g}} & \text{Period of a pendulum of length } L \end{cases}$$

$$\text{Kinematics of SHM}\begin{cases} x(t) = x_0 + A\cos 2\pi f(t - t_0) & \text{Position of an object in SHM of Amplitude } A \\ v(t) = -2\pi f A\cos 2\pi f(t - t_0) & \text{Velocity of an object in SHM of Amplitude } A \end{cases}$$

11.4 Examples

Example 1

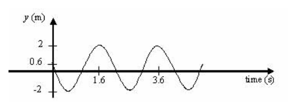
Question: The effective k of a diving board is 800N/m (we say effective because it bends in the direction of motion instead of stretching like a spring, but otherwise behaves the same). A pudgy diver is bouncing up and down at the end of the diving board. The y vs. t graph is shown below.

a) What is the distance between the lowest and the highest point of oscillation?

b) What is the y-position and velocity of the diver at $t = 2$?

c) What is the diver's mass?

d) Write the sinusoidal equation of motion for the diver.



Solution:

a) As we can see from the graph the highest point is 2m and the lowest point is -2m. Therefore the distance is

$$|2\text{m} - (-2\text{m})| = 4\text{m}$$

b) To find the y-position we will use the equation

$$y = y_i + Acos(2\pi f(t - t_i))$$

First we must solve for the frequency. We know that

$$f = \frac{1}{T}$$

From the graph we know that the period is 2 seconds, so the frequency is $\frac{1}{2}$hz. All we need to do now is plug in the values to find the position at $t = 2$.

$$y = y_i + A\cos(2\pi f(t - t_i)) = 0 + 2 \times \cos(\frac{2\pi}{2}(2 - 0) = 2 \times \cos\pi \times 2 = 4\text{m}$$

To find the velocity we take the equation

$$v = -2\pi f A \sin(2\pi f(t - t_i))$$

and plug in the known values.

$$v = -2\pi f A \sin(2\pi f(t - t_i)) = \frac{-2\pi}{\pi \text{s}} \times 2\text{m} \times \sin(\frac{2\pi}{\pi \text{s}}(2\text{s} - 0\text{s}) = 3.0\text{m/s}$$

Despite the fact that we have a negative value for the displacement (-1.3m) it makes sense that we would get a positive velocity because, as we can see from the graph, the diving board is still moving down at $t = 2$.

c) To find the diver's mass we will use the equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and solve for m. Then it is a simple matter to plug in the known values to get the mass.

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T}{2\pi} = \sqrt{\frac{m}{k}} \Rightarrow (\frac{T}{2\pi})^2 = \frac{m}{k} \Rightarrow k(\frac{T}{2\pi})^2 = m$$

Now we plug in what we know.

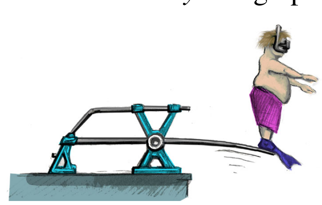
$$m = k(\frac{T}{2\pi})^2 = 800\frac{N}{m}(\frac{\pi\text{s}}{2\pi})^2 = 200\text{kg}$$

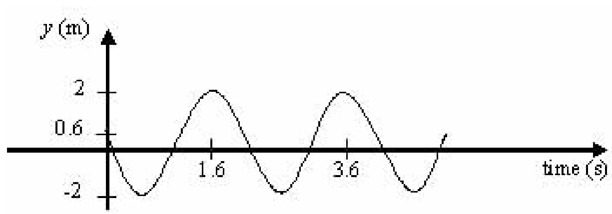
d) To get the sinusoidal equation we must first decide whether it is a cosine graph or a sine graph. Then we must find the amplitude (A), vertical shift (D), horizontal shift (C), and period (B). Cosine is easier in this case so we will work with it instead of sine. As we can see from the graph, the amplitude is 2, the vertical shift is 0, and the horizontal shift is $-.4$. We solved for the period already. Therefore, we can write the sinusoidal equation of this graph.

$$AcosB(x-C)+D = 2cos\pi(x+.4)$$

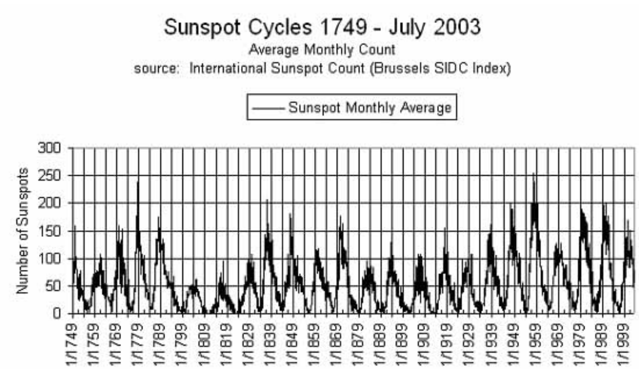
11.5 SHM Problem Set

1. While treading water, you notice a buoy way out towards the horizon. The buoy is bobbing up and down in simple harmonic motion. You only see the buoy at the most upward part of its cycle. You see the buoy appear 10 times over the course of one minute.
   a. What is the force that is leading to simple harmonic motion?
   b. What is the period (T) and frequency (f) of its cycle? Use the proper units.
2. A rope can be considered as a spring with a very high spring constant k, so high, in fact, that you don't notice the rope stretch at all before it "pulls back."
   a. What is the k of a rope that stretches by 1 mm when a 100 kg weight hangs from it?
   b. If a boy of 50 kg hangs from the rope, how far will it stretch?
   c. If the boy kicks himself up a bit, and then is bouncing up and down ever so slightly, what is his frequency of oscillation? Would he notice this oscillation? If so, how? If not, why not?
3. If a 5.0 kg mass attached to a spring oscillates 4.0 times every second, what is the spring constant k of the spring?
4. A horizontal spring attached to the wall is attached to a block of wood on the other end. All this is sitting on a frictionless surface. The spring is compressed 0.3 m. Due to the compression there is 5.0 J of energy stored in the spring. The spring is then released. The block of wood experiences a maximum speed of 25 m/s.
   a. Find the value of the spring constant.
   b. Find the mass of the block of wood.
   c. What is the equation that describes the position of the mass?
   d. What is the equation that describes the speed of the mass?
   e. Draw three complete cycles of the block's oscillatory motion on an x vs. t graph.
5. Give some everyday examples of simple harmonic motion.
6. Why doesn't the period of a pendulum depend on the mass of the pendulum weight? Shouldn't a heavier weight feel a stronger force of gravity?
7. The pitch of a Middle C note on a piano is 263 Hz. This means when you hear this note, the hairs in your ears wiggle back and forth at this frequency.
   a. What is the period of oscillation for your ear hairs?
   b. What is the period of oscillation of the struck wire within the piano?
8. The effective k of the diving board shown here is 800 N/m. (We say effective because it bends in the direction of motion instead of stretching like a spring, but otherwise behaves the same.) A pudgy diver is bouncing up and down at the end of the diving board, as shown. The y vs t graph is shown below.





a. What is the distance between the lowest and highest points of oscillation?
b. What is the $y-$position of the diver at times $t = 0$ s, $t = 2$ s, and $t = 4.6$ s?
c. Estimate the man's period of oscillation.
d. What is the diver's mass?
e. Write the sinusoidal equation of motion for the diver.



9. The Sun tends to have dark, Earth-sized spots on its surface due to kinks in its magnetic field. The number of visible spots varies over the course of years. Use the graph of the sunspot cycle above to answer the following questions. (Note that this is real data from our sun, so it doesn't look like a *perfect* sine wave. What you need to do is estimate the *best* sine wave that fits this data.)
   a. Estimate the period T in years.
   b. When do we expect the next "solar maximum?"
10. The pendulum of a small clock is 1.553 cm long. How many times does it go back and forth before the second hand goes forward one second?
11. On the moon, how long must a pendulum be if the period of one cycle is one second? The acceleration of gravity on the moon is one sixth that of Earth.
12. A spider of 0.5 g walks to the middle of her web. The web sinks by 1.0 mm due to her weight. You may assume the mass of the web is negligible.
   a. If a small burst of wind sets her in motion, with what frequency will she oscillate?
   b. How many times will she go up and down in one s? In 20 s?
   c. How long is each cycle?
   d. Draw the x vs t graph of three cycles, assuming the spider is at its highest point in the cycle at $t = 0$ s.
13. A mass on a spring on a frictionless horizontal surface undergoes SHM. The spring constant is 550 N/m and the mass is 0.400 kg. The initial amplitude is 0.300 m.
   a. At the point of release find:
      a. the potential energy
      b. the horizontal force on the mass
      c. the acceleration as it is released
   b. As the mass reaches the equilibrium point find:
      a. the speed of the mass
      b. the horizontal force on the mass
      c. the acceleration of the mass
   c. At a point .150 m from the equilibrium point find:
      a. the potential and kinetic energy
      b. the speed of the mass
      c. the force on the mass
      d. the acceleration of the mass
   d. Find the period and frequency of the harmonic motion.

14. A pendulum with a string of 0.750 m and a mass of 0.250 kg is given an initial amplitude by pulling it upward until it is at a height of 0.100 m more than when it hung vertically. This is point P. When it is allowed to swing it passes through point Q at a height of .050 m above the equilibrium position, the latter of which is called point R.
    a. Draw a diagram of this pendulum motion and at points P, Q, and R draw velocity and acceleration vectors. If they are zero, state that also.
    b. At point P calculate the potential energy.
    c. At point R calculate the speed of the mass.
    d. At point Q calculate the speed of the mass.
    e. If the string were to break at points P, Q, and R draw the path the mass would take until it hits ground for each point.
    f. Find the tension in the string at point P.
    g. Find the tension in the string at point R.
    h. Find the period of harmonic motion.

Answers to Selected Problems

1. a. Buoyant force and gravity b. $T = 6$ s, $f = 1/6$ Hz
2. a. 9.8×10^5 N/m b. 0.5 mm c. 22 Hz, no,
3. 3.2×10^3 N/m
4. a. 110 N/m d. $v(t) = (25)\cos(83t)$
5. .
6. .
7. a. 0.0038 s b. 0.0038 s
8. .
9. .
10. 4 times
11. 0.04 m
12. a. 16 Hz b. 16 complete cycles but 32 times up and down, 315 complete cycles but 630 times up and down c. 0.063 s
13. a. 24.8 J, 165 N, 413 m/s^2 b. 11.1m/s, 0, 0 c. 6.2 J, 18.6 J, 9.49 m/s, 82.5 N, 206 m/s^2 d. .169 sec, 5.9 Hz
14. b. .245 J c. 1.40m/s d. 1.00 m/s f. 2.82 N g. 3.10 N

CHAPTER 12 Wave Motion and Sound Version 2

Chapter Outline

12.1 The Big Idea

Objects in motion that return to the same position after a fixed period of time are said to be in *harmonic motion.* Objects in harmonic motion have the ability to transfer some of their energy over large distances. They do so by creating waves in a medium. Imagine pushing up and down on the surface of a bathtub filled with water. Water acts as the medium that carries energy from your hand to the edges of the bathtub. *Waves transfer energy over a distance without direct contact with the initial source.* Since waves are disturbances in an existing medium, they are considered phenomena and not actual objects.

12.2 Key Concepts

- A *medium* is the substance through which the wave travels. For example, water acts as the medium for ocean waves, while air molecules act as the medium for sound waves.
- When a wave passes through a medium, the medium is only temporarily disturbed. When an ocean wave travels from one side of the Mediterranean Sea to the other, no actual water molecules move this great distance. Only the *disturbance* propagates (moves) through the medium.
- An object oscillating with frequency f will create waves which oscillate with the same frequency f.
- The speed v and wavelength λ of a wave depend on the nature of the medium through which the wave travels.
- There are two main types of waves we will consider: longitudinal waves and transverse waves.
- In longitudinal waves, the vibrations of the medium are in the *same direction* as the wave motion. A classic example is a wave traveling down a line of standing dominoes: each domino will fall in the same direction as the motion of the wave. A more physical example is a sound wave. For sound waves, high and low pressure zones move both forward and backward as the wave moves through them.
- In transverse waves, the vibrations of the medium are *perpendicular* to the direction of motion. A classic example is a wave created in a long rope: the wave travels from one end of the rope to the other, but the actual rope moves up and down, and not from left to right as the wave does.
- Water waves act as a mix of longitudinal and transverse waves. A typical water molecule pretty much moves in a circle when a wave passes through it.
- Most wave media act like a series of connected oscillators. For instance, a rope can be thought of as a large number of masses (molecules) connected by springs (intermolecular forces). The speed of a wave through connected harmonic oscillators depends on the distance between them, the spring constant, and the mass. In this way, we can model wave media using the principles of simple harmonic motion.
- The speed of a wave on a string depends on the material the string is made of, as well as the tension in the string. This is why tightening a string on your violin or guitar will change the sound it produces.
- The speed of a sound wave in air depends subtly on pressure, density, and temperature, but is about 343m/s at room temperature.
- Resonance is a phenomenon that occurs when something that has a natural frequency of vibration (pendulum, guitar, glass, etc.) is shaken or pushed at a frequency that is equal to its natural frequency of vibration. A dramatic example of resonance is the Tacoma Narrows Bridge, which collapsed shortly after being built as a result of wind vibrating it at its natural frequency.

12.3 Key Equations

Basics

$T = \frac{1}{f}$	[1] Wave period
$v = \lambda f$	[2] Wave velocity

Common Frequencies

$f_{\text{beat}} = \|f_1 - f_2\|$	[3] Beat frequency from waves of frequency f_1 and f_2
$f_n = \frac{nv}{2L}$ | integer n	[4] Standing waves restricted or unrestricted at both ends
$f_n = \frac{nv}{4L}$ | odd integer n	[5] Standing waves restricted at one end

The Doppler Effect

When a source of a wave is moving towards you, the apparent frequency of the wave you detect is higher than that emitted. For instance, if an ambulance approaches you while blaring a siren at 500 Hz, the sound you hear will be slightly higher. This familiar phenomenon is known as the **Doppler Effect.** The opposite occurs for when the source is moving away. If the observer is moving also, it is the relative velocities that matter. There is a difference in the quantitative effect, depending on who is moving. (See the formulas below.) Note that these equations are for sound waves only. While the effect is similar for light and electromagnetic waves the formulas are not exactly the same as for sound.

Doppler Shifts:

$$f_o = f\frac{v+v_o}{v-v_s} f_o \text{ (observed frequency) is shifted up when source and observer moving closer}$$

$$f_o = f\frac{v-v_o}{v+v_s} f_o \text{ (observed frequency) is shifted down when source and observer moving apart, where}$$

v is the speed of sound, v_s is the speed of the source, and v_o is the speed of the observer

12.4 Key Applications

- *Constructive interference* occurs when two waves combine to create a larger wave. This occurs when the peaks of two waves line up.
- *Destructive interference* occurs when two waves combine and cancel each other out. This occurs when a peak in one wave lines up with a trough in the other wave.
- When waves of two different frequencies interfere, a phenomenon known as *beats* occur. The frequency of a beat is the difference of the two frequencies.
- When a wave meets a barrier, it is reflected and travels back the way it came. If the reflected wave interferes with the initial wave in such a way that the nodes do not move, a *standing wave* can be created. The types of standing waves that can form depend strongly on the speed of the wave and the size of the region in which it is traveling.
- A typical standing wave is shown below. This is the motion of a simple jump-rope. *Nodes* are the places where the rope doesn't move at all; *antinodes* occur where the motion is greatest.

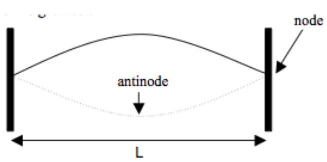


FIGURE 12.1
For this wave, the wavelength is . Since , the frequency of oscillation is .

- **Higher harmonics** can also form. Note that each end, where the rope is attached, must always be a node. Below is an example of a rope in a 5^{th} harmonic standing wave.

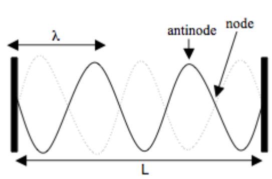


FIGURE 12.2
In general, the frequency of oscillation is , where n is the number of antinodes. The thick, dotted lines represent the wave *envelope*: these are the upper and lower limits to the motion of the string.

- Importantly, each of the above standing wave examples can also apply to sound waves in a closed tube, electromagnetic waves in a wire or fiber optic cable, and so on. In other words,the standing wave examples can apply to *any* kind of wave, as long as nodes are forced at both ends by whatever is containing/reflecting the wave back on itself.
- If a node is forced at one end, but an antinode is forced at the other end, then a different spectrum of standing waves is produced. For instance, the fundamental standing sound wave produced in a tube closed at one end is shown below. In this case, the amplitude of the standing wave is referring to the magnitude of the air pressure variations.

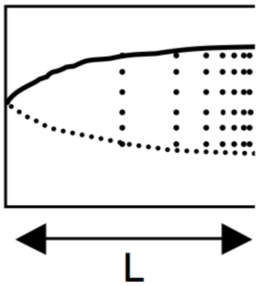


FIGURE 12.3

For this standing wave, the wavelength is . Since , the frequency of oscillation is . In general, the frequency of oscillation is , where is always odd.

12.5 Examples

Wavelength, Frequency, and Velocity

Question: A 120cm long string vibrates as a standing wave with four antinodes. The wave speed on the string is 48m/s. Find the wavelength and frequency of the standing wave.

Answer: We will solve for the wavelength first. The wavelength will then allow us to solve for the frequency.

Since there are 4 antinodes, there are two complete waves (see diagram above). Therefore, one complete wavelength must equal to half the length of the string

$$\lambda = \frac{120\text{cm}}{2} = 60\text{cm}$$

Now that we have both the velocity and the wavelength of the wave we can solve for the frequency using the equation

$$v = f\lambda$$

Now we simply solve for the frequency and then plug in the known values.

$$v = f\lambda = \frac{v}{\lambda}$$

Before we plug in the known values, we need to convert the wavelength from centimeters to meters. This will allow us to cancel the units.

$$\lambda = 60\text{cm} \times \frac{1\text{m}}{100\text{cm}} = .6\text{m}$$

Now we can solve for the frequency.

$$f = \frac{v}{\lambda} = \frac{48\text{m/s}}{.6\text{m}} = 80\text{Hz}$$

The Doppler Effect

Question: How fast would a student playing an A note (440Hz) have to move towards you in order for you to hear a G note (784Hz)?

Answer: We will use the Doppler shift equation for when the objects are getting closer together and solve for the speed of the student (the source).

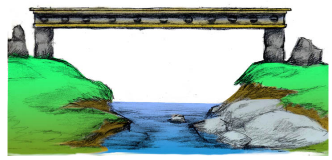
$$f_o = f\left(\frac{v+v_o}{v-v_s}\right) \Rightarrow f_o \times (v-v_s) = f \times (v+v_o) \Rightarrow vf_o - v_s f_o = f \times (v+v_o) \Rightarrow v_s = -\left(\frac{f \times (v+v_o) - vf_o}{f_o}\right)$$

Now we plug in the known values to solve for the velocity of the student.

$$v_s = -\left(\frac{f \times (v+v_o) - vf_o}{f_o}\right) = -\left(\frac{440\text{Hz} \times (343\text{m/s} + 0\text{m/s}) - 343\text{m/s} \times 784\text{Hz}}{784\text{Hz}}\right) = 151\text{m/s}$$

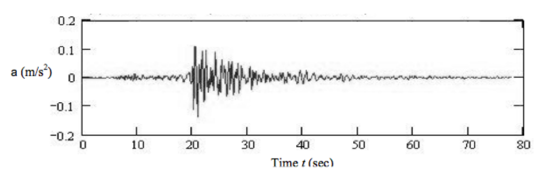
12.6 Wave Motion Problem Set

1. A violin string vibrates, when struck, as a standing wave with a frequency of 260 Hz. When you place your finger on the same string so that its length is reduced to $2/3$ of its original length, what is its new vibration frequency?
2. The simple bridge shown here oscillated up and down pretty violently four times every second as a result of an earthquake.



   a. What was the frequency of the shaking in Hz?
   b. Why was the bridge oscillating so violently?
   c. Calculate two other frequencies that would be considered "dangerous" for the bridge.
   d. What could you do to make the bridge safer?

3. The speed of water waves in deep oceans is proportional to the wavelength, which is why tsunamis, with their huge wavelengths, move at incredible speeds. The speed of water waves in shallow water is proportional to depth, which is why the waves "break" at shore. Draw a sketch which accurately portrays these concepts.
4. Below you will find actual measurements of acceleration as observed by a seismometer during a relatively small earthquake. An earthquake can be thought of as a whole bunch of different waves all piled up on top of each other.



   a. Estimate (using a ruler) the approximate period of oscillation T of the minor aftershock which occurs around $t = 40$ sec.
   b. Convert your estimated period from part (a) into a frequency f in Hz.
   c. Suppose a wave with frequency f from part (b) is traveling through concrete as a result of the earthquake. What is the wavelength λ of that wave in meters? (The speed of sound in concrete is approximately $v = 3200$ m/s.)

5. The length of the western section of the Bay Bridge is 2.7 km.

   a. Draw a side-view of the western section of the Bay Bridge and identify all the 'nodes' in the bridge.
   b. Assume that the bridge is concrete (the speed of sound in concrete is 3200 m/s). What is the lowest frequency of vibration for the bridge? (You can assume that the towers are equally spaced, and that the central support is equidistant from both middle towers. The best way to approach this problem is by drawing in a wave that "works.")

c. What might happen if an earthquake occurs that shakes the bridge at precisely this frequency?

6. The speed of sound v in air is approximately $331.4 \text{ m/s} + 0.6T$, where T is the temperature of the air in Celsius. The speed of light c is $300,000$ km/sec, which means it travels from one place to another on Earth more or less instantaneously. Let's say on a cool night (air temperature $10°$ Celsius) you see lightning flash and then hear the thunder rumble five seconds later. How far away (in *km*) did the lightning strike?
7. Human beings can hear sound waves in the frequency range 20 Hz – 20 kHz. Assuming a speed of sound of 343 m/s, answer the following questions.

   a. What is the shortest wavelength the human ear can hear?
   b. What is the longest wavelength the human ear can hear?

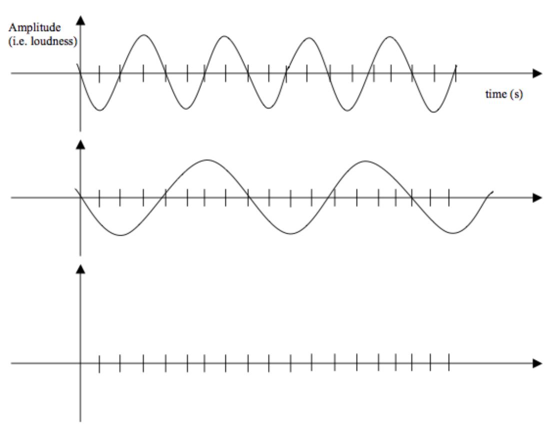
8. The speed of light c is $300,000$ km/sec.

   a. What is the frequency in Hz of a wave of red light ($\lambda = 0.7$? 10^{-6} m)?
   b. What is the period T of oscillation (in seconds) of an electron that is bouncing up and down in response to the passage of a packet of red light? Is the electron moving rapidly or slowly?

9. Radio signals are carried by electromagnetic waves (i.e. light waves). The radio waves from San Francisco radio station KMEL (106.1 FM) have a frequency of 106.1 MHz. When these waves reach your antenna, your radio converts the motions of the electrons in the antenna back into sound.

   a. What is the wavelength of the signal from KMEL?
   b. What is the wavelength of a signal from KPOO (89.5 FM)?
   c. If your antenna were broken off so that it was only 2 cm long, how would this affect your reception?

10. Add together the two sound waves shown below and sketch the resultant wave. Be as exact as possible – using a ruler to line up the waves will help. The two waves have different frequencies, but the same amplitude. What is the frequency of the resultant wave? How will the resultant wave sound different?



11. Aborigines, the native people of Australia, play an instrument called the didgeridoo like the one shown above. The didgeridoo produces a low pitch sound and is possibly the world's oldest instrument. The one shown above is about 1.3 m long and open at both ends.

    a. Knowing that when a tube is open at both ends there must be an antinode at both ends, draw the first 3 harmonics for this instrument.
    b. Derive a generic formula for the frequency of the nth standing wave mode for the didgeridoo, as was done for the string tied at both ends and for the tube open at one end.

12. Reread the difference between *transverse* and *longitudinal* waves. For each of the following types of waves, tell what type it is and why. (Include a sketch for each.)

    - sound waves
    - water waves in the wake of a boat
    - a vibrating string on a guitar
    - a swinging jump rope
    - the vibrating surface of a drum
    - the "wave" done by spectators at a sports event
    - slowly moving traffic jams

13. At the Sunday drum circle in Golden Gate Park, an Indian princess is striking her drum at a frequency of 2 Hz. You would like to hit your drum at another frequency, so that the sound of your drum and the sound of her drum "beat" together at a frequency of 0.1 Hz. What frequencies could you choose?
14. A guitar string is 0.70 m long and is tuned to play an E note $(f = 330\text{ Hz})$. How far from the end of this string must your finger be placed to play an A note $(f = 440\text{ Hz})$?
15. Piano strings are struck by a hammer and vibrate at frequencies that depend on the length of the string. A certain piano string is 1.10 m long and has a wave speed of 80 m/s. Draw sketches of each of the four lowest frequency nodes. Then, calculate their wavelengths and frequencies of vibration.
16. Suppose you are blowing into a soda bottle that is 20 cm in length and closed at one end.

    a. Draw the wave pattern in the tube for the lowest four notes you can produce.
    b. What are the frequencies of these notes?

17. You are inspecting two long metal pipes. Each is the same length; however, the first pipe is open at one end, while the other pipe is closed at both ends.

    a. Compare the wavelengths and frequencies for the fundamental tones of the standing sound waves in each of the two pipes.
    b. The temperature in the room rises. What happens to the frequency and wavelength for the open-on-one-end pipe?

18. A train, moving at some speed lower than the speed of sound, is equipped with a gun. The gun shoots a bullet forward at precisely the speed of sound, relative to the train. An observer watches some distance down the tracks, with the bullet headed towards him. Will the observer hear the sound of the bullet being fired before being struck by the bullet? Explain.
19. A 120 cm long string vibrates as a standing wave with four antinodes. The wave speed on the string is 48 m/s. Find the wavelength and frequency of the standing wave.
20. A tuning fork that produces a frequency of 375 Hz is held over pipe open on both ends. The bottom end of the pipe is adjustable so that the length of the tube can be set to whatever you please.

    a. What is the shortest length the tube can be and still produce a standing wave at that frequency?
    b. The second shortest length?
    c. The one after that?

21. The speed of sound in hydrogen gas at room temperature is 1270 m/s. Your flute plays notes of 600, 750, and 800 Hz when played in a room filled with normal air. What notes would the flute play in a room filled with hydrogen gas?
22. A friend plays an A note (440 Hz) on her flute while hurtling toward you in her imaginary space craft at a speed of 40 m/s. What frequency do you hear just before she rams into you?
23. How fast would a student playing an A note (440 Hz) have to move towards you in order for you to hear a G note (784 Hz)?
24. Students are doing an experiment to determine the speed of sound in air. The hold a tuning fork above a large empty graduated cylinder and try to create resonance. The air column in the graduated cylinder can be adjusted by putting water in it. At a certain point for each tuning fork a clear resonance point is heard. The students adjust the water finely to get the peak resonance then carefully measure the air column from water to

top of air column. (The assumption is that the tuning fork itself creates an anti-node and the water creates a node.) The following data table was developed:

TABLE 12.1:

Frequency OF tuning fork (Hz)	Length of air column (cm)	Wavelength (m)	Speed of sound (m/s)
184	46		
328	26		
384	22		
512	16		
1024	24		

Answers to Selected Problems

1. 390 Hz
2. a. 4 Hz b. It was being driven near its resonant frequency. c. 8 Hz, 12 Hz d. (Note that earthquakes rarely shake at more than 6 Hz).
3. .
4. .
5. a. 7 nodes including the 2 at the ends b. 3.6 Hz
6. 1.7 km
7. a. 1.7 cm b. 17 m
8. a. 4.3×10^{14} Hz b. 2.3×10^{-15} s$-$ man that electron is moving fast
9. a. 2.828 m b. 3.352 m c. $L = 1/4\,\lambda$ so it would be difficult to receive the longer wavelengths.
10. Very low frequency
11. b. Same as closed at both ends
12. .
13. 1.9 Hz or 2.1 Hz.
14. 0.53 m
15. 2.2 m, 36 Hz; 1.1 m, 73 Hz; 0.733 m, 110 Hz; 0.55 m, 146 Hz
16. 430 Hz; 1.3×10^3 Hz; 2.1×10^3 Hz; 3.0×10^3 Hz;
17. a. The tube closed at one end will have a longer fundamental wavelength and a lower frequency. b. If the temperature increases the wavelength will not change, but the frequency will increase accordingly.
18. struck by bullet first.
19. 80 Hz; 0.6 m
20. a. 0.457 m b. 0.914 m c. 1.37 m
21. 2230 Hz; 2780 Hz; 2970 Hz
22. 498 Hz
23. 150 m/s

12.7 References

1. CK-12 Foundation. . Public Domain
2. CK-12 Foundation. . Public Domain
3. CK-12 Foundation. . Public Domain

CHAPTER 13 Electricity Version 2

Chapter Outline

13.1 The Big Idea

Conservation of charge is the fourth of the five conservation laws in physics. There are two types of charge: positive and negative; the law of conservation of electric charge states that the net charge of the universe remains constant. As with momentum and energy, in any closed system charge can be transferred from one body to another and can move within the system but cannot leave the system.

Electromagnetism is associated with charge and is a fundamental force of nature, like gravity (which for us is associated with mass). If charges are static, the only manifestation of electromagnetism is the *Coulomb electric force.* In the same way the gravitational force that an object exerts upon other objects, and that other objects exert on it, depends on the amount of mass it possesses, ***the Coulomb electric force*** *that an object experiences depends on the amount of* ***electric charge*** the object possesses. Like gravity, the Coulomb electric force decreases with the square of the distance. The Coulomb electric force is responsible for many of the forces we discussed previously: the normal force, contact forces such as friction, and so on — *all* of these forces arise in the mutual attraction and repulsion of charged particles.

Although the law determining the magnitude of the Coulomb electric force has the same form as the law of gravity, the electric constant is 20 orders of magnitude greater than the gravitational constant. That is why electricity normally dominates gravity at the atomic and molecular level. Since there is only one type of mass but two opposite types of electric charge, gravity will dominate in large bodies *unless there is a separation of charge.*

13.2 Electric Forces and Fields

13.3 The Coulomb Force Law

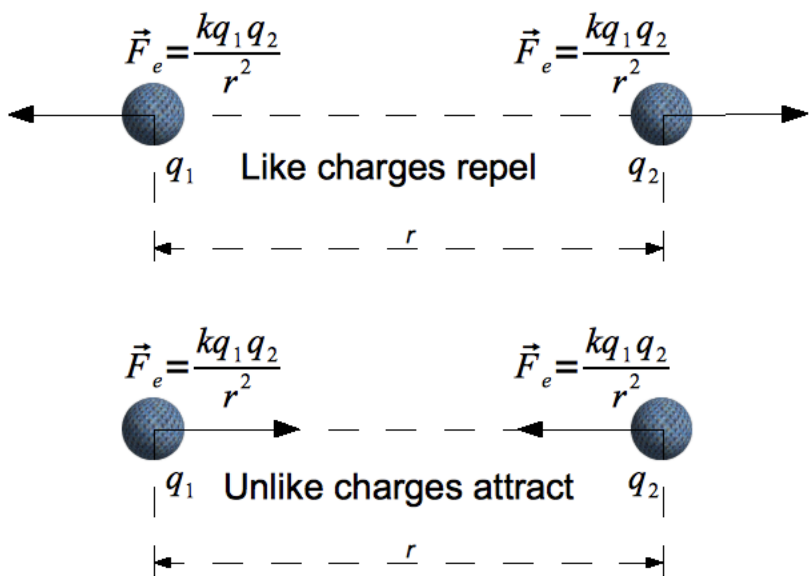
The Coulomb Force Law states that any two charged particles (q_1, q_2) — with charge measured in units of Coulombs — at a distance r from each other will experience a force of repulsion or attraction along the line joining them equal to:

$$\vec{F}_e = \frac{kq_1q_2}{r^2} \qquad \text{The Coulomb Force [1]}$$

Where

$$k = 8.987 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2}. \qquad \text{The Electric Constant}$$

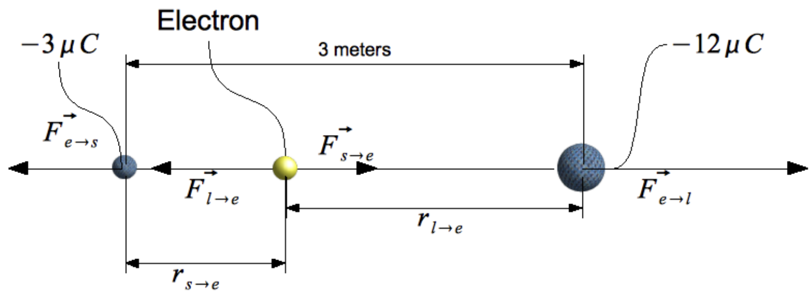
This looks a lot like the Law of Universal Gravitation, which deals with attraction between objects with mass. The big difference is that while any two masses experience mutual *attraction*, two charges can either attract or repel each other, depending on whether the signs of their charges are alike:



Like gravitational (and all other) forces, Coulomb forces add as vectors. Thus to find the force on a charge from an arrangement of charges, one needs to find the vector sum of the force from each charge in the arrangement.

Example 1

Question: Two negatively charged spheres (one with -12μC; the other with -3μC) are 3m apart. Where could you place an electron so that it will be suspended in space between them with a net force of zero (for this problem we will ignore the force of repulsion between the two charges because they are held in place)?



Answer: Consider the diagram above; here $r_{s\to e}$ is the distance between the electron and the small charge, while $\vec{F}_{s\to e}$ is the force the electron feels due to it. For the electron to be balanced in between the two charges, the forces of repulsion caused by the two charges on the electron would have to be balanced. To do this, we will set the equation for the force exerted by two charges on each other equal and solve for a distance ratio. We will denote the difference between the charges through the subscripts "s" for the smaller charge, "e" for the electron, and "l" for the larger charge.

$$\frac{kq_s q_e}{r^2_{s\to e}} = \frac{kq_l q_e}{r^2_{e\to l}}$$

Now we can cancel. The charge of the electron cancels. The constant k also cancels. We can then replace the large and small charges with the numbers. This leaves us with the distances. We can then manipulate the equation to produce a ratio of the distances.

$$\frac{-3\mu\text{C}}{r^2_{s\to e}} = \frac{-3\mu\text{C}}{r^2_{e\to l}} \Rightarrow \frac{r^2_{s\to e}}{r^2_{e\to l}} = \frac{-12\mu\text{C}}{-12\mu\text{C}} \Rightarrow \frac{r_{s\to e}}{r_{e\to l}} = \sqrt{\frac{1\mu\text{C}}{4\mu\text{C}}} = \frac{1}{2}$$

Given this ratio, we know that the electron is twice as far from the large charge (-12μC) as from the small charge (-12μC). Given that the distance between the small and large charges is 3m, we can determine that the electron must be located 2m away from the large charge and 1m away from the smaller charge.

Electric Fields and Electric Forces

Gravity and the Coulomb force have a nice property in common: they can be represented by **fields**. Fields are a kind of bookkeeping tool used to keep track of forces. Take the electromagnetic force between two charges given above:

$$\vec{F}_e = \frac{kq_1q_2}{r^2}$$

If we are interested in the acceleration of the first charge only — due to the force from the second charge — we can rewrite this force as the product of q_1 and $\frac{kq_2}{r^2}$. The first part of this product only depends on properties of the object we're interested in (the first charge), and the second part can be thought of as a property of the point in space where that object is.

In fact, the quantity $\frac{kq_2}{r^2}$ captures everything about the electromagnetic force on any object possible at a distance **r** from q_2. If we had replaced q_1 with a different charge, q_3, we would simply multiply q_3 by $\frac{kq_2}{r^2}$ to find the new force on the new charge. Such a quantity, $\frac{kq_2}{r^2}$ here, is referred to as the electric field from charge q_2 at that point: in this case, it is the electric field due to a single charge:

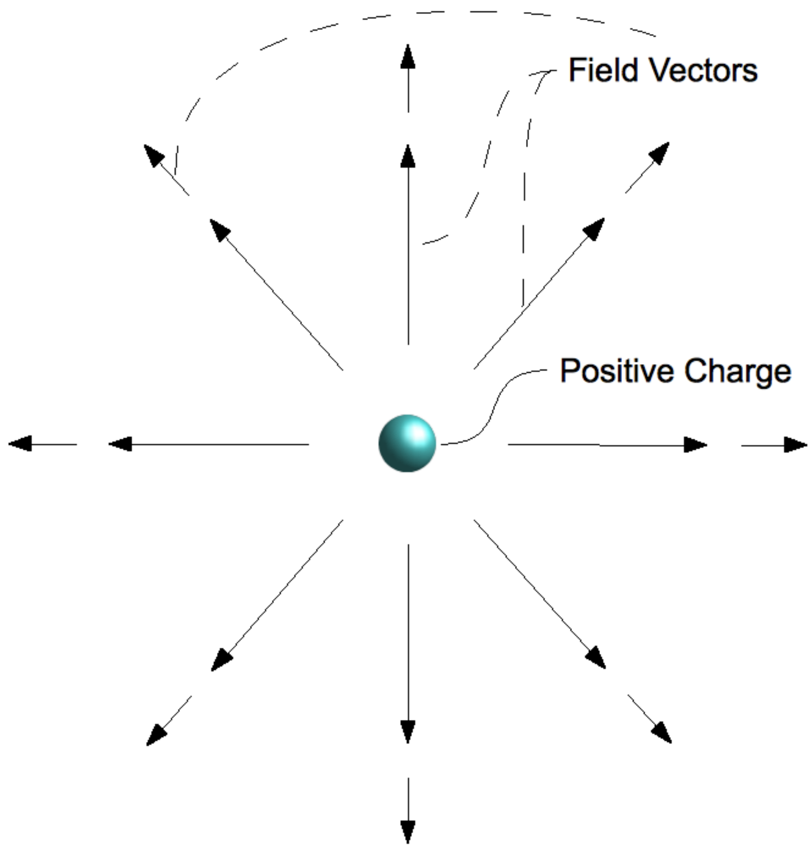
$$\vec{E}_f = \frac{kq}{r^2} \qquad \text{[2] Electric field due to point charge } q, \text{ distance } r \text{ away}$$

The electric field is a vector quantity, and points in the direction that a force felt by a positive charge at that point would. If we are given the electric field at some point, it is just a matter of multiplication — as illustrated above — to find the force any charge q_0 would feel at that point:

$$\underbrace{\vec{F}_e}_{\text{Force on charge } q_0} = \underbrace{\vec{E}_f}_{\text{Field}} \times \underbrace{q_0}_{\text{Charge}} \qquad \text{Force on charge } q_0 \text{ in an electric field}$$

Note that this is true for *all* electric fields, not just those from point charges. In general, the **electric field** at a point *is the force a* ***positive test charge of magnitude 1*** *would feel at that point*. Any other charge will feel a force along the same line (but possibly in the other direction) in proportion to its magnitude. In other words, the electric field can be though of as "force per unit charge".

In the case given above, the field was due to a single charge. Such a field is shown in the figure below. Notice that this a field due to a positive charge, since the field arrows are pointing outward. The field produced by a point charge will be radially symmetric i.e., the strength of the field only depends on the distance, r, from the charge, not the direction; the lengths of the arrows represent the strength of the field.



Example 2

Question: Calculate the electric field a distance of 4.0mm away from a -2.0μC charge. Then, calculate the force on a -8.0μC charge placed at this point.

Answer: To calculate the electric field we will use the equation

$$E = \frac{kq}{r^2}$$

Before we solve for the electric field by plugging in the values, we convert all of the values to the same units.

$$4.0\text{mm} \times \frac{1\text{m}}{1000\text{mm}} = .004\text{m}$$
$$-2.0\mu\text{C} \times \frac{1\text{C}}{1000000\mu\text{C}} = -2.0 \times 10^{-6}\text{C}$$

Now that we have consistent units we can solve the problem.

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9\text{Nm}^2/\text{C}^2 \times -2.0 \times 10^{-6}\text{C}}{(.004\text{m})^2} = -1.1 \times 10^9\text{N/C}$$

To solve for the force at the point we will use the equation

$$F = Eq$$

We already know all of the values so all we have to do is convert all of the values to the same units and then plug in the values.

$$-8.0\mu\text{C} \times \frac{1\text{C}}{1000000\mu\text{C}} = -8.0 \times 10^{-6}\text{C}$$
$$F = Eq = -8.0 \times 10^{-6}\text{C} \times -1.1 \times 10^9\text{N/C} = 9000\text{N}$$

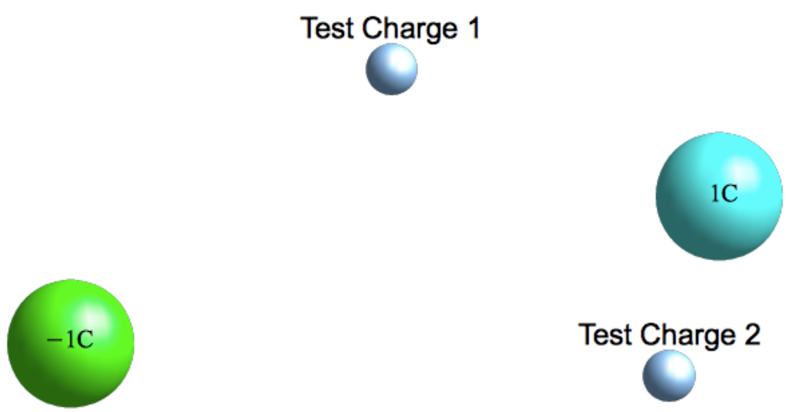
13.4 Fields Due to Several Charges

To find the field at a point due to an arrangement of charges — in fact, all electric fields arise due to *some* arrangement of charges — we find the vector sum of the individual fields:

$$\vec{E_{net}} = \sum_i \vec{E}_i \qquad \text{[3] Net Electric Field}$$

Electric fields are used more frequently than gravitational ones because there are two types of charge, which makes electric force and potential energy harder to keep track of than their gravitational counterparts. To apply this approach to gravitational forces — that is, to find a net gravitational field — one needs to repeat the steps above, with mass in place of charge (left for the reader).

Example 3



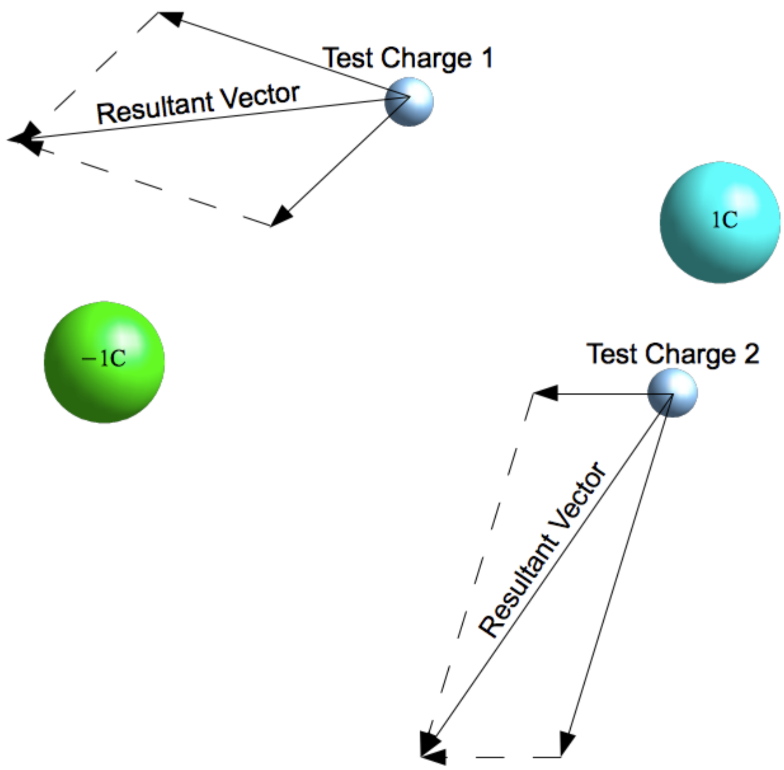
Question: For the diagram above, draw (qualitatively) the electric field vectors at the points shown using the test charge method.

Answer: We will start with Test Charge 1. Test charges are always positive and have magnitude 1. Therefore we know that the test charge will want to go toward the negative charge and away from the positive charge (like charges repel and opposite charges attract). The strength of the electric field felt by the test charge is dependent on the inverse square of the distance of the charges as shown by the equation

$$E = \frac{kq}{r^2}$$

The farther away from the source of the field, the weaker the field becomes. Therefore Test Charge 1 will experience a stronger field from the 1C charge. Because the distance from Test Charge 1 to the -1C is only slightly longer than the distance from Test Charge 1 to the 1C charge, the vectors will be similar in length. Once we have determined the relative scale of each vector, we can add them using the parallelogram method. The resultant vector is the electric field at that point.

Finding the electric field at Test Charge 2 will involve all of the same steps. First we must determine which charge Test Charge 2 is closer to. Like Test Charge 1, Test Charge 2 is closer to the 1C charge. However, Test Charge 2 is drastically closer whereas Test Charge 1 was only slightly closer. Therefore, the electric field that Test Charge 2 experiences as a result of the 1C charge will be strong, thus resulting in a longer arrow. The distance between the $-1C$ and Test Charge 2 is large and therefore the electric field experienced by Test Charge 2 as a result of the $-1C$ charge will be small. The resultant vectors will look something like this.



13.5 Electric Potential

Like gravity, the electric force can do work and has a potential energy associated with it. But like we use fields to keep track of electromagnetic forces, we use **electric potential**, or **voltage** to keep track of electric potential energy. So instead of looking for the potential energy of specific objects, we define it in terms of properties of the space where the objects are.

The ***electric potential difference**, or **voltage difference** (often just called voltage) between two points (A and B) in the presence of an electric field is defined as the work it would take to move a **positive test charge of magnitude 1** from the first point to the second against the electric force provided by the field*. For any other charge q, then, the relationship between potential difference and work will be:

$$\Delta V_{AB} = \frac{W_{AB}}{q} \quad \text{[4] Electric Potential}$$

Rearranging, we obtain:

$$\underbrace{W}_{\text{Work}} = \underbrace{\Delta V_{AB}}_{\text{Potential Difference}} \times \underbrace{q}_{\text{Charge}}$$

The potential of electric forces to do work corresponds to electric potential energy:

$$\Delta U_{E,AB} = q\Delta V_{AB} \quad \text{[5] Potential energy change due to voltage change}$$

The energy that the object gains or loses when traveling through a potential difference is supplied (or absorbed) by the electric field — there is nothing else there. Therefore, it follows that *electric fields contain energy*.

To summarize: just as an electric field denotes force per unit charge, so electric potential differences represent potential energy differences *per unit charge*. A useful mnemonic is to consider a cell phone: the battery has the potential to do work for you, but it needs a charge! Actually, the analogy there is much more rigorous than it at first seems; we'll see why in the chapter on current. Since voltage is a quantity proportional to work it is a scalar, and can be positive or negative.

13.6 Electric Field of a Parallel Plate Capacitor

Suppose we have two parallel metal plates set a distance d from one another. We place a positive charge on one of the plates and a negative charge on the other. In this configuration, there will be a uniform electric field between the plates pointing from, and normal to, the plate carrying the positive charge. The magnitude of this field is given by

$$E = \frac{V}{d}$$

where V is the potential difference (voltage) between the two plates.

The amount of charge, Q, held by each plate is given by

$$Q = CV$$

where again V is the voltage difference between the plates and C is the capacitance of the plate configuration. Capacitance can be thought of as the capacity a device has for storing charge . In the parallel plate case the capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}$$

where A is the area of the plates, d is the distance between the plates, and ε_0 is the permittivity of free space whose value is $8.84 \times 10^{-12} C/V \cdot m$.

The electric field between the capacitor plates stores energy. The electric potential energy, U_C, stored in the capacitor is given by

$$U_C = \frac{1}{2}CV^2$$

Where does this energy come from? Recall, that in our preliminary discussion of electric forces we assert that "like charges repel one another". To build our initial configuration we had to place an excess of positive and negative charges, respectively, on each of the metal plates. Forcing these charges together on the plate had to overcome the mutual repulsion that the charges experience; this takes work. The energy used in moving the charges onto the plates gets stored in the field between the plates. It is in this way that the capacitor can be thought of as an energy storage device. This property will become more important when we study capacitors in the context of electric circuits in the next chapter.

Note: Many home-electronic circuits include capacitors; for this reason, it can be dangerous to mess around with old electronic components, as the capacitors may be charged even if the unit is unplugged. For example, old computer monitors (not flat screens) and TVs have capacitors that hold dangerous amounts of charge hours after the power is turned off.

More on Electric and Gravitational Potential

There are several differences between our approach to gravity and electricity that could cause confusion. First, with gravity we usually used the concept of "energy", rather than "energy difference". Second, we spoke about it in absolute terms, rather than "per unit mass".

To address the first issue: when we dealt with gravitational potential energy we had to set some reference height $h = 0$ where it is equal to $mg \times 0 = 0$. In this sense, we were really talking about potential energy differences rather than absolute levels then also: at any point, we compared the gravitational potential energy of an object to the energy it would have had at the reference level $h = 0$. When we used the formula

$$U_g = mg\Delta h$$

we implicitly set the initial point as the zero: no free lunch! For the same reason, we use the concept of electric potential difference between two points — or we need to set the potential at some point to 0, and use it as a reference. This is not as easy in this case though; usually a point very far away ("infinitely" far) is considered to have 0 electric potential.

Regarding the second issue: in the chapter on potential energy, we could have gravitational potential difference between two points at different heights as $g\Delta h$. This, of course, is the work required to move an object of mass one a height Δh against gravity. To find the work required for any other mass, we would multiply this by its magnitude. In other words,

$$\underbrace{W}_{\text{Work}} = \underbrace{m}_{\text{Mass}} \times \underbrace{g\Delta h}_{\text{Potential Difference}}$$

Which is exactly analogous to the equation above.

13.7 Summary of Relationships

The following table recaps the relationships discussed in this chapter.

TABLE 13.1: Relationship between "per Coulomb" and absolute quantities.

Property of Object.	Property of Space.	Combine Into:
Charge (Coulombs)	Field* (Newtons/Coulomb)	Force (Newtons)
Charge (Coulombs)	Potential* (Joules/C)	Potential Energy (Joules)

- An advanced note: for a certain class of forces called *conservative* forces e.g., gravity and the electromagnetic force, a specific potential distribution corresponds to a unique field. Conversely, a field corresponds to a unique potential distribution up to an additive constant. Remember though, it's relative potential between points not absolute potential that is physically relevant. In effect the field corresponds to a unique potential. In particular, we see that in the case of conservative forces the scalar potential (one degree of freedom per point) carries all information needed to determine the vector electric field (three degrees of freedom per point. The potential formulation is even more useful than it at first seems.

13.8 Key Concepts

- Electrons have negative charge and protons have positive charge. The magnitude of the charge is the same for both: $e = 1.6 \times 10^{-19}$C.
- In any closed system, electric charge is conserved. The total electric charge of the universe does not change. Therefore, electric charge can only be transferred not lost from one body to another.
- Normally, electric charge is transferred when electrons leave the outer orbits of the atoms of one body (leaving it positively charged) and move to the surface of another body (causing the new surface to gain a negative net charge). *In a **plasma**, the fourth state of matter, all electrons are stripped from the atoms, leaving positively charged ions and free electrons.*
- Similarly-charged objects have a repulsive force between them. Oppositely charged objects have an attractive force between them.
- The value of the electric field tells you the force that a charged object would feel *if* it entered this field. Electric field lines tell you the direction a positive charge would go if it were placed in the field.
- Electric potential is measured in units of Volts (V) thus electric potential is often referred to as voltage. Electric potential is the source of the electric potential energy.
- Positive charges move towards lower electric potential; negative charges move toward higher electric potential. If you are familiar with a contour map then positive charges go 'downhill' and negative charges go 'uphill'.
- Faraday cages consist of a metal box. All of your sensitive electronics are encased in a metal box called a Faraday cage. The Faraday cage protects everything inside from external electric fields. Basically the electrons in the metal box move around to cancel out the electric field, thus preventing it from coming inside the box and thus preventing movement of charge and possible blown out electronic chips. Cars and airplanes, being enclosed in metal, are also Faraday cages and thus the safest place to be in a lightning storm.

MEDIA

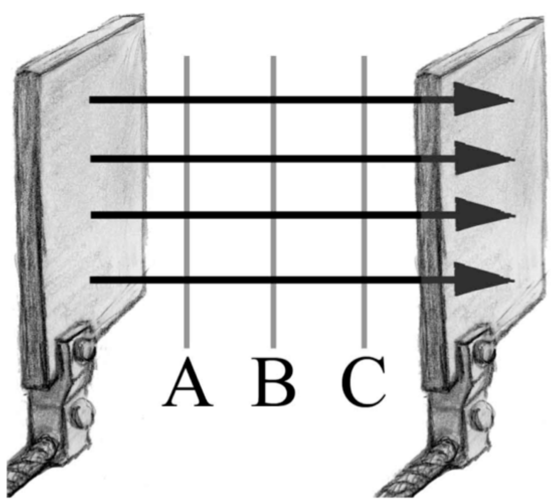
Click image to the left for more content.

13.9 Key Applications

- In problems that ask for excess negative or positive charge, remember that each electron has one unit of the fundamental charge $e = 1.6 \times 10^{-19}$C.
- To find the speed of a particle after it traverses a voltage difference, use the equation for the conservation of energy: $q\Delta V = \frac{1}{2}mv^2$
- Force and electric field are vectors. Use your vector math skills (i.e. keep the x and y directions separate) when solving two-dimensional problems.

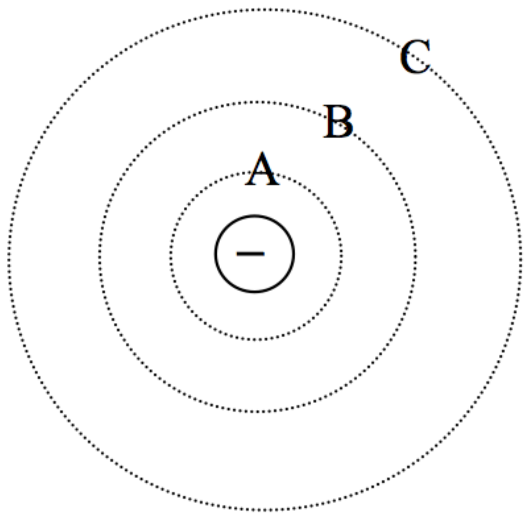
13.10 Electricity Problem Set

1. After sliding your feet across the rug, you touch the sink faucet and get shocked. Explain what is happening.
2. What is the net charge of the universe? Of your toaster?
3. As you slide your feet along the carpet, you pick up a net charge of $+4$ mC. Which of the following is true?
   a. You have an excess of 2.5×10^{16} electrons
   b. You have an excess of 2.5×10^{19} electrons
   c. You have an excess of 2.5×10^{16} protons
   d. You have an excess of 2.5×10^{19} protons
4. You rub a glass rod with a piece of fur. If the rod now has a charge of $-0.6\ \mu C$, how many electrons have been added to the rod?
   a. 3.75×10^{18}
   b. 3.75×10^{12}
   c. 6000
   d. 6.00×10^{12}
   e. Not enough information
5. What is the direction of the electric field if an electron initially at rest begins to move in the North direction as a result of the field?
   a. North
   b. East
   c. West
   d. South
   e. Not enough information

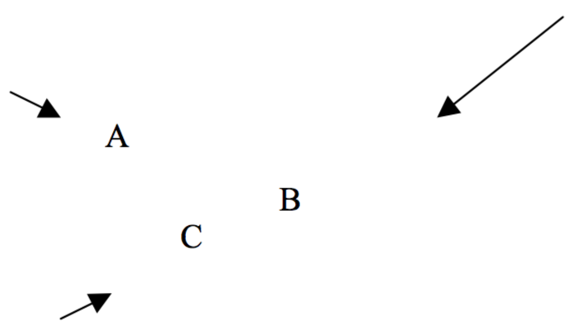


6. Two metal plates have gained excess electrons in differing amounts through the application of rabbit fur. The arrows indicate the direction of the electric field which has resulted. Three electric potential lines, labeled A, B, and C are shown. Order them from the greatest electric potential to the least.
   a. A, B, C
   b. C, B, A

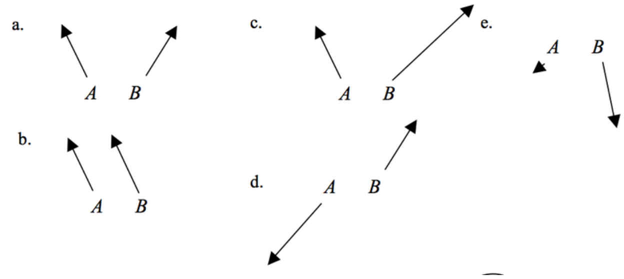
c. B, A, C
d. B, C, A
e. $A = B = C\dots$ theyÕre all at the same potential



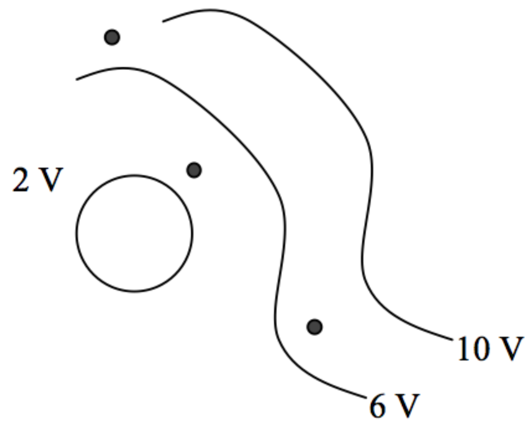
7. The diagram to the right shows a negatively charged electron. Order the electric potential lines from greatest to least.
   a. A, B, C
   b. C, B, A
   c. B, A, C
   d. B, C, A
   e. $A = B = C\dots$ theyÕre all at the same electric potential
8. The three arrows shown here represent the magnitudes of the electric field and the directions at the tail end of each arrow. Consider the distribution of charges which would lead to this arrangement of electric fields. Which of the following is most likely to be the case here?



   a. A positive charge is located at point A
   b. A negative charge is located at point B
   c. A positive charge is located at point B and a negative charge is located at point C
   d. A positive charge is located at point A and a negative charge is located at point C
   e. Both answers a) and b) are possible
9. Particles A and B are both positively charged. The arrows shown indicate the direction of the *forces* acting on them due to an applied electric field (not shown in the picture). For each, draw in the electric field lines that would best match the observed force.

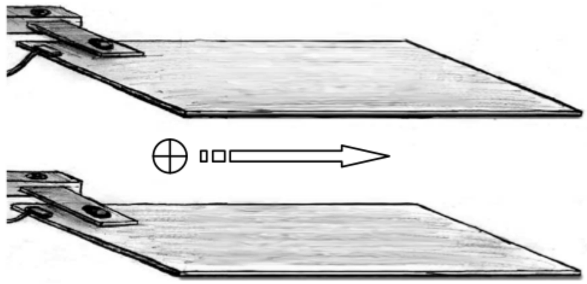


10. To the right are the electric potential lines for a certain arrangement of charges. Draw the direction of the electric field for all the black dots.



11. A suspended pith ball possessing $+10\ \mu\text{C}$ of charge is placed 0.02 m away from a metal plate possessing $-6\ \mu\text{C}$ of charge.
    a. Are these objects attracted or repulsed?
    b. What is the force on the negatively charged object?
    c. What is the force on the positively charged object?
12. Calculate the electric field a distance of 4.0 mm away from a $-2.0\ \mu\text{C}$ charge. Then, calculate the force on a $-8.0\ \mu\text{C}$ charge placed at this point.
13. Consider the hydrogen atom. Does the electron orbit the proton due to the force of gravity or the electric force? Calculate both forces and compare them. (You may need to look up the properties of the hydrogen atom to complete this problem.)
14. As a great magic trick, you will float your little sister in the air using the force of opposing electric charges. If your sister has 40 kg of mass and you wish to float her 0.5 m in the air, how much charge do you need to deposit both on her and on a metal plate directly below her? Assume an equal amount of charge on both the plate and your sister.
15. Copy the arrangement of charges below. Draw the electric field from the -2 C charge in one color and the electric field from the $+2$ C charge in a different color. Be sure to indicate the directions with arrows. Now take the individual electric field vectors, add them together, and draw the resultant vector. This is the electric field created by the two charges together.

16. A proton traveling to the right moves in between the two large plates. A vertical electric field, pointing downwards with magnitude 3.0 N/C, is produced by the plates.

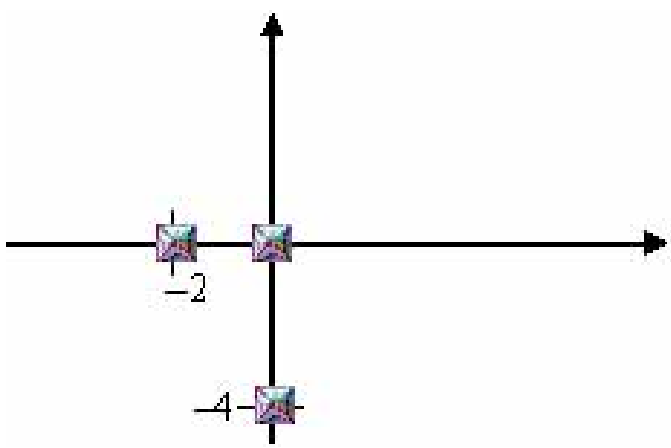


a. What is the direction of the force on the proton?
b. Draw the electric field lines on the diagram.
c. If the electric field is 3.0 N/C, what is the acceleration of the proton in the region of the plates?
d. Pretend the force of gravity doesnÕt exist; then sketch the path of the proton.
e. We take this whole setup to another planet. If the proton travels straight through the apparatus without deflecting, what is the acceleration of gravity on this planet?

17. A molecule shown by the square object shown below contains an excess of 100 electrons.
    a. What is the direction of the electric field at point A, 2.0×10^{-9} m away?

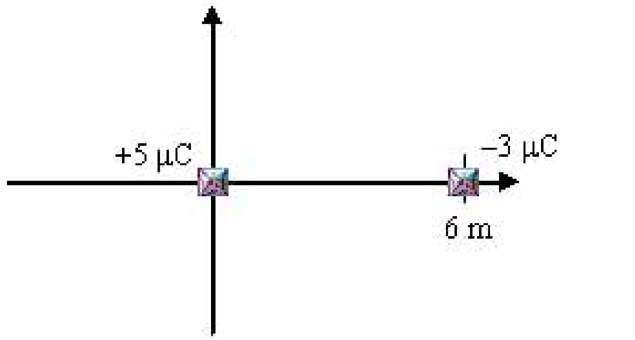
    b. What is the value of the electric field at point A?
    c. A molecule of charge 8.0 μC is placed at point A. What are the magnitude and direction of the force acting on this molecule?

18. Two negatively charged spheres (one with -12 μC; the other with -3 μC) are 3 m apart. Where could you place an electron so that it will be suspended in space between them with zero net force? *For problems 19, 20, and 21 assume* 3$-$*significant digit accuracy in all numbers and coordinates. All charges are positive.*
19. Find the direction and magnitude of the force on the charge at the origin (see picture). The object at the origin has a charge of 8 μC, the object at coordinates (-2 m, 0) has a charge of 12 μC, and the object at coordinates $(0, -4$ m) has a charge of 44 μC. All distance units are in meters.

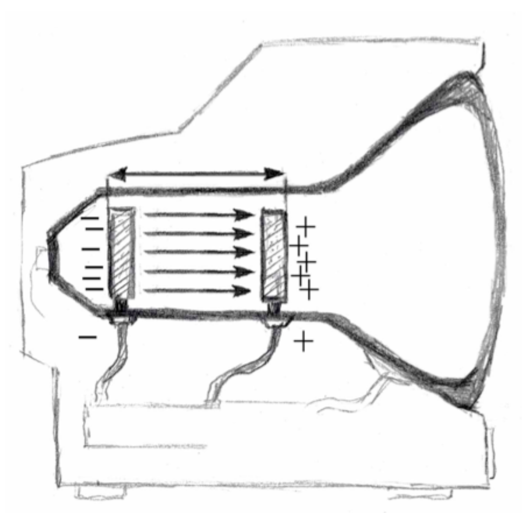


20. A 2 C charge is located at the origin and a 7 C charge is located at $(0, 6$ m$)$. Find the electric field at the coordinate $(10$ m$, 0)$. It may help to draw a sketch.
21. A metal sphere with a net charge of $+5$ μC and a mass of 400 g is placed at the origin and held fixed there.
    a. Find the electric potential at the coordinate $(6$ m$, 0)$.

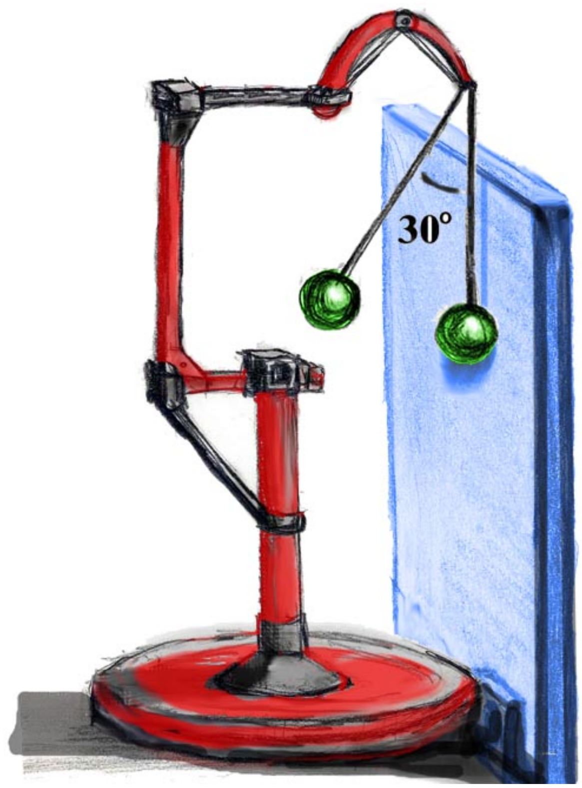
b. If another metal sphere of $-3\ \mu$C charge and mass of 20 g is placed at the coordinate $(6\text{ m}, 0)$ and left free to move, what will its speed be just before it collides with the metal sphere at the origin?



22. Collisions of electrons with the surface of your television set give rise to the images you see. How are the electrons accelerated to high speed? Consider the following: two metal plates (The right hand one has small holes allow electrons to pass through to the surface of the screen.), separated by 30 cm, have a uniform electric field between them of 400 N/C.



a. Find the force on an electron located at a point midway between the plates
b. Find the voltage difference between the two plates
c. Find the change in electric potential energy of the electron when it travels from the back plate to the front plate
d. Find the speed of the electron just before striking the front plate (the screen of your TV)



23. Two pith balls of equal and like charges are repulsed from each other as shown in the figure below. They both have a mass of 2 g and are separated by 30°. One is hanging freely from a 0.5 m string, while the other, also hanging from a 0.5 m string, is stuck like putty to the wall.
    a. Draw the free body diagram for the hanging pith ball
    b. Find the distance between the leftmost pith ball and the wall (this will involve working a geometry problem)
    c. Find the tension in the string (Hint: use $y-$direction force balance)
    d. Find the amount of charge on the pith balls (Hint: use $x-$direction force balance)

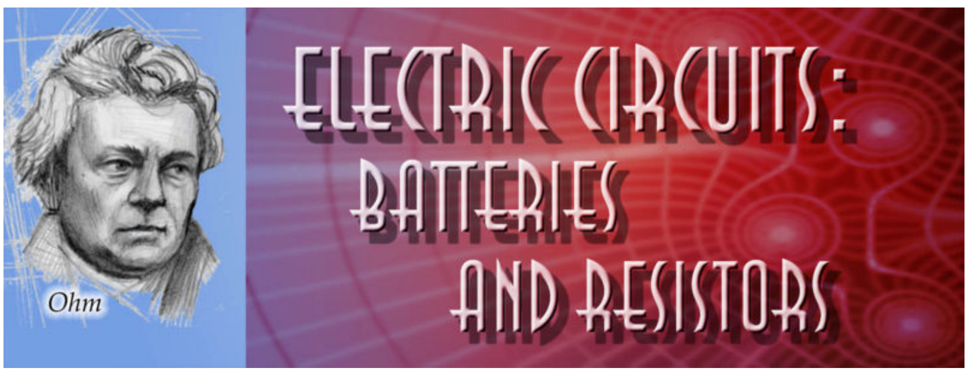
Answers to Selected Problems

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. b. 1350 N c. 1350 N
12. a. 1.1×10^{9} N/C b. 9000 N
13. $F_g = 1.0 \times 10^{-47}$ N and $F_e = 2.3 \times 10^{-8}$ N. The electric force is 39 orders of magnitudes bigger.
14. $1.0 \times 10^{-4} C$
15. .

16. a. down b. Up $16c, 5.5 \times 10^{11}$ m/s^2 e. 2.9×10^8 m/s^2
17. a. Toward the object b. 3.6×10^4 N/C to the left with a force of 2.8×10^{-7} N
18. Twice as close to the smaller charge, so 2 m from 12μC charge and 1 m from 3μC charge.
19. 0.293 N and at 42.5°
20. 624 N/C and at an angle of $-22.4°$ from the $+x-$ *axis.*
21. a. 7500V b. 1.5 m/s
22. a. 6.4×10^{-17} N b. 1300V c. 2.1×10^{-16} J d. 2.2×10^7 m/s
23. b. 0.25m c. $F_T = 0.022$ N d. 0.37μC

CHAPTER 14 Electric Circuits Version 2

Chapter Outline



14.1 The Big Ideas

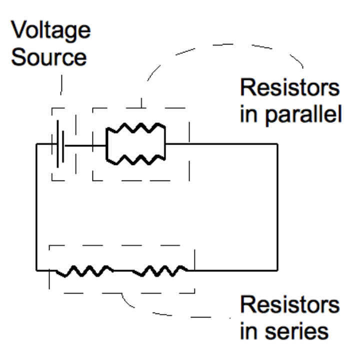
In the last chapter, we looked at static configurations of charges. In general, problems with moving charges are very difficult to solve; the field that deals with these is called **electrodynamics**. In this chapter, we consider how charge can flow through conducting wires connecting opposite ends of a battery. Such a setup, called a **circuit** usually involves a current, a voltage source, and resistors.

Conductors have an effectively infinite supply of charge, so when they are placed in an electric field, a **separation of charge** occurs. A battery with a potential drop across the ends creates such an electric field; when the ends are connected with a wire, charge will flow across it. The term given to the flow of charge is **electric current**, and it is measured in Amperes (A) — Coulombs per second. Current is analogous to a river of water, but instead of water flowing, charge does.

Voltage is the electrical energy density (energy divided by charge) and differences in this density (voltage) cause electric current. **Batteries** often provide a voltage difference across the ends of a circuit, but other **voltage sources** exist. If current is a river, differences in voltage can be thought of as pipes coming out of a water dam at different heights. The lower the pipe along the dam wall, the larger the water pressure, thus the higher the voltage.

Resistance is the amount a device in the wire resists the flow of current by converting electrical energy into other forms of energy. A resistor could be a light bulb, transferring electrical energy into heat and light or an electric motor that converts electric energy into mechanical energy. The difference in energy density across a resistor or other electrical device is called *voltage drop.* Resistance is analogous to rocks and other objects that impede the flow of water, transforming the water's kinetic energy into heat, sound, and other forms of energy through contact forces.

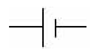
This is what a typical circuit looks like:



14.2 Circuit Basics

We use the following symbols to represent the quantities discussed above:

TABLE 14.1: Circuit Quantities

Name	Symbol	Electrical Symbol	Units	Everyday device
Voltage	V		Volts (V)	Battery, the plugs in your house, etc.
Current (flow of charge)	$I = \frac{\Delta q}{\Delta t}$		Amps (A) $\mathrm{A} = \mathrm{C}/\mathrm{s}$	Whatever you plug into your wall sockets draws current
Resistance	R		Ohm (Ω)	Light bulb, Toaster, etc.

Loop and Junction Rules for Voltage/Current

In electric *circuits* (closed loops of wire with resistors and constant voltage sources) energy must be conserved. It follows that changes in energy density, the algebraic sum of voltage drops and voltage sources, around any closed loop will equal zero.

In an electric *junction* or *node* there is more than one possible path for current to flow. For charge to be conserved at a junction the current into the junction must equal the current out of the junction.

Ohm's Law

The resistance of an object — described above — is quantified as the ratio of the voltage drop across it to the amount of current that will flow from that voltage. Note that the current depends on the voltage drop; here, as above we use V instead of ΔV to mean voltage difference (both are accepted ways).

$$R = \frac{V}{I} \qquad \text{[1] Definition of Resistance}$$

Generally, more current flowing through a resistor will cause a higher voltage drop. For the special class of resistors discussed in this class this ratio is a constant — the current flowing across these resistors will rise at the same rate as the voltage difference supplied. In other words, the *resistance does not depend on the amount of current that flows through the resistor, or the voltage drop across it*. This relationship is known as **Ohm's Law**, for a constant current it is usually written as

$$V = IR \qquad \text{[2] Ohm's Law}$$

For sources of constant voltage, such as batteries, the current varies with resistance:

$$I = \frac{V}{R} \qquad [3]$$

Unlike equation [1], where R varied with current, we can use equation [2] to find the current, voltage drop, or resistance across a resistor when given the other two. When dealing with a constant current, use equation [2], but when dealing with a battery driven circuit (a source of constant voltage difference), use equation [3].

Power

Power is the rate at which energy is lost by a system. The units of power are Watts (W), which equal Joules per second(1W = 1J/s). Therefore, a 60 W light bulb releases 60 Joules of energy every second.

The equation used to calculate the **power** dissipated in a circuit or across a resistor is:

$$P = IV = \underbrace{I^2R}_{\text{Since } V=IR} \qquad \text{[4] Power Dissipated Through a Voltage Drop}$$

As with OhmÕs Law, one must be careful not to mix apples with oranges. If you want the power of the entire circuit, then you multiply the *total* voltage of the power source by the *total* current coming out of the power source. If you want the power dissipated (i.e. released) by a light bulb, then you multiply the *voltage drop* across the light bulb by the *current going through that light bulb*.

Resistors in Series and in Parallel

Sometimes, circuits have many resistors in various geometrical arrangements. When **in series**, two or more resistors are connected end to end (See picture). In this case the resistors receive the same current, but since they can have different resistances they may have different voltage drops across them. Analogously, there may be more rocks at some points in the river than in others, but if there is only one way for the river to flow, the current has to be the same at all points. It follows from Ohm's law that

$$R_{total} = R_1 + R_2 + R_3 + \ldots \qquad \text{[5] Resistors in Series}$$

Since the total resistance will *increase* with each resistor added in series, adding resistors in series will cause the less current to flow at a set voltage (according to Ohm's Law for constant voltage sources, [3]).

When two or more resistors are connected together at both ends, they are said to be "in parallel" (see picture). There are many rivers (the river splits into streams), so all resistors receive different amounts of current. But since they all connect the same points on the circuit, the voltage drops across them have to be equal. The rule for combining resistors in parallel is

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad \text{[6] Resistors in Parallel}$$

Since the total resistance will *decrease* with the number or resistors in parallel, adding resistors in parallel to existing ones will cause more current to flow through a circuit.

Ohm's Law and Total Quantities

Ohm's law is the main relationship for electric circuits but it is often misused. In order to calculate the voltage drop across a light bulb — or any single resistor — use the formula:

$$V_{lightbulb} = I_{lightbulb}R_{lightbulb}$$

.

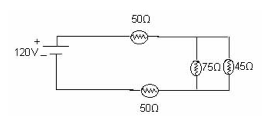
Using the formulas and the rules above, a circuit with any number of resistors (and voltage sources) can be modeled as a circuit with just one voltage source and one resistor, for which Ohm's Law also holds. For the *total* current flowing out of the power source, you need the *total* resistance of the circuit and the *total* current:

$$V_{total} = I_{total}R_{total} \qquad [7]$$

This concept is illustrated below.

Example on Circuit Math

Question: Analyze the diagram below.



a) Find the current going out of the power supply.

b) How many Joules per second of energy is the power supply giving out?

c) Find the current going through the 75Ω light bulb.

d) Order the light bulbs in terms of brightness.

e) If the light bulbs were all wired in parallel, order them in terms of brightness.

Answer

a) To find the current going out of the power supply, we will use equation [7], $V_{total} = I_{total}R_{total}$. We already have the total voltage drop (120V) and we are trying to solve for the current. This means that we need to know the total resistance before we can find the current.

To solve for the resistance we will apply the two rules for resistors (series and parallel) because we have both in are circuit. First, we must combine the two resistors in parallel so that we can treat the entire circuit as a series. According to equation [6],

$$\frac{1}{R}$$

par$==1\frac{}{75\Omega+\frac{1}{45\Omega}=\frac{120}{3375}\Omega}$

Because $\frac{120}{3375}\Omega$ is equal to $\frac{1}{R}$

par $,weneedtoflipthefraction to get$ R_total .

Now that we have three resistors in series (the two in parallel can be counted as one), we simply need to add them to get the total resistance.

R

total=$50\Omega+\frac{3375}{120}\Omega+50\Omega=128.125\Omega$

We can now solve for the current by using equation [7]

$$I_{total}=\frac{V_{total}}{R_{total}}=\frac{120\text{V}}{128.125\Omega}=.94\text{A}$$

This is total net current through the circuit; it's also the current across the 50Ω resistors, but not the ones connected in parallel.

b) To find the power dissipated, we will use equation [4].

$$P=I\times\Delta\text{V}=.94\text{A}\times120\text{V}=112\text{W}$$

c) To find the current going through the 75Ω light bulb, we must realize that a total of .94A goes through the two light bulbs in parallel; according to the junction rule above, the currents across the two light bulbs must add to this. Now we must find what proportion of the current the 75Ω light bulb gets. To do this, we use our knowledge that resistors in parallel have the same voltage drop and Ohm's Law:

$$V_{75}=I_{75}\times75\Omega=V_{45}=I_{45}\times45\Omega$$
$$I_{45}+I_{75}=.94\text{A, so } I_{45}=.94\text{A}-I_{75}$$

Therefore,

$$I_{75}\times75\Omega=(.94\text{A}-I_{75})\times45\Omega$$

Solving for the needed current, we find:

$$I_{75}\approx.35\text{A}$$

d) The brightness is determined by the power dissipated. More power means a brighter lightbulb. According to equation [4], the power dissipated by a resistor can be written as I^2R. Since we know the resistance of and current across every resistor, we can simply calculate this quantity for each one. The order is 50Ω's, 45Ω, then 75Ω. The 50Ω is brighter than the 45Ω because the 50Ω gets considerably more current.

e) When the bulbs are wired entirely in parallel, the voltage drops across them will be the same. Since $P=IV$, the way to determine the brightest bulb is to look at the currents across them, which will be inversely related with their resistances. So, the bulb with the lowest resistance will be the brightest, the one with the second lowest resistance will be second, and so on. Therefore the order is 45Ω, 50Ω, and finally 75Ω.

14.3 Capacitors in Circuits (Steady-State)

When a capacitor is placed in a circuit, current does not actually travel across it. Rather, equal and opposite charge begins to build up on opposite sides of the capacitor — mimicking a current — until the electric field in the capacitor creates a potential difference across it that balances the voltage drop across any parallel resistors or the voltage source itself (if there are no resistors in parallel with the capacitor). The ratio of charge on a capacitor to potential difference across it is called capacitance:

$$C = \frac{Q}{V}$$ [1] Definition of Capacitance

14.4 Capacitors in Series and in Parallel

Like resistors, combinations of capacitors in circuits can be combined into one 'effective' capacitor. The rules for combining them are reversed from resistors:

$$C_{\text{parallel}} = C_1 + C_2 + C_3 + \ldots$$ [5] Capacitors in parallel add like resistors in series

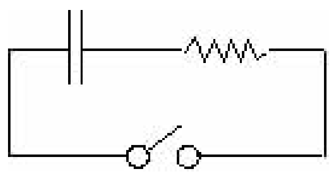
$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$ [6] Capacitors in series add like resistors in parallel

14.5 Charging and Discharging Capacitors (Transient)

When a capacitor is initially uncharged, it is very easy to increase the amount of charge on its plates. As charge builds, the charge present repels new charge with more and more force. Due to this effect, the charging of a capacitor follows a logarithmic curve. When a circuit passes current through a resistor into a capacitor, the capacitor eventually Òfills upÓ and *no more current flows across it*. A typical RC circuit is shown below; when the switch is closed, the capacitor discharges with an exponentially decreasing current:

$Q(t) = Q_0 e^{\frac{-t}{\tau}}$ [7] Discharge rate of a capacitor, where $\tau = RC$ and $Q_0 = V\dot{C}$

$Q(t) = Q_0(1 - e^{\frac{-t}{\tau}})$ [8] Charge rate of a capacitor, where $\tau = RC$ and $Q_0 = V\dot{C}$

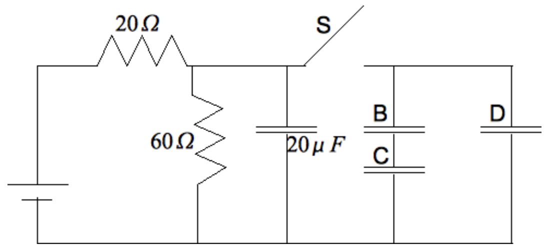


$I(t) = I_0(e^{\frac{-t}{\tau}}$ [9] Discharge and Charge rate for current, where $\tau = RC$ and$I_0 = \frac{V}{R}$

Charging a capacitor involves moving charges through a potential difference; as we saw in the electricity chapter, this results in electric potential energy being stored in the capacitor:

$U = \frac{1}{2}CV^2$ [10] Potential energy stored in a capacitor

14.6 Capacitor Example



Question: Consider the figure above when switch S is open. Note that the power supply is set to 24 V.

a) What is the voltage drop across the 20Ω resistor?

b) What current flows through the 60Ω resistor?

c) What is the voltage drop across the 20 microfarad capacitor?

d) What is the charge on the capacitor?

e} How much energy is stored in that capacitor?

Answer:

a) When the capacitor is charged — in the steady state — no current flows across it, and we basically have a circuit with two resistors in series. Accordingly, the voltage drop across the 20Ω resistor will be in the same proportion to the net voltage across the circuit as its resistance is to the net resistance (see circuits chapter):

$$\frac{20\Omega}{20\Omega + 60\Omega} = .25$$

This means that the voltage drop across the resistor is

$$.25 \times 24\text{V} = 6\text{V}$$

b) Since there is only one path for the current to take, its value is the same everywhere on the circuit; all we have to do is find the total current. This will then also be the amount of current that flows through the 60Ω resistor. We can find it by applying Ohm's Law for the circuit:

$$R_{total} = 60\Omega + 20\Omega = 80\Omega$$

Since we have the total resistance and the total voltage, we can solve for the total current using Ohm's law.

$$V = RI \Rightarrow I = \frac{V}{R} = \frac{24\text{V}}{80\Omega} = .3\text{A}$$

The current flowing through the resistor is therefore .3A.

c) We can find the voltage drop across the $20\mu F$ capacitor by realizing that the voltage drop across any parallel paths in a circuit have to be equal; otherwise the loop rule would be violated. Therefore, the voltage drop across the capacitor is the same as the voltage drop across the 60Ω resistor. We can find this analogously to how we found the voltage drop across the other resistor:

$$\frac{60\Omega}{20\Omega + 60\Omega} = .75$$
$$.75 \times 24\text{V} = 18\text{V}$$

d) To find the charge stored in the capacitor we will use the equation

$$Q = CV$$

First we must convert the capacitor into the correct units for the equation. Then we can substitute in the values and solve for the charge stored.

$$20\mu\text{F} \times \frac{1\text{F}}{1000000\mu F} = 2.0 \times 10^{-5}\text{F}$$
$$Q = CV = 2.0 \times 10^{-5}\text{F} \times 18\text{V} = 3.6 \times 10^{-4}\text{C}$$

e) The potential energy stored in a capacitor is

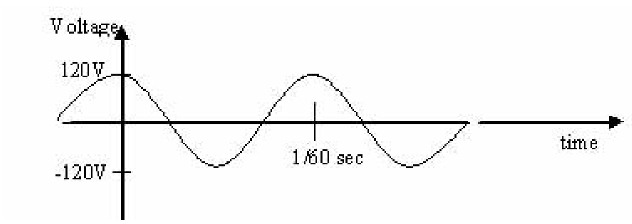
$$U = \frac{1}{2}CV^2$$

All we need to do is plug in the known values and get the potential energy.

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2.0 \times 10^{-5}\text{F} \times (18\text{V})^2 = 3.2 \times 10^{-3}$$

14.7 Key Terms

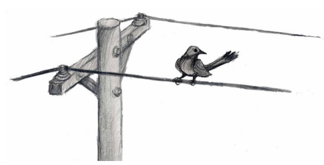
- **DC Power:** Voltage and current flow in one direction. Examples are batteries and the power supplies we use in class.
- **AC Power:** Voltage and current flow in alternate directions. In the US they reverse direction 60 times a second. (This is a more efficient way to transport electricity and electrical devices do not care which way it flows as long as current is flowing. Note: your TV and computer screen are actually flickering 60 times a second due to the alternating current that comes out of household plugs. Our eyesight does not work this fast, so we never notice it. However, if you film a TV or computer screen the effect is observable due to the mismatched frame rates of the camera and TV screen.) Electrical current coming out of your plug is an example.



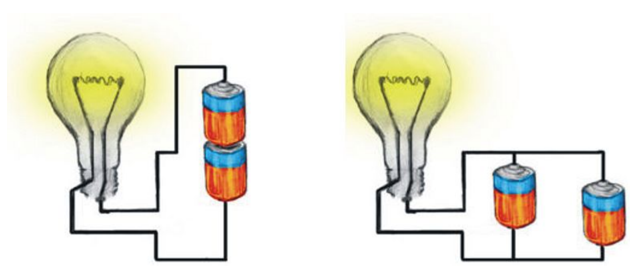
- **Ammeter:** A device that measures electric current. You must break the circuit to measure the current. Ammeters have very low resistance; therefore you must wire them in series.
- **Voltmeter:** A device that measures voltage. In order to measure a voltage difference between two points, place the probes down on the wires for the two points. Do not break the circuit. Volt meters have very high resistance; therefore you must wire them in parallel.
- **Voltage source:** A power source that produces fixed voltage regardless of what is hooked up to it. A battery is a real-life voltage source. A battery can be thought of as a perfect voltage source with a small resistor (called internal resistance) in series. The electric energy density produced by the chemistry of the battery is called **emf,** but the amount of voltage available from the battery is called **terminal voltage.** The terminal voltage equals the emf minus the voltage drop across the internal resistance (current of the external circuit times the internal resistance.)

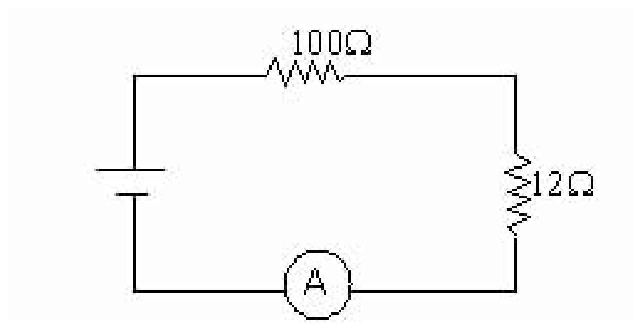
14.8 Electric Circuits Problem Set

1. The current in a wire is 4.5 A.
    a. How many coulombs per second are going through the wire?
    b. How many electrons per second are going through the wire?
2. A light bulb with resistance of 80 Ω is connected to a 9 V battery.
    a. What is the electric current going through it?
    b. What is the power (i.e. wattage) dissipated in this light bulb with the 9 V battery?
    c. How many electrons leave the battery every hour?
    d. How many Joules of energy leave the battery every hour?
3. A 120 V, 75 W light bulb is shining in your room and you ask yourselfÉ
    a. What is the resistance of the light bulb?
    b. How bright would it shine with a 9 V battery (i.e. what is its power output)?

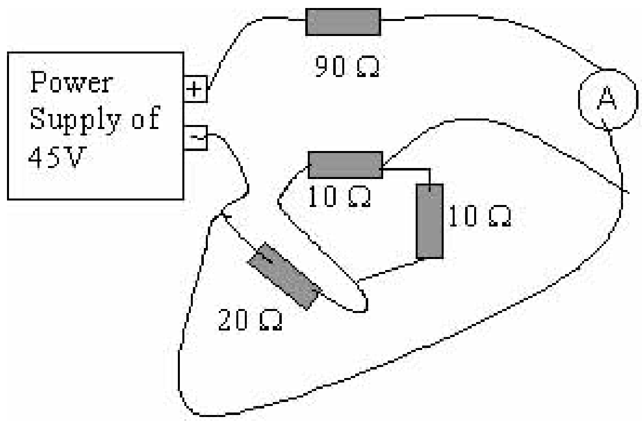


4. A bird is standing on an electric transmission line carrying 3000 A of current. A wire like this has about 3.0×10^{-5} Ω of resistance per meter. The birdÕs feet are 6 cm apart. The bird, itself, has a resistance of about 4×10^{5} Ω.
    a. What voltage does the bird feel?
    b. What current goes through the bird?
    c. What is the power dissipated by the bird?
    d. By how many Joules of energy does the bird heat up every hour?
5. Which light bulb will shine brighter? Which light bulb will shine for a longer amount of time? Draw the schematic diagram for both situations. Note that the objects on the right are batteries, not resistors.

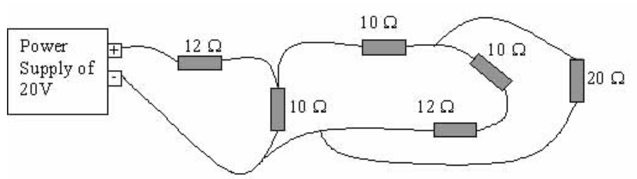




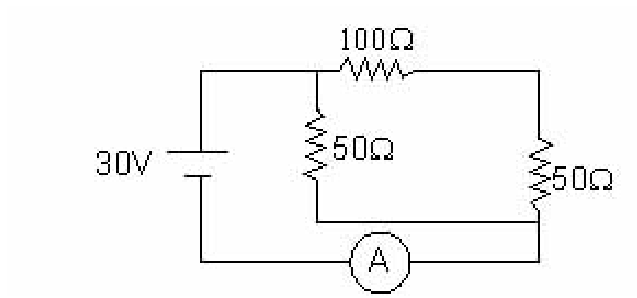
6. Regarding the circuit to the right.
    a. If the ammeter reads 2 A, what is the voltage?
    b. How many watts is the power supply supplying?
    c. How many watts are dissipated in each resistor?
7. Three 82 Ω resistors and one 12 Ω resistor are wired in parallel with a 9 V battery.
    a. Draw the schematic diagram.
    b. What is the total resistance of the circuit?
8. What will the ammeter read for the circuit shown to the right?



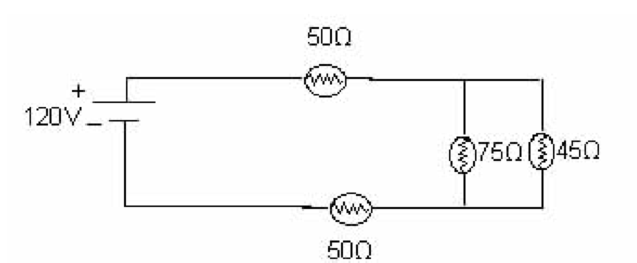
9. Draw the schematic of the following circuit.



10. What does the ammeter read and which resistor is dissipating the most power?



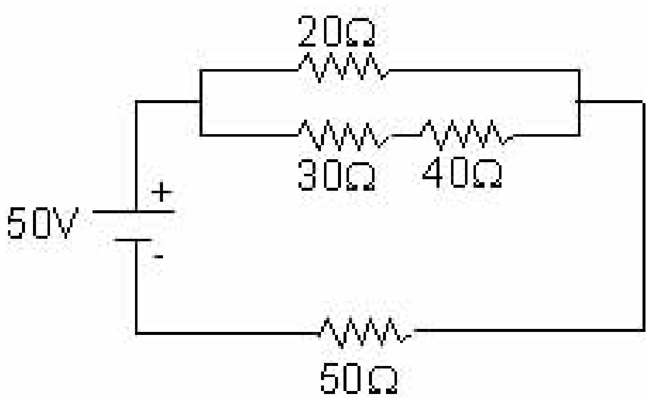
11. Analyze the circuit below.



    a. Find the current going out of the power supply

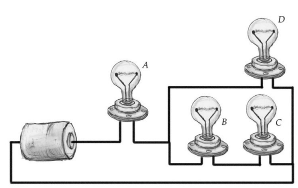
b. How many Joules per second of energy is the power supply giving out?
c. Find the current going through the 75 Ω light bulb.
d. Find the current going through the 50 Ω light bulbs (hint: it's the same, why?).
e. Order the light bulbs in terms of brightness
f. If they were all wired in parallel, order them in terms of brightness.

12. Find the total current output by the power supply and the power dissipated by the 20 Ω resistor.



13. You have a 600 V power source, two 10 Ω toasters that both run on 100 V and a 25 Ω resistor.

a. Show me how you would wire them up so the toasters run properly.
b. What is the power dissipated by the toasters?
c. Where would you put the fuses to make sure the toasters don't draw more than 15 Amps?
d. Where would you put a 25 Amp fuse to prevent a fire (if too much current flows through the wires they will heat up and possibly cause a fire)?



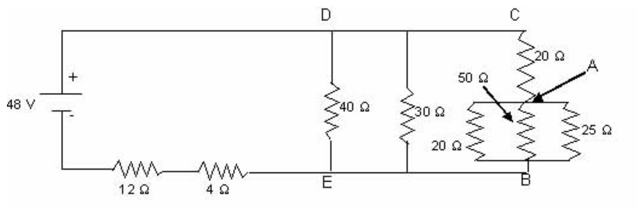
14. Look at the following scheme of four identical light bulbs connected as shown. Answer the questions below giving a justification for your answer:

a. Which of the four light bulbs is the brightest?
b. Which light bulbs are the dimmest?
c. Tell in the following cases which other light bulbs go out if:
d. bulb *A* goes out (ii). bulb *B* goes out (iii). bulb *D* goes out
e. Tell in the following cases which other light bulbs get dimmer, and which get brighter if:
f. bulb *B* goes out (ii). bulb *D* goes out

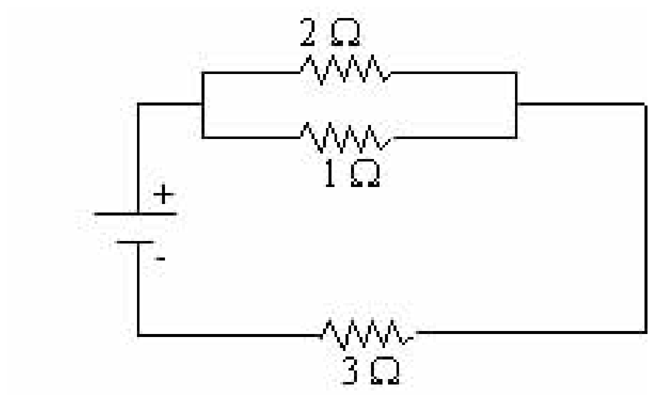
15. Refer to the circuit diagram below and answer the following questions.

a. What is the resistance between *A* and *B*?
b. What is the resistance between *C* and *B*?
c. What is the resistance between *D* and *E*?
d. What is the the total equivalent resistance of the circuit?
e. What is the current leaving the battery?
f. What is the voltage drop across the 12 Ω resistor?
g. What is the voltage drop between *D* and *E*?
h. What is the voltage drop between *A* and *B*?
i. What is the current through the 25 Ω resistor?

j. What is the total energy dissipated in the 25 Ω if it is in use for 11 hours?



16. In the circuit shown here, the battery produces an *emf* of 1.5 V and has an internal resistance of 0.5 Ω.



a. Find the total resistance of the external circuit.
b. Find the current drawn from the battery.
c. Determine the terminal voltage of the battery
d. Show the proper connection of an ammeter and a voltmeter that could measure voltage across and current through the 2 Ω resistor. What measurements would these instruments read?

17. Students measuring an unknown resistor take the following measurements:

TABLE 14.2:

Voltage (v)	Current (a)
15	.11
12	.08
10	.068
8	.052
6	.04
4	.025
2	.01

(a) Show a circuit diagram with the connections to the power supply, ammeter and voltmeter.

(b) Graph voltage vs. current; find the best-fit straight line.

(c) Use this line to determine the resistance.

(d) How confident can you be of the results?

(e) Use the graph to determine the current if the voltage were 13 V.

18. Students are now measuring the terminal voltage of a battery hooked up to an external circuit. They change the external circuit four times and develop the following table of data:

TABLE 14.3:

Terminal Voltage (v)	Current (a)
14.63	.15
14.13	.35
13.62	.55
12.88	.85

(a) Graph this data, with the voltage on the vertical axis.

(b) Use the graph to determine the emf of the battery.

(c) Use the graph to determine the internal resistance of the battery.

(d) What voltage would the battery read if it were not hooked up to an external circuit?

19. Students are using a variable power supply to quickly increase the voltage across a resistor. They measure the current and the time the power supply is on. The following table of data is developed:

TABLE 14.4:

Time(sec)	Voltage (v)	Current (a)
0	0	0
2	10	1.0
4	20	2.0
6	30	3.0
8	40	3.6
10	50	3.8
12	60	3.5
14	70	3.1
16	80	2.7
18	90	2.0

(a) Graph voltage vs. current

(b) Explain the probable cause of the anomalous data after 8 seconds

(c) Determine the likely value of the resistor and explain how you used the data to support this determination.

(d) Graph power vs. time

(e) Determine the total energy dissipation during the 18 seconds.

20. You are given the following three devices and a power supply of exactly 120 v. * Device X is rated at 60 V and 0.5 A* Device Y is rated at 15 w and 0.5 A* Device Z is rated at 120 V and 1800 w Design a circuit that obeys the following rules: you may only use the power supply given, one sample of each device, and an extra, single resistor of any value (you choose). Also, each device must be run at their rated values.
21. Given three resistors, 200 $\Omega, 300$ Ω and 600 Ω and a 120 V power source connect them in a way to heat a container of water as rapidly as possible.
    a. Show the circuit diagram
    b. How many joules of heat are developed after 5 minutes?
22. Construct a circuit using the following devices: a 120 V power source. Two 9 Ω resistors, device A rated at 1 A, 6 V; device B rated at 2 A, 60 V; device C rated at 225 w, 3 A; device D rated at 15 w, 15 V.
23. You have a battery with an emf of 12 V and an internal resistance of 1.00 Ω. Some 2.00 A are drawn from the external circuit.

a. What is the terminal voltage
b. The external circuit consists of device X, 0.5 A and 6 V; device Y, 0.5 A and 10 V, and two resistors. Show how this circuit is connected.
c. Determine the value of the two resistors.

24. Students use a variable power supply an ammeter and three voltmeters to measure the voltage drops across three unknown resistors. The power supply is slowly cranked up and the following table of data is developed:

TABLE 14.5:

Current (ma)	Voltage R_1 (v)	Voltage R_2 (v)	Voltage R_3 (v)
100	2.1	3.6	5.1
150	3.0	5.0	7.7
200	3.9	7.1	10.0
250	5.0	8.9	12.7
300	6.2	10.8	15.0
350	7.1	12.7	18.0
400	7.9	14.3	20.0
450	9.0	16.0	22.0
500	10.2	18.0	25.0
600	12.5	21.0	31.0
700	14.0	25.0	36.0

(a) Draw a circuit diagram, showing the ammeter and voltmeter connections.

(b) Graph the above data with voltage on the vertical axis.

(c) Use the slope of the best-fit straight line to determine the values of the three resistors.

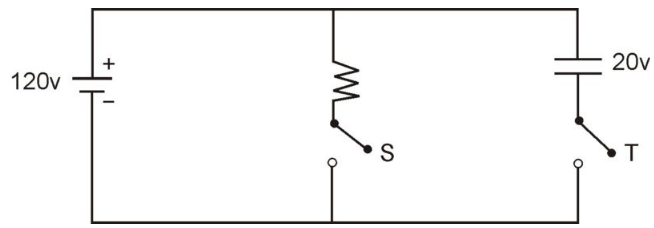
(d) Quantitatively discuss the confidence you have in the results

(e) What experimental errors are most likely might have contributed to any inaccuracies.

25. Design a parallel plate capacitor with a capacitance of 100 mF. You can select any area, plate separation, and dielectric substance that you wish.
26. You have a 5μF capacitor.
    a. How much voltage would you have to apply to charge the capacitor with 200 C of charge?
    b. Once you have finished, how much potential energy are you storing here?
    c. If all this energy could be harnessed to lift you up into the air, how high would you be lifted?
27. Show, by means of a sketch illustrating the charge distribution, that two identical parallel-plate capacitors wired in parallel act exactly the same as a single capacitor with twice the area.
28. A certain capacitor can store 5 C of charge if you apply a voltage of 10 V.
    a. How many volts would you have to apply to store 50 C of charge in the same capacitor?
    b. Why is it harder to store more charge?
29. A certain capacitor can store 500 J of energy (by storing charge) if you apply a voltage of 15 V. How many volts would you have to apply to store 1000 J of energy in the same capacitor? (Important: why isn't the answer to this just 30 V?)
30. Marciel, a bicycling physicist, wishes to harvest some of the energy he puts into turning the pedals of his bike and store this energy in a capacitor. Then, when he stops at a stop light, the charge from this capacitor can flow out and run his bicycle headlight. He is able to generate 18 V of electric potential, on average, by pedaling (and using magnetic induction).
    a. If Mars wants to provide 0.5 A of current for 60 seconds at a stop light, how big a 18 V capacitor should he buy (i.e. how many farads)?

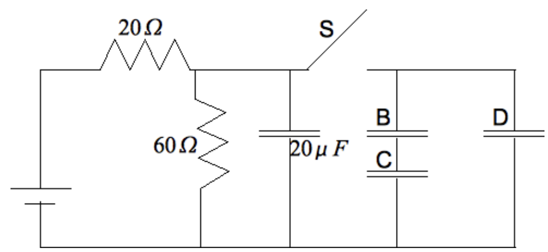
b. How big a resistor should he pass the current through so the RC time is three minutes?

31. Given a capacitor with 1 cm between the plates a field of $20,000$ N/C is established between the plates.



a. What is the voltage across the capacitor?
b. If the charge on the plates is 1μC, what is the capacitance of the capacitor?
c. If two identical capacitors of this capacitance are connected in series what it the total capacitance?
d. Consider the capacitor connected in the following circuit at point B with two switches S and T, a 20Ω resistor and a 120 V power source:

i. Calculate the current through and the voltage across the resistor if S is open and T is closed ii. Repeat if S is closed and T is open Figure for Problems 32-34:



32. Consider the figure above with switch, S, initially open and the power supply set to 24 V:

a. What is the voltage drop across the 20Ω resistor?
b. What current flows thru the 60Ω resistor?
c. What is the voltage drop across the 20 microfarad capacitor?
d. What is the charge on the capacitor?
e. How much energy is stored in that capacitor?
f. Find the capacitance of capacitors B, C, and D if compared to the 20μF capacitor where...

(i). B has twice the plate area and half the plate separation (ii). C has twice the plate area and the same plate separation (iii). D has three times the plate area and half the plate separation

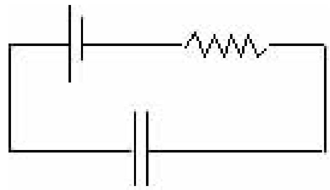
33. Now the switch in the previous problem is closed.

a. What is the total capacitance of branch II?
b. What is the total capacitance of branches I, II, and III taken together?
c. What is the voltage drop across capacitor B?

34. Reopen the switch in the previous problem and look at the 20μF capacitor. It has a plate separation of $2.0mm$.

a. What is the magnitude and direction of the electric field?
b. If an electron is released in the center to traverse the capacitor and given a speed $2/3$ the speed of light parallel to the plates , what is the magnitude of the force on that electron?
c. What would be its acceleration in the direction perpendicular to its motion?
d. If the plates are 1.0 cm long, how much time would it take to traverse the plate?
e. What displacement toward the plates would the electron undergo?
f. With what angle with respect to the direction of motion does the electron leave the plate?

35. Design a circuit that uses capacitors, switches, voltage sources, and light bulbs that will allow the interior lights of your car to dim slowly once you get out.
36. Design a circuit that would allow you to determine the capacitance of an unknown capacitor.



37. The voltage source in the circuit below provides 10 V. The resistor is 200Ω and the capacitor has a value of 50μF. What is the voltage across the capacitor after the circuit has been hooked up for a long time?
38. A simple circuit consisting of a 39μF and a 10kΩ resistor. A switch is flipped connecting the circuit to a 12 V battery.
    a. How long until the capacitor has 2/3 of the total charge across it?
    b. How long until the capacitor has 99% of the total charge across it?
    c. What is the total charge possible on the capacitor?
    d. Will it ever reach the full charge in part c.?
    e. Derive the formula for V(t) across the capacitor.
    f. Draw the graph of V vs. t for the capacitor.
    g. Draw the graph of V vs. t for the resistor.
39. If you have a 39μF capacitor and want a time constant of 5 seconds, what resistor value is needed?

Answers to Selected Problems

1. a. 4.5C b. 2.8×10^{19} electrons
2. a. 0.11 A b. 1.0 W c. 2.5×10^{21} electrons d. 3636 W
3. a. 192 Ω b. 0.42 W
4. a. 5.4 mV b. 1.4×10^{-8} A c. 7.3×10^{-11} W, not a lot d. 2.6×10^{-7} J
5. left = brighter, right = longer
6. a. 224 V b. 448 W c. 400 W by 100 Ω and 48 W by 12 Ω
7. b. 8.3 W
8. 0.5A
9. .
10. 0.8A and the 50 Ω on the left
11. a. 0.94 A b. 112 W c. 0.35 A d. 0.94 A e. 50, 45, 75 Ω f. both 50 Ω resistors are brightest, then 45 Ω, then 75 Ω
12. a. 0.76 A b. 7.0 W
13. b. 1000 W
14. .
15. a. 9.1 Ω b 29.1 Ω c. 10.8 Ω d.26.8 Ω e. 1.8A f. 21.5V g. 19.4V h. 6.1V i. 0.24A j. 16 kW
16. a. 3.66 Ω b. 0.36A c. 1.32 V
17. .
18. .
19. .
20. .
21. .
22. .
23. .
24. .
25. .
26. .

27. .
28. .
29. .
30. .
31. .
32. a. 6V b. 0.3A c. 18V d. 3.6×10^{-4}C e. 3.2×10^{-3}J f. i) 80μF ii) 40μF iii) 120μF
33. a. 26.7μF b. 166.7μF
34. a. 19.0×10^{3} N/C b. 1.4×10^{-15} N c. 1.6×10^{15} m/s^2 d. 3.3×10^{-11} s e. 8.9×10^{-7} m f. 5.1×10^{-30}
35. .
36. .
37. a. 10V
38. a. 0.43 seconds b. 1.8 seconds c. 4.7×10^{-4}C d. No, it will asymptotically approach it. e. The graph is same shape as the Q(t) graph. It will rise rapidly and then tail off asymptotically towards 12 V. f. The voltage across the resistor is 12 V minus the voltage across the capacitor. Thus, it exponentially decreases approaching the asymptote of 0 V.
39. about 128kΩ

CHAPTER 15 Magnetism Version 2

Chapter Outline

15.1 The Big Idea

For static electric charges, the electromagnetic force is manifested by the Coulomb electric force alone. If charges are moving, an additional force emerges, called magnetism. The 19^{th} century realization that electricity and magnetism are dual aspects of the same force completely changed our understanding of the world we live in. As with electricity, we use a field formulation to keep track of magnetic forces. Magnetic fields are usually denoted by the letter $\vec{B}$ and are measured in Teslas, in honor of the Serbian physicist Nikola Tesla; like electric fields, they are vector fields that contain energy, unlike electric fields, they have three dimensional properties and require some special vector rules to understand.

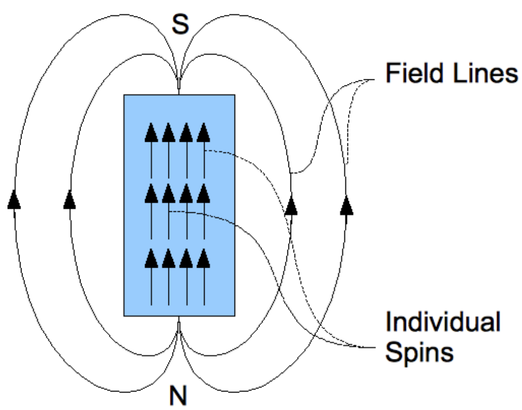
Insights due to Ampere, Gauss, and Maxwell led to the understanding that moving charges — electric currents — create magnetic fields. Varying magnetic fields create electric fields. Thus a loop of wire in a changing magnetic field will have current induced in it. This is called **electromagnetic induction.**

15.2 Sources of Magnetic Fields

In the electricity chapter, we learned that static electric fields have, as their source, some arrangement of charges. On the other hand, there are no sources of magnetic charge: every magnet, no matter how small, has a 'north' and 'south' pole. Nonetheless, there exist 'magnetic materials' that create fields and experience forces from other magnetic materials. In this chapter, we study magnetic fields produced by two different phenomena.

Permanent Magnets

Permanent magnets (like refrigerator magnets) consist of atoms, such as iron, for which the magnetic moments (roughly electron spin) of the electrons are "lined up" all across the atom. This means that their magnetic fields add up, rather than canceling each other out. The net effect is noticeable because so many atoms have lined up. The magnetic field of such a magnet always points from the north pole to the south. The magnetic field of a bar magnet, for example, is illustrated below:



If we were to cut the magnet above in half, it would still have north and south poles; the resulting magnetic field would be qualitatively the same as the one above (but weaker).

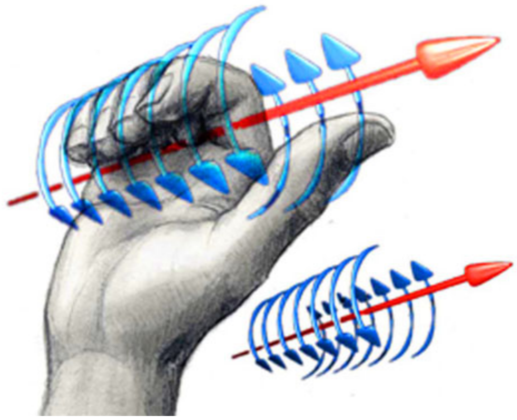
Charged Particles in Motion (Wires)

Charged particles in motion also generate magnetic fields. The most frequently used example is a current carrying wire, since current is literally moving charged particles. The magnitude of a field generated by a wire depends on distance to the wire and strength of the current (I) :

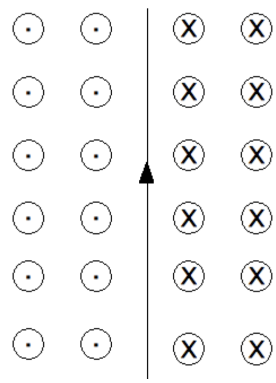
$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$ [1] Magnetic field at a distance r from a current-carrying wire

Where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A [2] Permeability of Vacuum (approximately same for air also)

Meanwhile, its direction can be found using the so called **first right hand rule**: point your thumb in the direction of the current. Then, curl your fingers around the wire. The direction your fingers will point in the same direction as the field. Be sure to use your right hand!



Sometimes, it is necessary to represent such three dimensional fields on a two dimensional sheet of paper. The following example illustrates how this is done.



In the example above, a current is running along a wire towards the top of your page. The magnetic field is circling the wire in loops that are pierced through the center by the current. Where these loops intersect this piece of paper, *we use the symbol* $\odot$ *to represent where the magnetic field is coming* ***out of the page*** *and the symbol* $\otimes$ *to represent where the magnetic field is going* ***into the page***. This convention can be used for all vector quantities: fields, forces, velocities, etc.

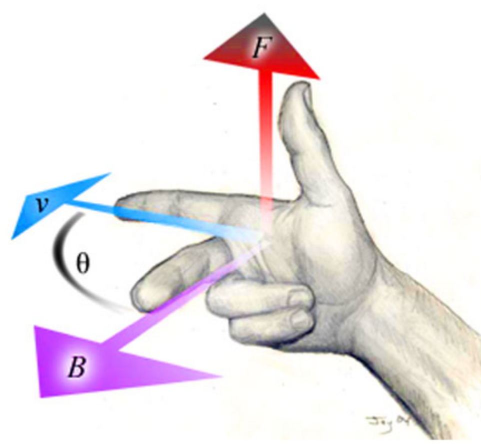
15.3 Effects of Magnetic Fields

Force on a Charged Particle

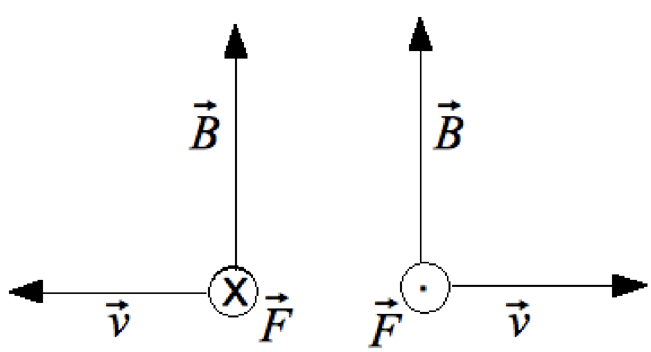
As moving charges create magnetic fields, so they experience forces from magnetic fields generated by other materials. The magnitude of the force experienced by a particle traveling in a magnetic field depends on the charge of the particle (q), the velocity of the particle (v), the strength of the field (B), and, importantly, the angle between their relative directions (θ):

$$F_B = qvB\sin\theta \qquad \text{[3] Force on a Charged Particle}$$

There is a **second right hand rule** that will show the direction of the force on a positive charge in a magnetic field: point your index finger along the direction of the particle's velocity . If your middle finger points along the magnetic field, your thumb will point in the direction of the force. NOTE: For negative charge reverse the direction of the force (or use your left hand)



For instance, if a positively charged particle is moving to the right, and it enters a magnetic field pointing towards the top of your page, it feels a force going out of the page, while if a positively charged particle is moving to the left, and it enters a magnetic field pointing towards the top of your page, it feels a force going into the page:



Example 1: Find the Magnetic Field

Question: An electron is moving to the east at a speed of $1.8 \times 10^6 \text{m/s}$. It feels a force in the upward direction with a magnitude of $2.2?10^{-12}\text{N}$. What is the magnitude and direction of the magnetic field this electron just passed through?

Answer: There are two parts to this question, the magnitude of the electric field and the direction. We will first focus on the magnitude.

To find the magnitude we will use the equation

$$F_B = qvBsin\theta$$

We were given the force of the magnetic field ($2.2?10^{-12}\text{N}$) and the velocity that the electron is traveling ($1.8 \times 10^6 \text{m/s}$). We also know the charge of the electron ($1.6 \times 10^{-19}\text{C}$). Also, because the electron's velocity is perpendicular to the field, we do not have to deal with $\sin\theta$ because $\sin\theta$ of 90 degrees is 1. Therefore all we have to do is solve for B and plug in the known values to get the answer.

$$F_B = qvB\sin\theta$$

Solving for B:

$$B = \frac{F_B}{qv\sin\theta}$$

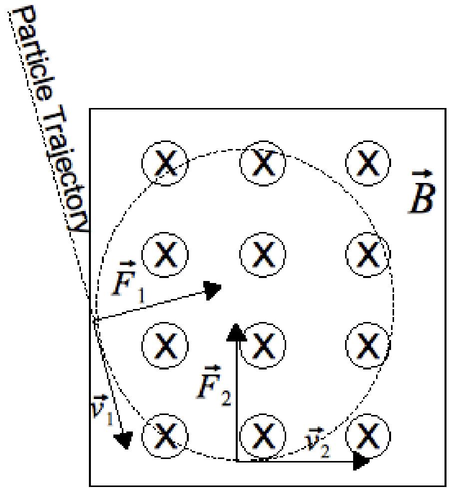
Now, plugging the known values we have

$$B = \frac{F_B}{qv\sin\theta} = \frac{2.2?10^{-12}\text{N}}{1.6 \times 10^{-19}\text{C} \times 1.8 \times 10^6 \text{m/s} \times 1} = 7.6\text{T}$$

Now we will find the direction of the field. We know the direction of the velocity (east) and the direction of the force due to the magnetic field (up, out of the page). Therefore we can use the second right hand rule (we will use the left hand, since an electron's charge is negative). Point the pointer finger to the right to represent the velocity and the thumb up to represent the force. This forces the middle finger, which represents the direction of the magnetic field, to point south. Alternatively, we could recognize that this situation is illustrated for a *positive* particle in the right half of the drawing above; for a negative particle to experience the same force, the field has to point in the opposite direction: south.

Example 2: Circular Motion in Magnetic Fields

Consider the following problem: a positively charged particle with an initial velocity of $\vec{v}_1$, charge q and mass m traveling in the plane of this page enters a region with a constant magnetic field $\vec{B}$ pointing into the page. We are interested in finding the trajectory of this particle.



Since the force on a charged particle in a magnetic field is always perpendicular to both its velocity vector and the field vector (check this using the second right hand rule above), a *constant* magnetic field will provide a centripetal force — that is, a constant force that is always directed perpendicular to the direction of motion. Two such force/velocity combinations are illustrated above. According to our study of rotational motion, this implies that as long as the particle does not leave the region of the magnetic field, it will travel in a circle. To find the radius of the circle, we set the magnitude of the centripetal force equal to the magnitude of the magnetic force and solve for r:

$$F_c = \frac{mv^2}{r} = F_B = qvB\sin\theta = qvB$$

Therefore,

$$r = \frac{mv^2}{qvB}$$

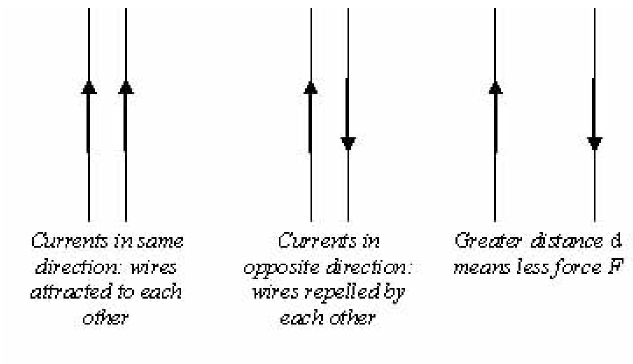
In the examples above, θ was conveniently 90 degrees, which made $\sin\theta = 1$. But that does not really matter; in a constant magnetic fields a different θ will simply decrease the force by a constant factor and will not change the qualitative behavior of the particle, since θ *cannot change due to such a magnetic force.* (Why? Hint: what is the force perpendicular to? Read the paragraph above.)

Force on a Wire

Since a wire is nothing but a collection of moving charges, the force it will experience in a magnetic field will simply be the vector sum of the forces on the individual charges. If the wire is straight — that is, all the charges are moving in the same direction — these forces will all point in the same direction, and so will their sum. Then, the direction of the force can be found using the second right hand rule, while its magnitude will depend on the length of the wire (denoted L), the strength of the current, the strength of the field, and the angle between their directions:

$$F_{\text{wire}} = LIB\sin(\theta) \qquad \text{[4] Force on a Current Carrying Wire}$$

Two current-carrying wires next to each other each generate magnetic fields and therefore exert forces on each other:



By plugging equation [1] into equation [4], one can find the exact formula for this force (left to the reader — make sure to remember that the two wires can have different currents).

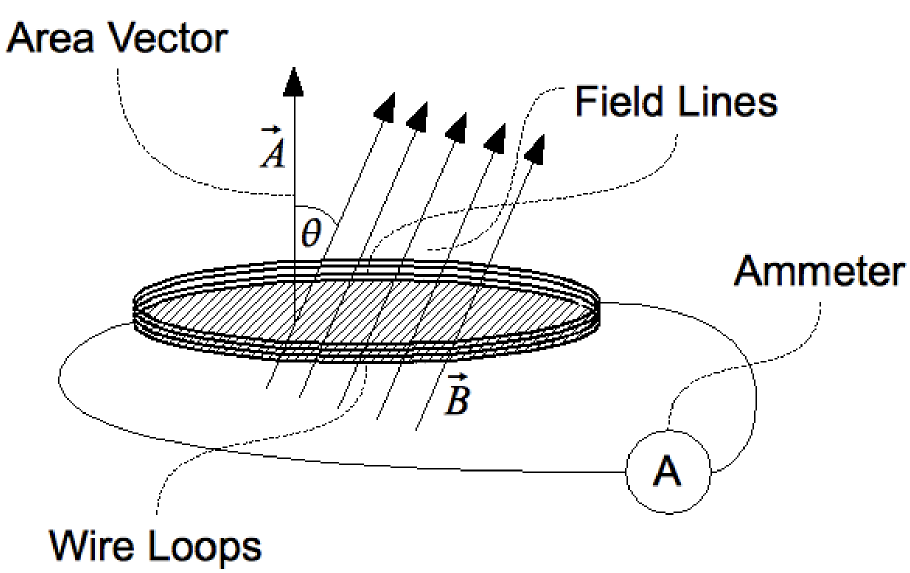
Electromagnetic Induction

Changing magnetic fields passing through a loop of wire generate currents in that wire; this is how electric power generators work. Likewise, a changing current in a wire will create a changing magnetic field; this is how speakers and electric motors work.

To understand induction, we need to introduce the concept of **electromagnetic flux**. If you have a closed, looped wire of area A (measured in m^2) and N loops, and you pass a magnetic field B through, the magnetic flux Φ is given by the formula below. Again, the relative direction of the loops and the field matter; this relationship is preserved by creating an 'area vector': a vector whose magnitude is equal to the area of the loop and whose direction is perpendicular to the plane of the loop. The directions' influence can then be conveniently captured through a dot product:

$$\Phi = N\vec{B}\cdot\vec{A} \qquad \text{[5] Electromagnetic Flux}$$

The units of magnetic flux are $\text{T} \times \text{m}^2$, also known as **Webers**(Wb).



In the example above, there are four loops of wire ($N = 4$) and each has area πr^2 (horizontally hashed). The magnetic field is pointing at an angle θ to the area vector. If the magnetic field has magnitude B, the flux through the loops will equal $4\cos\theta B\pi r^2$. Think of the magnetic flux as the part of the "bundle" of magnetic field lines "held" by the loop that points along the area vector.

If the magnetic flux through a loop or loops changes, electrons in the wire will feel a force, and this will generate a current. The *induced voltage* (also called **electromotive force, or emf**) that they feel is equal to the change in flux $\triangle\Phi$ divided by the amount of time $\triangle t$ that change took. This relationship is called Faraday's Law of Induction:

$$emf = -\frac{\Delta\Phi}{\Delta t} \qquad \text{[6] Faraday's Law of Induction}$$

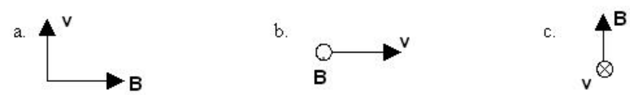
The direction of the induced current is determined as follows: the current will flow so as to generate a magnetic field that *opposes* the change in flux. This is called Lenz's Law. Note that the electromotive force described above is not actually a force, since it is measured in Volts and acts like an induced potential difference. It was originally called that since it caused charged particles to move — hence *electromotive* — and the name stuck (it's somewhat analogous to calling an increase in a particle's gravitational potential energy difference a gravitomotive force).

Since only a changing flux can produce an induced potential difference, one or more of the variables in equation [5] must be changing if the ammeter in the picture above is to register any current. Specifically, the following can all induce a current in the loops of wire:

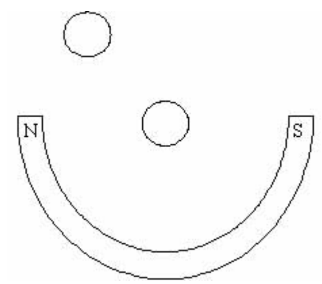
- Changing the direction or magnitude of the magnetic field.
- Changing the loops' orientation or area.
- Moving the loops out of the region with the magnetic field.

15.4 Magnetism Problem Set

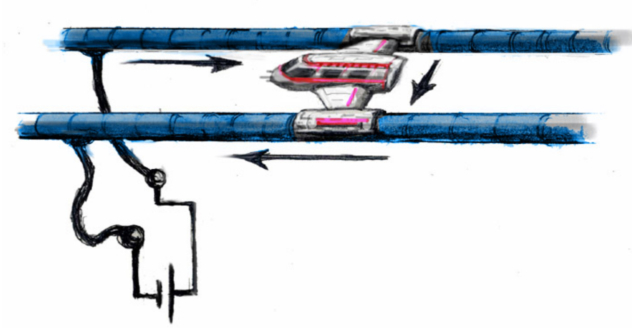
1. Can you set a resting electron into motion with a stationary magnetic field? With an electric field? Explain.
2. How is electrical energy produced in a dam using a hydroelectric generator? Explain in a short essay involving as many different ideas from physics as you need.
3. A speaker consists of a diaphragm (a flat plate), which is attached to a magnet. A coil of wire surrounds the magnet. How can an electrical current be transformed into sound? Why is a coil better than a single loop? If you want to make music, what should you do to the current?
4. For each of the arrangements of velocity v and magnetic field B below, determine the direction of the force. Assume the moving particle has a positive charge.



5. Sketch the magnetic field lines for the horseshoe magnet shown here. Then, show the direction in which the two compasses (shown as circles) should point considering their positions. In other words, draw an arrow in the compass that represents North in relation to the compass magnet.



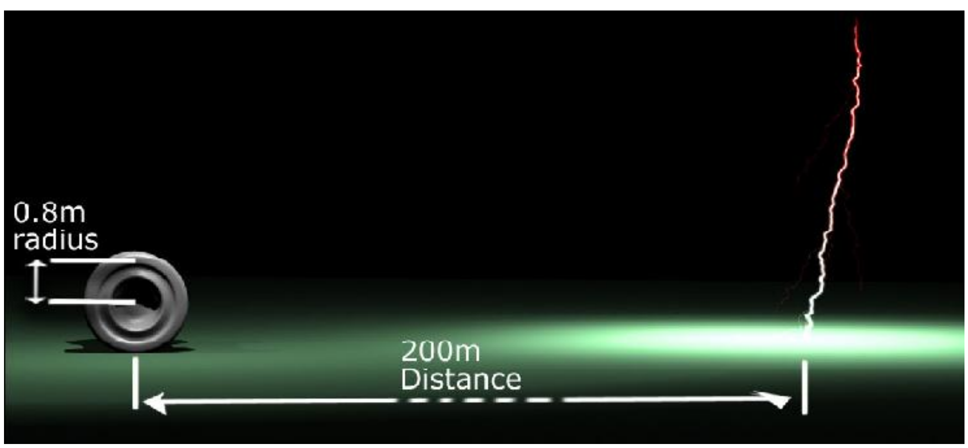
6. As an electron that is traveling in the positive $x-$direction encounters a magnetic field, it begins to turn in the upward direction (positive $y-$direction). What is the direction of the magnetic field?
   a. $-$direction
   b. $+$direction (towards the top of the page)
   c. $-$direction (i.e. into the page)
   d. $+$direction (i.e. out of the page)
   e. none of the above
7. A positively charged hydrogen ion turns upward as it enters a magnetic field that points into the page. What direction was the ion going before it entered the field?
   a. $-$direction
   b. $+$direction
   c. $-$direction (towards the bottom of the page)
   d. $+$direction (i.e. out of the page)
   e. none of the above
8. An electron is moving to the east at a speed of 1.8×10^6 m/s. It feels a force in the upward direction with a magnitude of 2.2×10^{-12} N. What is the magnitude and direction of the magnetic field this electron just passed through?
9. A vertical wire, with a current of 6.0 A going towards the ground, is immersed in a magnetic field of 5.0 T pointing to the right. What is the value and direction of the force on the wire? The length of the wire is 2.0 m.



10. A futuristic magneto-car uses the interaction between current flowing across the magneto car and magnetic fields to propel itself forward. The device consists of two fixed metal tracks and a freely moving metal car (see illustration above). A magnetic field is pointing downward with respect to the car, and has the strength of 5.00 T. The car is 4.70 m wide and has 800 A of current flowing through it. The arrows indicate the direction of the current flow.

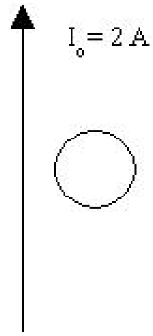
    a. Find the direction and magnitude of the force on the car.
    b. If the car has a mass of 2050 kg, what is its velocity after 10 s, assuming it starts at rest?
    c. If you want double the force for the same magnetic field, how should the current change?

11. A horizontal wire carries a current of 48 A towards the east. A second wire with mass 0.05 kg runs parallel to the first, but lies 15 cm below it. This second wire is held in suspension by the magnetic field of the first wire above it. If each wire has a length of half a meter, what is the magnitude and direction of the current in the lower wire?
12. Protons with momentum 5.1×10^{-20} kg$\cdot$ m/s are magnetically steered clockwise in a circular path. The path is 2.0 km in diameter. (This takes place at the Dann International Accelerator Laboratory, to be built in 2057 in San Francisco.) Find the magnitude and direction of the magnetic field acting on the protons.



13. A bolt of lightening strikes the ground 200 m away from a 100−turn coil (see above). If the current in the lightening bolt falls from 6.0×10^6 A to 0.0 A in 10 ms, what is the average *voltage*, ε, induced in the coil? What is the *direction* of the induced current in the coil? (Is it clockwise or counterclockwise?) Assume that the distance to the center of the coil determines the average magnetic induction at the coil's position. Treat the lightning bolt as a vertical wire with the current flowing toward the ground.
14. A coil of wire with 10 loops and a radius of 0.2 m is sitting on the lab bench with an electro-magnet facing into the loop. For the purposes of your sketch, assume the magnetic field from the electromagnet is pointing out of the page. In 0.035 s, the magnetic field drops from 0.42 T to 0 T.

    a. What is the voltage induced in the coil of wire?
    b. Sketch the direction of the current flowing in the loop as the magnetic field is turned off. (Answer as if you are looking down at the loop).



15. A wire has 2 A of current flowing in the upward direction.
    a. What is the value of the magnetic field 2 cm away from the wire?
    b. Sketch the direction of the magnetic field lines in the picture to the right.
    c. If we turn on a magnetic field of 1.4 T, pointing to the right, what is the value and direction of the force per meter acting on the wire of current?
    d. Instead of turning on a magnetic field, we decide to add a loop of wire (with radius 1 cm) with its center 2 cm from the original wire. If we then increase the current in the straight wire by 3 A per second, what is the direction of the induced current flow in the loop of wire?
16. An electron is accelerated from rest through a potential difference of 1.67×10^5 volts. It then enters a region traveling perpendicular to a magnetic field of 0.25 T.
    a. Calculate the velocity of the electron.
    b. Calculate the magnitude of the magnetic force on the electron.
    c. Calculate the radius of the circle of the electron's path in the region of the magnetic field
17. A beam of charged particles travel in a straight line through mutually perpendicular electric and magnetic fields. One of the particles has a charge, q; the magnetic field is B and the electric field is E. Find the velocity of the particle.
18. Two long thin wires are on the same plane but perpendicular to each other. The wire on the $y-$axis carries a current of 6.0 A in the $-y$ direction. The wire on the $x-$axis carries a current of 2.0 A in the $+x$ direction. Point, P has the co-ordinates of $(2.0, 2, 0)$ in meters. A charged particle moves in a direction of 45^o away from the origin at point, P, with a velocity of 1.0×10^7 m/s.
    a. Find the magnitude and direction of the magnetic field at point, P.
    b. If there is a magnetic force of 1.0×10^{-6} N on the particle determine its charge.
    c. Determine the magnitude of an electric field that will cancel the magnetic force on the particle.
19. A rectangular loop of wire 8.0 m long and 1.0 m wide has a resistor of 5.0 Ω on the 1.0 side and moves out of a 0.40 T magnetic field at a speed of 2.0 m/s in the direction of the 8.0 m side.
    a. Determine the induced voltage in the loop.
    b. Determine the direction of current.
    c. What would be the net force needed to keep the loop at a steady velocity?
    d. What is the electric field across the .50 m long resistor?
    e. What is the power dissipated in the resistor?
20. A positron (same mass, opposite charge as an electron) is accelerated through 35,000 volts and enters the center of a 1.00 cm long and 1.00 mm wide capacitor, which is charged to 400 volts. A magnetic filed is applied to keep the positron in a straight line in the capacitor. The same field is applied to the region (region II) the positron enters after the capacitor.
    a. What is the speed of the positron as it enters the capacitor?
    b. Show all forces on the positron.
    c. Prove that the force of gravity can be safely ignored in this problem.
    d. Calculate the magnitude and direction of the magnetic field necessary.

e. Show the path and calculate the radius of the positron in region II.
f. Now the magnetic field is removed; calculate the acceleration of the positron away from the center.
g. Calculate the angle away from the center with which it would enter region II if the magnetic field were to be removed.

21. A small rectangular loop of wire 2.00 m by 3.00 m moves with a velocity of 80.0 m/s in a non-uniform field that diminishes in the direction of motion uniformly by .0400 T/m. Calculate the induced emf in the loop. What would be the direction of current?
22. An electron is accelerated through 20,000 V and moves along the positive $x-$axis through a plate 1.00 cm wide and 2.00 cm long. A magnetic field of 0.020 T is applied in the $-z$ direction.

   a. Calculate the velocity with which the electron enters the plate.
   b. Calculate the magnitude and direction of the magnetic force on the electron.
   c. Calculate the acceleration of the electron.
   d. Calculate the deviation in the y direction of the electron form the center.
   e. Calculate the electric field necessary to keep the electron on a straight path.
   f. Calculate the necessary voltage that must be applied to the plate.

23. A long straight wire is on the $x-$axis and has a current of 12 A in the $-x$ direction. A point P, is located 2.0 m above the wire on the $y-$axis.

   a. What is the magnitude and direction of the magnetic field at P.
   b. If an electron moves through P in the$-x$ direction at a speed of 8.0×10^7 m/s what is the magnitude and direction of the force on the electron?
   c. What would be the magnitude and direction of an electric field to be applied at P that would counteract the magnetic force on the electron?

Answers to Selected Problems

1. No: if $v = 0$ then $F = 0$; yes: $F = qE$
2. .
3. .
4. a. Into the page b. Down the page c. Right
5. Both pointing away from north
6. .
7. .
8. 7.6 T, south
9. Down the page; 60 N
10. a. To the right, 1.88×10^4 N b. 91.7 m/s c. It should be doubled
11. East 1.5×10^4 A
12. 0.00016 T; if CCW motion, B is pointed into the ground.
13. 1.2×105 V, counterclockwise
14. a. 15 V b. Counter-clockwise
15. a. 2×10^{-5} T b. Into the page c. 2.8 N/m d. CW
16. a. 2.42×10^8 m/s b. 9.69×10^{-12} N c. .0055 m
17. E/B
18. a. 8×10^{-7} T b. 1.3×10^{-6} C
19. a. 0.8 V b. CCW c. .064 N d. .16 N/C e. .13 w
20. a. 1.11×10^8 m/s b. 9.1×10^{-30} N $<< 6.4 \times 10^{-14}$ N d. .00364 T e. .173 m f. 7.03×1016 m/s^2 g. 3.27°
21. 19.2 V
22. a.8.39×10^7 m/s b. 2.68×10^{-13} N, $-y$ c. 2.95×10^17 m/s^2 d. .00838 m e. 1.68×10^6 N/C f. 16,800 V
23. a. 1.2×10^{-6} T, $+z$ b. 1.5×10^{-17} N, $-y$ c. 96 N/C, $-y$

CHAPTER 16 Electric Circuits Advanced Topics Version 2

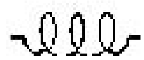
Chapter Outline

16.1 The Big Idea

Modern circuitry depends on much more than just resistors and capacitors. The circuits in your computer, cell phone, and iPod depend on circuit elements called diodes, inductors, transistors, and operational amplifiers, as well as on other chips. In particular the invention of the transistor made the small size of modern devices possible. Transistors and op amps are known as *active* circuit elements. An active circuit element needs an external source of power to operate. This differentiates them from diodes, capacitors, inductors and resistors, which are *passive* elements.

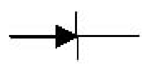
Key Concepts

- **Inductors** are made from coiled wires, normally wrapped around ferromagnetic material and operate according to the principles of magnetic induction presented in Magnetism. Inductors generate a *back-emf*. Back-emf is essentially an induced negative voltage which opposes changes in current. The amount of back-emf generated is proportional to how quickly the current changes. They can be thought of as automatic flow regulators that oppose any change in current. Thus electrical engineers call them *chokes*.



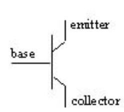
In a circuit diagram, an inductor looks like a coil. The resistance R and capacitance C of an inductor are very close to zero. When analyzing a circuit diagram, assume R and C are precisely zero.

- **Diodes** are passive circuit elements that act like one-way gates. Diodes allow current to flow one way, but not the other. For example, a diode that "turns on" at 0.6 V acts as follows: if the voltage drop across the diode is less than 0.6 V, no current will flow. Above 0.6 V, current flows with essentially no resistance. If the voltage drop is negative (and not extremely large), no current will flow.



Diodes have an arrow showing the direction of the flow.

- **Transistors** are active circuit elements that act like control gates for the flow of current. Although there are many types of transistors, let's consider just one kind for now. This type of transistor has three electrical leads: the base, the emitter, and the collector.



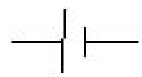
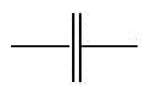
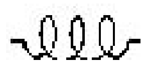
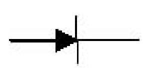
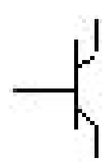
The voltage applied to the base controls the amount of current which flows from the emitter to the collector.

- For example, if the base voltage is more than 0.8 V above the collector voltage, then current can freely flow from the emitter to the collector, as if it were just a wire. If the base voltage is less than 0.8 V above the collector voltage, then current does not flow from the emitter to the collector. Thus the transistor acts as a switch. (This 0.8 V is known as a "diode drop" and varies from transistor to transistor.)

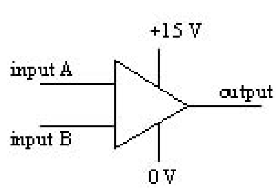
- Transistors have an infinite **output resistance**.. If you measure the resistance between the collector and the base (or between the emitter and the base), it will be extremely high. Essentially no current flows into the base from either the collector or the emitter; any current, if it flows, flows from the emitter to the collector..

- Transistors are used in **amplifier** circuits, which take an input voltage and magnify it by a large factor. Amplifiers typically run on the principle of positive and negative **feedback**. Feedback occurs when a small portion of an output voltage is used to influence the input voltage.

TABLE 16.1:

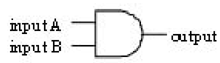
Circuit element	Symbol	Electrical symbol	Unit	Everyday device
Voltage Source	$?V$		Volts (V)	Batteries, electrical outlets, power stations
Resistor	R		Ohm (Ω)	Light bulbs, toasters, hair dryers
Capacitor	C		Farad (F)	Computer keyboards, timers
Inductor	L		Henry (H)	Electronic chokes, AC transformers
Diode	varies by type		none	Light-emitting diodes (LEDs)
Transistor	varies by type		none	Computer chips, amplifiers

- **An operational amplifier** or op-amp is an active circuit element that performs a specific function. The most common op-amp has five leads: two input leads, one output lead, and two fixed-voltage leads.

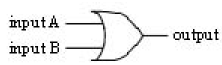


The job of an op-amp is to use the voltage it is supplied to adjust its output voltage. The op-amp will adjust its output voltage until the two input voltages are brought closer together. In other words, the output voltage will change as it needs to until $V_A - V_B = 0$. This won't happen unless the output voltage is somehow "fed back" into one of the inputs

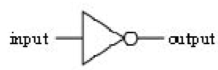
- **Digital circuits** only care about two voltages: for example, +5 V (known as "on") and 0 V (known as "off").
- **Logic devices**, which are active circuit elements, interpret voltages according to a simple set of mathematical rules known as Boolean logic. The most basic logic devices are the AND, OR, and NOT gates:



For an AND gate, the output will always be at an electric potential of 0 V (off) unless *both* the inputs are at 5 V (on), in which case the output will be at 5 V (on) as well.



For an OR gate, the output will always be at an electric potential of 0 V (off) unless *either* of the inputs are at 5 V (on), in which case the output will be at 5 V (on) as well.



For a NOT gate, the output will always be the *opposite* of the input. Thus, if the input is 5 V (on), the output will be 0 V (off) and vice-versa.

- **Alternating current** changes direction of current flow. The frequency is the number of times the current reverses direction in a second. Household AC is 60 Hz. In AC circuits the current is impeded but not stopped by elements like capacitors and inductors.

- **Capacitive Reactance** is a measure of how a capacitor impedes the current flow from a given voltage in an AC circuit and is inversely proportional to capacitance. *Inductive Reactance* is a measure of how an inductor in an AC circuit impedes the current flow from a given voltage and is directly proportional to inductance.

- *The total impedance of an AC circuit* depends on resitance, capacitive reactance and inductive reactance.

- If the capacitive reactance and inductive reactance are both zero or unequal the voltage and current are *out of phase*. That is they peak at different times in the cycle. The *phase angle* measures the lag or lead of current over voltage.

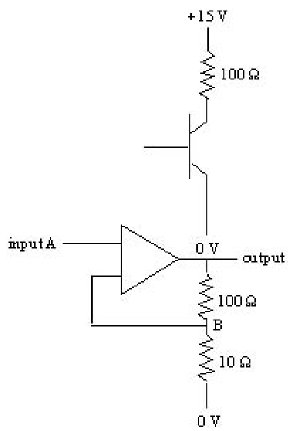
16.2 Key Equations

$$\begin{cases} \mathrm{E} = \frac{-L\Delta I}{\Delta t} & \text{Emf across an inductor with inductance L} \\ L = \frac{\mu_0 N^2 A}{I} & \text{Inductance of a solenoid with}N\text{turns, area }A\text{, in Henrys (H)} \\ X_L = 2\pi f C & \text{Inductive reactance for AC of frequency f} \\ X_C = \frac{1}{X_L} & \text{Capacitive reactance for AC of frequency f} \\ Z = \sqrt{R^2 + (X_L - X_C)^2} & \text{Impedance of an RC circuit (Pythagorean Theorem)} \\ \tan\phi = \frac{X_L - X_C}{R} & \text{Phase angle between peak current and voltage} \end{cases}$$

16.3 Examples

Example 1

Question: Consider the op-amp circuit diagram shown here. Note the fixed-voltage leads are omitted for clarity. (This is typical.)



Let's begin with an input voltage at point A of 0.5V. a) If the op-amp is "doing its job," what is the electric potential at point B? b) What current is flowing through the 10Ω resistor? c) What current must be flowing through the 100Ω resistor? d) What *must* the output voltage be? Now let's adjust the input voltage at point A to 0.75V. e) What is the output voltage now? f) By what factor is the op-amp amplifying the input voltage?

Answer:

a) The op-amp is supposed to make the two input voltages as close to equal as possible, or in other words,

$$V_A - V_B = 0$$

Therefore if the input voltage at point A is .5V, then the input voltage at point B should also be .5V.

b) We will use Ohm's Law to find the current going through the 10Ω resistor.

$$V = IR \Rightarrow \frac{V}{R} = I \Rightarrow I = \frac{.5\text{V}}{10\Omega} = .05\text{A}$$

c) Recall that no current ever flows *into* an op-amp. Therefore, the current must be the same as the current running through the 10Ω resistor, which is .05A.

d) We will again use Ohm's law. First we must find the total resistance and then we can plug in the known values to solve for the voltage.

$$R_{total} = R_1 + R_2 = 10\Omega + 100\Omega = 110\Omega$$
$$V = IR = .05\text{A} \times 110\Omega = 5.5\text{V}$$

e) We now want to find the output voltage given an input voltage of 0.75V at point A. Though the numbers are different, it is the same process as solving it when the input voltage was 0.5V. First we must find the current and the total resistance. We then use these values to solve for the output voltage.

$$V = IR \Rightarrow \frac{V}{R} = I \Rightarrow I = \frac{.75\text{V}}{10\Omega} = .075\text{A}$$

The total resistance remains unchanged.

$$R_{total} = R_1 + R_2 = 10\Omega + 100\Omega = 110\Omega$$

So the new output voltage is

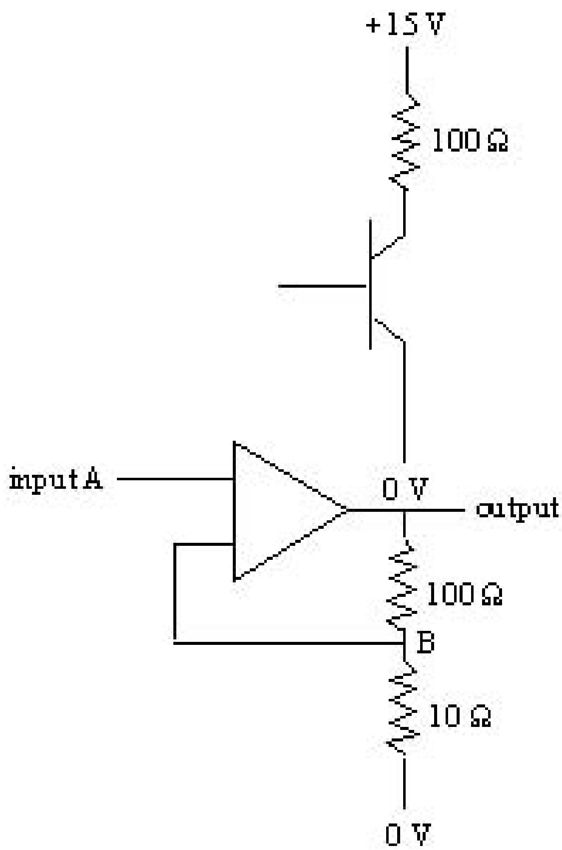
$$V = IR = .075\text{A} \times 110\Omega = 8.25\text{V}$$

f) This is a simple division problem. The output voltage divided by the input voltage will give us the factor by which the output voltage is greater than the input voltage.

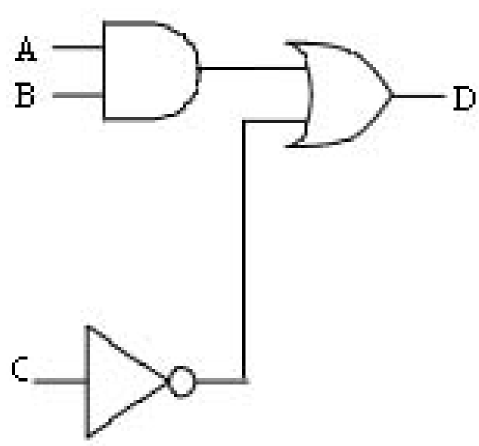
$$\frac{8.25\text{V}}{.75\text{V}} = 11$$

16.4 Advanced Topics Problem Set

1. You purchase a circular solenoid with 100 turns, a radius of 0.5 cm, and a length of 2.0 cm.
    a. Calculate the inductance of your solenoid in Henrys.
    b. A current of 0.5 A is passing through your solenoid. The current is turned down to zero over the course of 0.25 seconds. What voltage is induced in the solenoid?
2. What is the voltage drop across an inductor if the current passing through it is not changing with time? Does your answer depend on the physical makeup of the inductor? Explain.



3. Consider the transistor circuit diagram shown here. The resistor is a light bulb that shines when current passes through it.
    a. If the base is raised to a voltage of 5 V, will the light bulb shine?
    b. If the base is lowered to a voltage of 0 V, will the light bulb shine?
    c. Why are transistors sometimes called electronic switches?
4. Consider the op-amp circuit diagram shown here. Note the fixed-voltage leads are omitted for clarity. (This is typical.) Let's begin with an input voltage at point *A* of 0.5 V. (a) If the op-amp is "doing its job," what is the electric potential at point *B*? (b) What current is flowing through the 10 Ω resistor? (c) Recall that no current ever flows *into* an op-amp. What current must be flowing through the 100 Ω resistor? (d) What *must* the output voltage be? Now let's adjust the input voltage at point *A* to 0.75 V. (e) What is the output voltage now? (f) By what factor is the op-amp amplifying the input voltage? (g) What are some practical applications for such a device?



5. Consider the logic circuit shown here.
   a. If A, B, and C are all off, what is the state of D?
   b. If A, B, and C are all on, what is the state of D?
   c. Fill out the entire "logic table" for this circuit.

TABLE 16.2:

State of A	*State of B*	*State of C*	*State of D*
on	on	on	
on	on	off	
on	off	on	
on	off	off	
off	on	on	
off	off	on	
off	on	off	
off	off	off	

6. A series circuit contains the following elements: a 125 Ω resistor, a 175 mH inductor, two 30.0μF capacitors and a 40.0μF capacitor. Voltage is provided by a 235 V_m generator operating at 75.0 Hz.
   a. Draw a schematic diagram of the circuit.
   b. Calculate the total capacitance of the circuit.
   c. Calculate the capacitive reactance.
   d. Calculate the impedance.
   e. Calculate the peak current.
   f. Calculate the phase angle.
   g. Resonance occurs at the frequency when peak current is maximized. What is that frequency?

Answers to Selected Problems

1.
   1. 4.9×10^{-5} H
   2. -9.8×10^{-5} V
2. Zero
   1. Yes
   2. No

3. Because they turn current flow on and off.

1. 0.5 V
2. 0.05 A
3. 0.05 A
4. 5.5 V
5. 8.25V
6. 11×

1. *On*
2. *On*
3. $On, on, off, on, off, off, on, on$

3. (b) 10.9μ F (c) 195 Ω (d) 169 Ω (e) 1.39 A (f) −42° (g) 115Hz

CHAPTER 17 Light Version 2

Chapter Outline

17.1 The Big Idea

Light is a *wave* of changing electric and magnetic fields. Light waves are caused by disturbances in an electromagnetic field, like the acceleration of charged particles (such as electrons). Light has a dual nature; at times, it acts like waves, while at other times it acts like particles, called *photons*. Light travels through space at the maximum speed allowed by the laws of physics, called the speed of light. Light has no mass, but it carries energy and momentum. Fermat's principle states that *light will always take the path that takes the least amount of time* (not distance).

Fermat's Principle governs the paths light will take and explains the familiar phenomena of reflection, refraction, diffraction, scattering and color absorption and dispersion. Light rarely travels in a straight line path. When photons interact with electrons in matter, the time it takes for this interaction determines the path. For example, higher frequency blue light is refracted more than red because blue interacts more frequently with electrons. Also, the path of least time is achieved when blue light bends more than red light so that it gets out of the 'slow' region faster. Fermat's Principle explains the many fascinating phenomena of light from rainbows to sunsets to the halos around the moon.

Key Concepts

- When charged particles *accelerate*, changing electric and magnetic fields radiate outward. The traveling electric and magnetic fields of an accelerating (often oscillating) charged particle are known as electromagnetic radiation or light.
- The color of light that we observe is a measure of the frequency of the light: the *smaller* the frequency, the *redder* the light.
- The spectrum of electromagnetic radiation can be roughly broken into the following ranges:

TABLE 17.1:

EM wave	Wavelength range	Comparison size
gamma-ray ($\gamma-$ ray)	10^{-11} m and shorter	atomic nucleus
$x-$ray	10^{-11} m – 10^{-8} m	hydrogen atom
ultraviolet (UV)	10^{-8} m – 10^{-7} m	small molecule
violet (visible)	$\sim 4 \times 10^{-7}$ m(400 nm)*	typical molecule
blue (visible)	~ 450 nm	typical molecule
green (visible)	~ 500 nm	typical molecule
red (visible)	~ 650 nm	typical molecule
infrared (IR)	10^{-6} m – 1 mm	human hair
microwave	1 mm – 10 cm	human finger
radio	Larger than 10 cm	car antenna

- Light can have any wavelength. Our *vision* is restricted to a very narrow range of colors between red and violet.
- Fermat's Principle makes the angle of incident light equal to the angle of reflected light. This is the *law of reflection*.
- When light travels from one type of material (like air) into another (like glass), its effective speed is reduced due to interactions between photons and electrons. If the ray enters the material at an angle, Fermat's Principle dictates that the light will change the direction of its motion. One way to think about this is that light takes the

path of least time to get from points A to point B, thus it takes a more direct path through 'slower' mediums, so it can get out of the slow part faster. Light does not always travel in a straight line, it travels on the path of least time. This is called *refraction*.

- White light consists of a mixture of all the visible colors: red, orange, yellow, green, blue, indigo, and violet (ROYGBIV). Our perception of the color black is tied to the *absence* of light.
- Different frequencies of light (and hence different colors in the visible spectrum) will travel at slightly different speeds in materials, like glass, and thus have slightly different refracting angles. This phenomena gives rise to rainbows.
- Our eyes include color-sensitive and brightness-sensitive cells. The three different color-sensitive cells (cones) can have sensitivity in three colors: red, blue, and green. Our perception of other colors is made from the *relative amounts* of each color that the cones register from light reflected from the object we are looking at. Our brightness-sensitive cells work well in low light. This is why things look 'black and white' at night.
- The chemical bonds in pigments and dyes – like those in a colorful shirt – absorb light at frequencies that correspond to certain colors. When you shine white light on these pigments and dyes, some colors are absorbed and some colors are reflected. We only see the colors objects *reflect*.

Color Addition

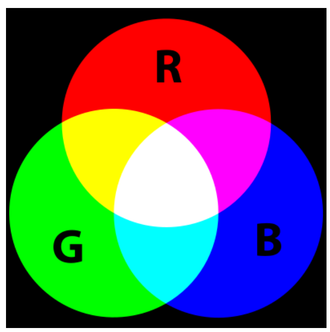


TABLE 17.2:

Red	Green	Blue	Perceived color
√	√	√	white
			black
√		√	magenta
√	√		yellow
	√	√	cyan

Key Applications

- *Total internal reflection* occurs when light goes from a slow (high index of refraction) medium to a fast (low index of refraction) medium. With total internal reflection, light refracts so much it actually refracts back into the first medium. This is how fiber optic cables work: no light leaves the wire.

- *Rayleigh scattering* occurs when light interacts with our atmosphere. The shorter the wavelength of light, the more strongly it is disturbed by collisions with atmospheric molecules. Accordingly, blue light from the Sun is preferentially *scattered* by these collisions into our line of sight. This is why the sky appears blue.

- *Beautiful sunsets* are another manifestation of Rayleigh scattering that occurs when light travels long distances through the atmosphere. The blue light and some green is scattered away, making the sun appear red.

- *Lenses*, made from curved pieces of glass, focus or de-focus light as it passes through. Lenses that focus light are called *converging* lenses, and these are the ones used to make telescopes and cameras. Lenses that de-focus light are called *diverging* lenses.

Converging lens

Diverging lens

- Lenses can be used to make visual representations, called *images*.

- *Mirrors* are made from highly reflective metal that is applied to a curved or flat piece of glass. Converging mirrors can be used to focus light – headlights, telescopes, satellite TV receivers, and solar cookers all rely on this principle. Like lenses, mirrors can create images.

- The *focal length*, f, of a lens or mirror is the distance from the surface of the lens or mirror to the place where the light is focused. This is called the *focal point* or *focus*. For diverging lenses or mirrors, the focal length is negative.

- When light rays converge in front of a mirror or behind a lens, a *real* image is formed. Real images are useful in that you can place photographic film at the physical location of the real image, expose the film to the light, and make a two-dimensional representation of the world, a photograph.

- When light rays diverge in front of a mirror or behind a lens, a *virtual* image is formed. A virtual image is a trick, like the person you see "behind" a mirror's surface when you brush your teeth. Since virtual images aren't actually "anywhere," you can't place photographic film anywhere to capture them.
- Real images are upside-down, or *inverted*. You can make a real image of an object by putting it farther from a mirror or lens than the focal length. Virtual images are typically right-side-up. You can make virtual images by moving the mirror or lens closer to the object than the focal length.
- Waves are characterized by their ability to constructively and destructively *interfere*. Light waves which interfere with themselves after interaction with a small aperture or target are said to *diffract*.
- Light creates interference patterns when passing through holes ("slits") in an obstruction such as paper or the surface of a CD, or when passing through a thin film such as soap.

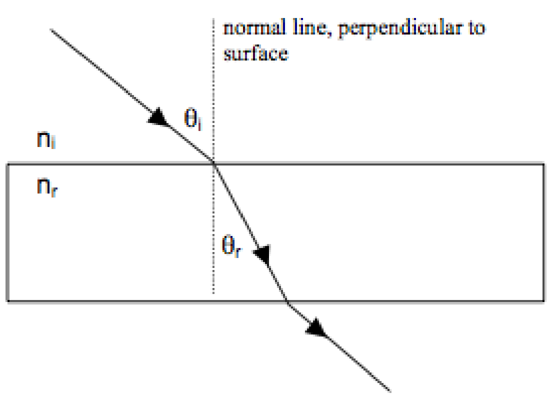
Key Equations

$$\lambda f = c$$

The product of the wavelength λ of the light (in meters) and the frequency f of the light (in Hz, or $1/\text{sec}$) is always equal to a constant, namely the speed of light $c = 300,000,000 \text{ m/s}$.

$$n = \frac{c}{u}$$

The index of refraction, n, is the ratio of its speed (c) in a vacuum to the slower speed (u) it travels in a material. n can depend slightly on wavelength.



$$n_i \sin\theta_i = n_f \sin\theta_f$$

$$m\lambda = d \sin\theta$$

Double slit interference maxima. m is the order of the interference maximum in question, d is the distance between slits. and θ is the angular position of the maximum.

$$m\lambda = d\sin\theta$$

Single slit interference maxima. m and θ are defined as above and d is the width of the slit.

$$m\lambda = d\sin\theta$$

Diffraction grating interference maxima. m and θ are defined as above and d is the distance between the lines on the grating.

$$m\lambda = 2nd$$

Thin film interference: n is the index of refraction of the film, d is the thickness of the film, and m is an integer. In the film interference, there is a $\lambda/2$ delay (phase change) if the light is reflected from an object with an index of refraction greater than that of the incident material.

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

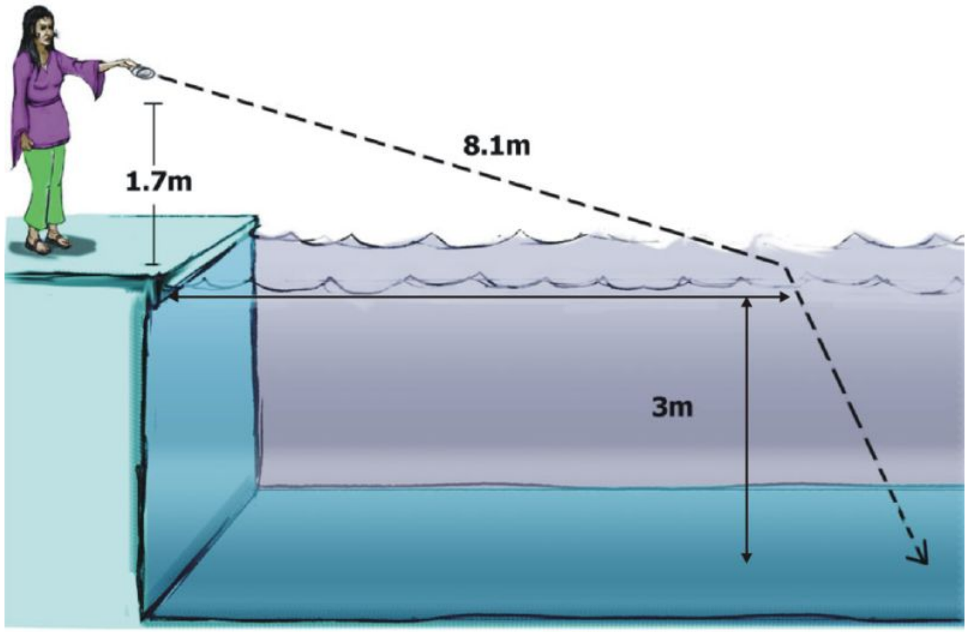
For lenses, the distance from the center of the lens to the focus is f. Focal lengths for foci behind the lens are positive in sign. The distance from the center of the lens to the object in question is d_0, where distances to the left of the lens are positive in sign. The distance from the center of the lens to the image is d_i. This number is positive for real images (formed to the right of the lens), and negative for virtual images (formed to the left of the lens). For mirrors, the same equation holds! However, the object and image distances are both positive for real images formed to the left of the mirror. For virtual images formed to the right of the mirror, the image distance is negative

$$M = \frac{-d_i}{d_0}$$

The size of an object's image is larger (or smaller) than the object itself by its magnification, M. The level of magnification is proportional to the ratio of d_i and d_o. An image that is double the size of the object would have magnification $M = 2$.

$$R = 2f$$

The radius of curvature of a mirror is twice its focal length



Question: Nisha stands at the edge of an aquarium 3.0m deep. She shines a laser at a height of 1.7m that hits the water of the pool 8.1m from the edge. Draw a diagram of this situation. Label all known lengths.

a) How far from the edge of the pool will the light hit bottom?

b) If her friend, Marc, were at the bottom and shined a light back, hitting the same spot as Nisha's, how far from the edge would he have to be so that the light never leaves the water?

Answer:

a) To solve for the distance from the edge we must first solve for the distance from the laser to the pool surface and then add that to the distance from the pool surface to the bottom of the pool. We can find the distance from the laser to the pool by using the Pythagorean Theorem.

$$a^2 + b^2 = c^2 \Rightarrow b = \sqrt{c^2 - a^2} = 8.1^2\text{m} - 1.7^2\text{m} = 7.9\text{m}$$

Now that we have the length from the laser to the pull all we need is the length from the surface of the pool to the bottom of it.

To find this value we will use the equation

$$n_a \times sin\theta_a = n_w \times sin\theta_w$$

Once we have solved for θ_w, we will be able to use trigonometry to solve for the distance from the surface of the pool to the bottom of the pool. We know that $n_a = 1.00029$ and that $n_w = 1.33$. So once we solve for θ_a, we can solve for θ_w.

$$sin^{-1}\frac{1.7}{8.1} = 12.1^o$$

This is the complement of θ_a, so

$$90^o - 12.1^o = 77.9^o$$

Now we can solve for θ_w.

$$n_a \times sin\theta_a = n_w \times sin\theta_w \Rightarrow \theta_w = sin^{-1}(\frac{n_a \times sin\theta_a}{n_w}) = sin^{-1}(\frac{1.00029 \times .98}{1.33}) = 47.5^o$$

Now we can use trigonometry.

$$tan\theta = \frac{\text{opposite}}{\text{adjacent}} \Rightarrow \text{opposite} = \text{adjacent} \times tan\theta = 3\text{m} \times tan47.5^o = 3.3\text{m}$$

Now we simply need to add the two distances together to get our answer.

$$3.3\text{m} + 7.9\text{m} = 11.2\text{m}$$

b) For the beam of light to never leave the water, $sin^{-1}\theta_a = 90$. For this to be true, $\theta_a = 1$. So, we will use following equation and then substitute 1 for θ_a. This will allow us to solve for θ_w which will, in turn, allow for us to solve for James' distance from the edge of the pool.

$$n_a \times sin\theta_a = n_w \times sin\theta_w \Rightarrow n_a \times 1 = n_w \times sin\theta_w \Rightarrow \theta_w = sin^{-1}(\frac{n_a}{n_w}) = sin^{-1}(\frac{1.00029}{1.33}) = 48.8^o$$

Now we can use trigonometry to get our answer.

$$tan\theta = \frac{\text{opposite}}{\text{adjacent}} \Rightarrow \text{opposite} = \text{adjacent} \times tan\theta = 3\text{m} \times tan48.8^o = 3.4\text{m}$$

$$3.4\text{m} + 7.9\text{m} = 11.3\text{m}$$

Light Problem Set

1. Which corresponds to light of longer wavelength, UV rays or IR rays?
2. Which corresponds to light of lower frequency, $x-$rays or millimeter-wavelength light?
3. Approximately how many blue wavelengths would fit end-to-end within a space of one millimeter?
4. Approximately how many short ("hard") $x-$rays would fit end-to-end within the space of a single red wavelength?
5. Calculate the frequency in Hz of a typical green photon emitted by the Sun. What is the physical interpretation of this (very high) frequency? (That is, what is oscillating?)
6. FM radio stations list the frequency of the light they are emitting in MHz, or millions of cycles per second. For instance, 90.3 FM would operate at a frequency of 90.3×10^6 Hz. What is the wavelength of the radio-frequency light emitted by this radio station? Compare this length to the size of your car's antenna, and make an argument as to why the length of a car's antenna should be about the wavelength of the light you are receiving.
7. Consult the color table for human perception under the 'Key Concepts' section and answer the questions which follow.
   a. Your coat looks magenta in white light. What color does it appear in blue light? In green light?
   b. Which secondary color would look black under a blue light bulb?
   c. You look at a cyan-colored ribbon under white light. Which of the three primary colors is your eye *not* detecting?
8. Consider the following table, which states the indices of refraction for a number of materials.

TABLE 17.3:

Material	*n*
vacuum	1.00000
air	1.00029

TABLE 17.3: (continued)

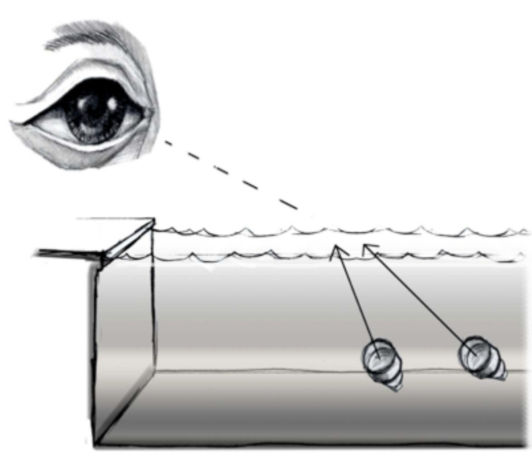
Material	n
water	1.33
typical glass	1.52
cooking oil	1.53
heavy flint glass	1.65
sapphire	1.77
diamond	2.42

(a) For which of these materials is the speed of light *slowest*?

(b) Which two materials have the most similar indices of refraction?

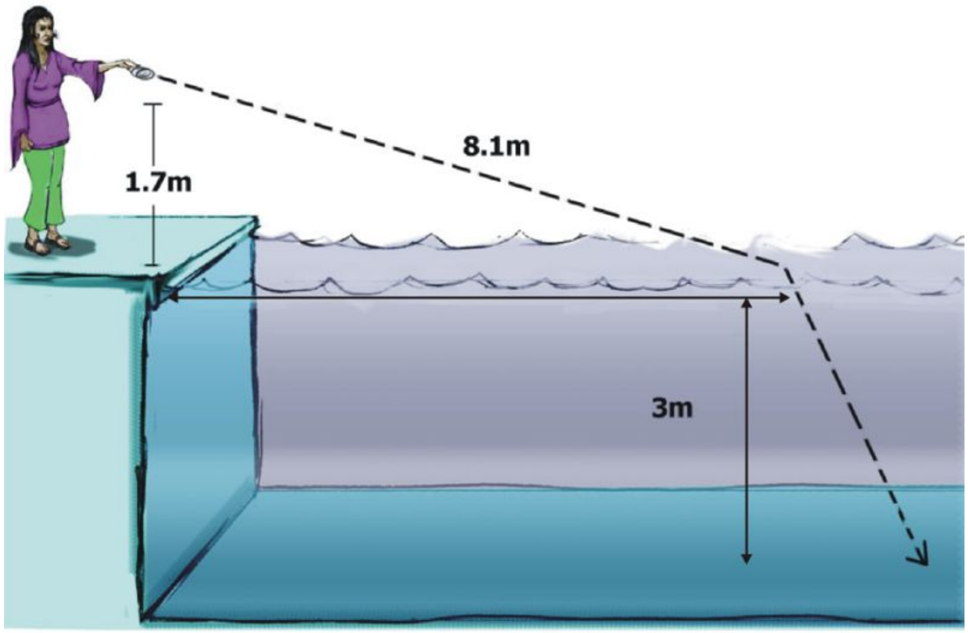
(c) What is the speed of light in cooking oil?

9. A certain light wave has a frequency of 4.29×10^{14} Hz. What is the wavelength of this wave in empty space? In water?
10. A light ray bounces off a fish in your aquarium. It travels through the water, into the glass side of the aquarium, and then into air. Draw a sketch of the situation, being careful to indicate how the light will change directions when it refracts at each interface. Include a brief discussion of why this occurs.
11. Why is the sky blue? Find a family member who doesn't know why the sky is blue and explain it to them. Ask them to write a short paragraph explaining the situation and include a sketch.
12. Describe the function of the dye in blue jeans. What does the dye do to each of the various colors of visible light?
13. A light ray goes from the air into the water. If the angle of incidence is 34°, what is the angle of refraction?
14. In the "disappearing test tube" demo, a test tube filled with vegetable oil vanishes when placed in a beaker full of the same oil. How is this possible? Would a diamond tube filled with water and placed in water have the same effect?
15. Imagine a thread of diamond wire immersed in water. Can such an object demonstrate total internal reflection? If so, what is the critical angle? Draw a picture along with your calculations.



16. A light source sits in a tank of water, as shown.
    a. If one of the light rays coming from inside the tank of water hits the surface at 35.0°, as measured from the normal to the surface, at what angle will it enter the air?
    b. Now suppose the incident angle in the water is 80° as measured from the normal. What is the refracted angle? What problem arises?

c. Find the *critical angle* for the water-air interface. This is the incident angle that corresponds to the largest possible refracted angle, 90°.

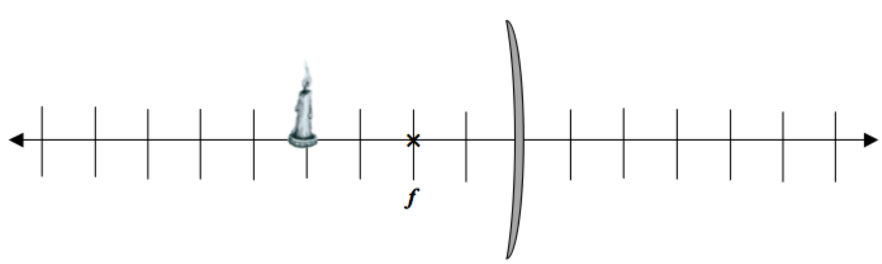


17. Nisha stands at the edge of an aquarium 3.0 m deep. She shines a laser at a height of 1.7 m that hits the water of the pool 8.1 m from the edge.
    a. Draw a diagram of this situation. Label all known lengths.
    b. How far from the edge of the pool will the light hit bottom?
    c. If her friend, James, were at the bottom and shined a light back, hitting the same spot as Nisha's, how far from the edge would he have to be so that the light never leaves the water?
18. Here's an example of the "flat mirror problem." Marjan is looking at herself in the mirror. Assume that her eyes are 10 cm below the top of her head, and that she stands 180 cm tall. Calculate the minimum length flat mirror that Marjan would need to see her body from eye level all the way down to her feet. Sketch at least 3 ray traces from her eyes showing the topmost, bottommost, and middle rays. *In the following five problems, you will do a careful ray tracing with a ruler (including the extrapolation of rays for virtual images). It is best if you can use different colors for the three different ray tracings. When sketching diverging rays, you should use dotted lines for the extrapolated lines behind a mirror or in front of a lens in order to produce the virtual image. When comparing measured distances and heights to calculated distances and heights, values within 10% are considered "good." Use the following cheat sheet as your guide*.

TABLE 17.4:

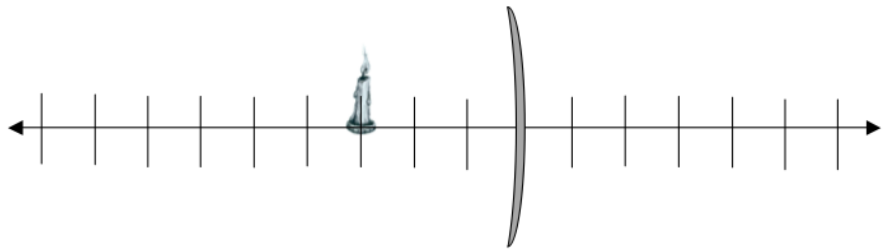
CONVERGING(CONCAVE)MIRRORS	Ray #1: Leaves tip of candle, travels parallel to optic axis, reflects back through focus. Ray #2: Leaves tip, travels through focus, reflects back parallel to optic axis. Ray #3: Leaves tip, reflects off center of mirror with an angle of reflection equal to the angle of incidence.
DIVERGING (CONVEX) MIRRORS	Ray #1: Leaves tip, travels parallel to optic axis, reflects OUTWARD by lining up with focus on the OPPOSITE side as the candle. Ray #2: Leaves tip, heads toward the focus on the OPPOSITE side, and emerges parallel to the optic axis. Ray #3: Leaves tip, heads straight for the mirror center, and reflects at an equal angle.
CONVERGING (CONVEX) LENSES	Ray #1: Leaves tip, travels parallel to optic axis, refracts and travels through to the focus. Ray #2: Leaves tip, travels through focus on same side, travels through lens, and exits lens parallel to optic axis on opposite side. Ray #3: Leaves tip, passes straight through center of lens and exits without bending.
DIVERGING (CONCAVE) LENSES	Ray #1: Leaves tip, travels parallel to optic axis, refracts OUTWARD by lining up with focus on the SAME side as the candle. Ray #2: Leaves tip, heads toward the focus on the OPPOSITE side, and emerges parallel from the lens. Ray #3: Leaves tip, passes straight through the center of lens and exits without bending.

19. Consider a concave mirror with a focal length equal to two units, as shown below. (a) *Carefully* trace three rays coming off the top of the object in order to form the image.



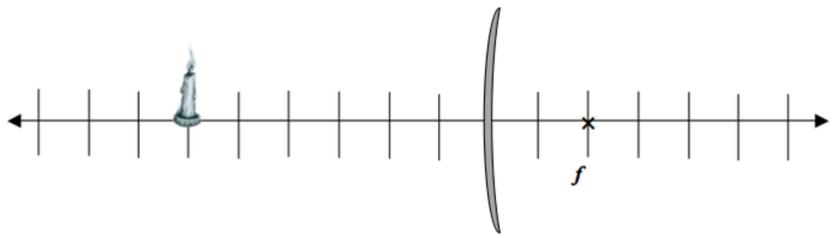
(b) Measure d_o and d_i. (c) Use the mirror/lens equation to calculate d_i. (d) Find the percent difference between your measured d_i and your calculated d_i. (e) Measure the magnification M and compare it to the calculated magnification.

20. Consider a concave mirror with unknown focal length that produces a virtual image six units behind the mirror. (a) Calculate the focal length of the mirror and draw an $\times$ at the position of the focus. (b) *Carefully* trace three rays coming off the top of the object and show how they converge to form the image.



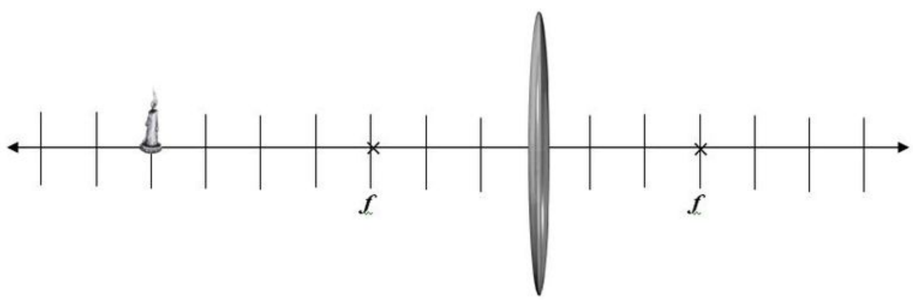
(c) Does your image appear bigger or smaller than the object? Calculate the expected magnification and compare it to your sketch.

21. Consider a convex mirror with a focal length equal to two units. (a) *Carefully* trace three rays coming off the top of the object and form the image.



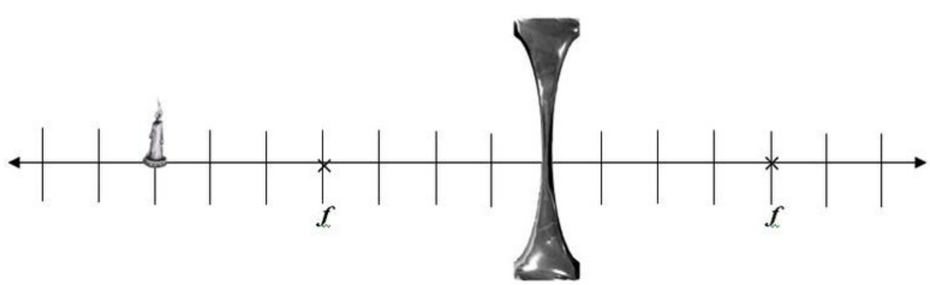
(b) Measure d_o and d_i. (c) Use the mirror/lens equation to calculate d_i. (d) Find the percent difference between your measured d_i and your calculated d_i. (e) Measure the magnification M and compare it to the calculated magnification.

22. Consider a converging lens with a focal length equal to three units. (a) *Carefully* trace three rays coming off the top of the object and form the image.



(b) Measure d_o and d_i. (c) Use the mirror/lens equation to calculate d_i. (d) Find the percent difference between your measured d_i and your calculated d_i. (e) Measure the magnification M and compare it to the calculated magnification.

23. Consider a diverging lens with a focal length equal to four units.
    a. *Carefully* trace three rays coming off the top of the object and show where they converge to form the image.



    b.

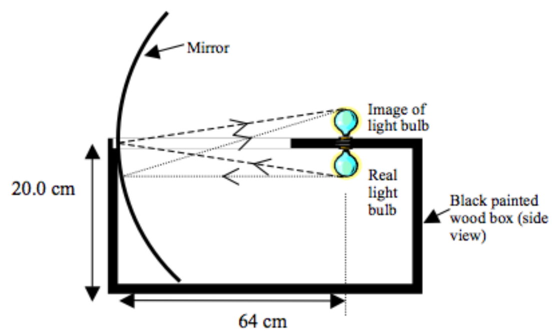
c. Measure d_o and d_i.
d. Use the mirror/lens equation to calculate d_i.
e. Find the percent difference between your measured d_i and your calculated d_i.
f. Measure the magnification M and compare it to the calculated magnification.

24. A piece of transparent goo falls on your paper. You notice that the letters on your page appear smaller than they really are. Is the goo acting as a converging lens or a diverging lens? Explain. Is the image you see real or virtual? Explain.
25. An object is placed 30 mm in front of a lens. An image of the object is located 90 mm behind the lens.

a. Is the lens converging or diverging? Explain your reasoning.
b. What is the focal length of the lens?

26. Little Red Riding Hood (*aka* $R-$Hood) gets to her grandmother's house only to find the Big Bad Wolf (*aka* BBW) in her place. $R-$Hood notices that BBW is wearing her grandmother's glasses and it makes the wolf's eyes look magnified (bigger).

a. Are these glasses for near-sighted or far-sighted people? For full credit, explain your answer thoroughly. You may need to consult some resources online.
b. Create a diagram of how these glasses correct a person's vision.



27. To the right is a diagram showing how to make a "ghost light bulb." The real light bulb is below the box and it forms an image of the exact same size right above it. The image looks very real until you try to touch it. What is the focal length of the concave mirror?
28. In your laboratory, light from a 650 nm laser shines on two thin slits. The slits are separated by 0.011 mm. A flat screen is located 1.5 m behind the slits.

a. Find the angle made by rays traveling to the third maximum off the optic axis.
b. How far from the center of the screen is the third maximum located?
c. How would your answers change if the experiment was conducted underwater?

29. Again, in your laboratory, 540 nm light falls on a pinhole 0.0038 mm in diameter. Diffraction maxima are observed on a screen 5.0 m away.

a. Calculate the distance from the central maximum to the first interference maximum.
b. Qualitatively explain how your answer to (a) would change if you . . .

i. . . . move the screen closer to the pinhole. ii. . . . increase the wavelength of light. iii. . . . reduce the diameter of the pinhole.

30. You are to design an experiment to determine the index of refraction of an unknown liquid. You have a small square container of the liquid, the sides of which are made of transparent thin plastic. In addition you have a screen, laser, ruler and protractors. Design the experiment. Give a detailed procedure; include a diagram of the experiment. Tell which equations you would use and give some sample calculations. Finally, tell in detail what level of accuracy you can expect and explain the causes of lab error in order of importance.

31. Students are doing an experiment with a Helium-neon laser, which emits 632.5 nm light. They use a diffraction grating with 8000 lines/cm. They place the laser 1 m from a screen and the diffraction grating, initially, 95 cm from the screen. They observe the first and then the second order diffraction peaks. Afterwards, they move the diffraction grating closer to the screen.

(a) Fill in the table below with the *expected* data based on your understanding of physics. Hint: find the general solution through algebra *before* plugging in any numbers.

TABLE 17.5:

Distance of diffraction grating to screen (cm)	Distance from central maximum to first order peak (cm)
95	
75	
55	
35	
15	

(b) Plot a graph of the first order distance as a function of the distance between the grating and the screen.

(c) How would you need to manipulate this data in order to create a *linear* plot?

(d) In a real experiment what could cause the data to deviate from the expected values? Explain.

(e) What safety considerations are important for this experiment?

(f) Explain how you could use a diffraction grating to calculate the unknown wavelength of another laser.

32. An abalone shell, when exposed to white light, produces an array of cyan, magenta and yellow. There is a thin film on the shell that both refracts and reflects the light. Explain clearly why these and only these colors are observed.
33. A crystal of silicon has atoms spaced 54.2 nm apart. It is analyzed as if it were a diffraction grating using an $x-$ray of wavelength 12 nm. Calculate the angular separation between the first and second order peaks from the central maximum.
34. Laser light shines on an oil film $(n = 1.43)$ sitting on water. At a point where the film is 96 nm thick, a 1^{st} order dark fringe is observed. What is the wavelength of the laser?
35. You want to design an experiment in which you use the properties of thin film interference to investigate the variations in thickness of a film of water on glass.
    a. List all the necessary lab equipment you will need.
    b. Carefully explain the procedure of the experiment and draw a diagram.
    c. List the equations you will use and do a sample calculation using realistic numbers.
    d. Explain what would be the most significant errors in the experiment and what effect they would have on the data.

Answers to Selected Problems

1. .
2. .
3. 2200 blue wavelengths
4. 65000 $x-$rays
5. 6×10^{14} Hz6.3.3 m

6. .
7. .
8. (b) vacuum air (c) 1.96×10^8 m/s
9. 6.99×10^{-7} m; $5.26 \times 10^{-}7$ m
10. .
11. .
12. Absorbs red and green.
13. 25°
14. .
15. 33.3°
    1. 49.7°
    2. No such angle
    3. 48.8°
16. see example problem
17. 85 cm
18. (c) +4 units (e) −1
    1. 6 units
    2. bigger; $M = 3$
19. (c) 1.5 units (d) 2/3
20. (c) 3 units (e) −2/3
21. (c) 5.3 units
22. .
23. (b) 22.5 mm
24. .
25. 32 cm
    1. 10.2°
    2. 27 cm
    3. 20 cm
26. a. 0.72 m
27. .
28. 54 cm, 44 cm, 21 cm, 8.8 cm
29. .
30. 13.5°
31. 549 nm

Chapter 18 Fluids Version 2

Chapter Outline

18.1 The Big Idea

In studying fluids we apply the concepts of force, momentum, and energy – which we have learned previously – to new phenomena. Since fluids are made from a large number of individual molecules, we have to look at their behavior as an ensemble and not individually. For this reason, we use the concept of conservation of *energy density* in place of conservation of energy. Energy density is energy divided by volume.

Key Concepts

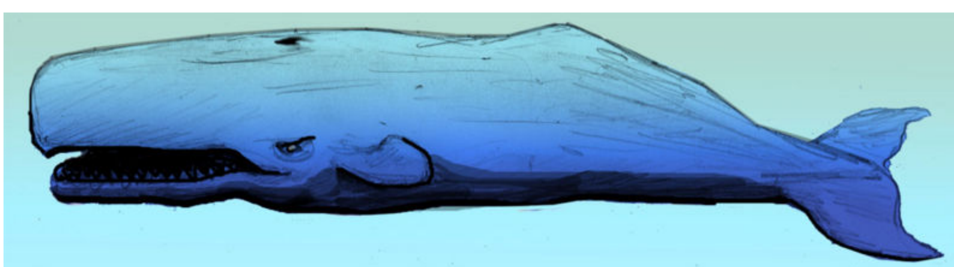
- The *pressure* of a fluid is a measure of the forces exerted by a large number of molecules when they collide and bounce off its boundary. The unit of pressure is the Pascal (Pa).
- Mass density represents the amount of mass in a given volume. We also speak of fluids as having gravitational potential energy *density*, kinetic energy *density*, and momentum *density*. These represent the amount of energy or momentum possessed by a given volume of fluid. If we multiply these quantities by a volume, they will be completely identical to their versions from earlier chapters.
- Pressure and energy density have the same units: $1 \text{ Pa} = 1 \text{ N/m}^2 = 1 \text{ J/m}^3$. The pressure of a fluid can be thought of as an arbitrary level of energy density.
- For static fluids and fluids flowing in a steady state all locations in the connected fluid system must have the same total energy density. This means that the algebraic sum of pressure, kinetic energy density and gravitational energy density equals zero. Changes in fluid pressure must be equal to changes in energy density (kinetic and/or gravitational).
- Liquids obey a *continuity equation* which is based on the fact that liquids are very difficult to compress. This means that the total volume of a fluid will remain constant in most situations. Imagine trying to compress a filled water balloon!
- The *specific gravity* of an object is the ratio of the density of that object to the density of water. Objects with specific gravities greater than 1.0 (*i.e.,* greater than water) will sink in water; otherwise, they will float. The density of fresh water is 1000 kg/m^3.

Key Equations and Definitions

$$\begin{cases} \rho = \frac{M}{V} & \text{Mass density, in kg/m}^3 \\ u_g = \rho g h & \text{Gravitational potential energy density of a fluid per unit volume} \\ k = \frac{1}{2}\rho v^2 & \text{Kinetic energy density of a fluid per unit volume} \\ P = \frac{F}{A} & \text{Pressure is force per unit area} \\ \Delta P + \Delta k + \Delta u_g = 0 & \text{Bernoulli's principle} \\ \Phi = A \cdot v & \text{Flux of fluid with velocity}v\text{through area }A \\ F_{\text{buoy}} = \rho_{\text{water}} g V_{\text{displaced}} & \text{Archimedes' principle} \end{cases}$$

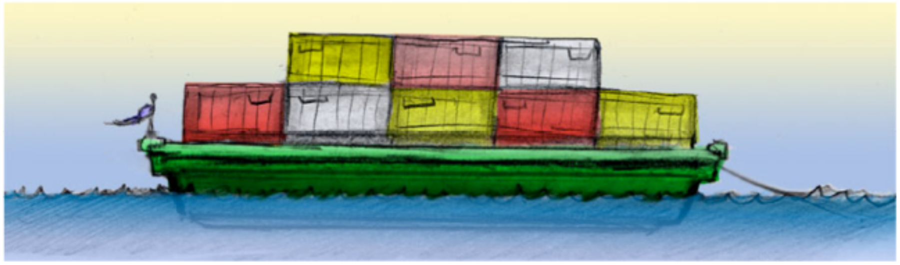
Key Applications

- In a fluid at rest, pressure increases linearly with depth – this is due to the weight of the water above it.
- *Archimedes' Principle* states that the upward buoyant force on an object in the water is equal to the weight of the displaced volume of water. The reason for this upward force is that the bottom of the object is at lower depth, and therefore higher pressure, than the top. If an object has a higher density than the density of water, the weight of the displaced volume will be less than the object's weight, and the object will sink. Otherwise, the object will float.
- *Pascal's Principle* reminds us that, for a fluid of uniform pressure, the force exerted on a small area in contact with the fluid will be smaller than the force exerted on a large area. Thus, a small force applied to a small area in a fluid can create a large force on a larger area. This is the principle behind hydraulic machinery.
- *Bernoulli's Principle* is a restatement of the conservation of energy, but for fluids. The sum of pressure, kinetic energy density, and gravitational potential energy density is conserved. In other words, $\Delta P + \Delta k + \Delta u_g$ equals zero. One consequence of this is that a fluid moving at higher speed will exhibit a *lower* pressure, and vice versa. There are a number of common applications for this: when you turn on your shower, the moving water and air reduce the pressure in the shower stall, and the shower curtain is pulled inward; when a strong wind blows outside your house, the pressure decreases, and your shutters are blown open; due to the flaps on airplane wings, the speed of the air below the wing is lower than above the wing, which means the pressure below the wing is higher, and provides extra lift for the plane during landing. There are many more examples.
- Conservation of flux, Φ, means that a smaller fluid-carrying pipe requires a faster moving fluid. Bernoulli's Principle, which says that fast-moving fluids have low pressure, provides a useful result: pressure in a smaller pipe must be lower than pressure in a larger pipe.
- If the fluid is not in a steady state, energy can be lost in fluid flow. The loss of energy is related to *viscosity*, or deviation from smooth flow. Viscosity is related to *turbulence*, the tendency of fluids to become chaotic in their motion. In a high viscosity fluid, energy is lost from a fluid in a way that is quite analogous to energy loss due to current flow through a resistor. A pump can add energy to a fluid system also. The full Bernoulli Equation takes these two factors, viscosity and pumps, into account.



Fluids Problem Set

1. A block of wood with a density of 920 $\mathrm{kg/m^3}$ is floating in a fluid of density 1100 $\mathrm{kg/m^3}$. What fraction of the block is submerged, and what fraction is above the surface?



2. A rectangular barge 17 m long, 5 m wide, and 2.5 m in height is floating in a river. When the barge is empty, only 0.6 m is submerged. With its current load, however, the barge sinks so that 2.2 m is submerged. Calculate the mass of the load.
3. The density of ice is 90% that of water.

   a. Why does this fact make icebergs so dangerous?
   b. A form of the liquid naphthalene has a specific gravity of 1.58. What fraction of an ice cube would be submerged in a bath of naphthalene?

4. A cube of aluminum with a specific gravity of 2.70 and side length 4.00 cm is put into a beaker of methanol, which has a specific gravity of 0.791.

   a. Draw a free body diagram for the cube.
   b. Calculate the buoyant force acting on the cube.
   c. Calculate the acceleration of the cube toward the bottom when it is released.

5. A cube of aluminum (specific gravity of 2.70) and side length 4.00 cm is put in a beaker of liquid naphthalene (specific gravity of 1.58). When the cube is released, what is its acceleration?
6. Your class is building boats out of aluminum foil. One group fashions a boat with a square 10 cm by 10 cm bottom and sides 1 cm high. They begin to put 2.5 g coins in the boat, adding them until it sinks. Assume they put the coins in evenly so the boat doesn't tip. How many coins can they put in? (You may ignore the mass of the aluminum boat ... assume it is zero.)

7. You are riding a hot air balloon. The balloon is a sphere of radius 3.0 m and it is filled with hot air. The density of hot air depends on its temperature: assume that the density of the hot air is $0.925\ \mathrm{kg/m^3}$, compared to the usual $1.29\ \mathrm{kg/m^3}$ for air at room temperature. The balloon and its payload (including you) have a combined mass of 100 kg.

    a. Draw a free body diagram for the cube.
    b. Is the balloon accelerating upward or downward?
    c. What is the magnitude of the acceleration?
    d. Why do hot air ballooners prefer to lift off in the morning?
    e. What would limit the maximum height attainable by a hot air balloon?

8. You are doing an experiment in which you are slowly lowering a tall, empty cup into a beaker of water. The cup is held by a string attached to a spring scale that measures tension. You collect data on tension as a function of depth. The mass of the cup is 520 g, and it is long enough that it never fills with water during the experiment. The following table of data is collected:

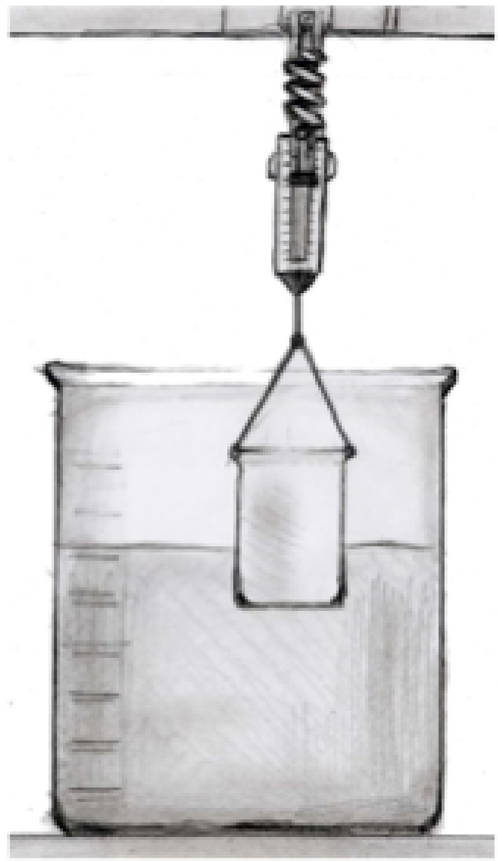


TABLE 18.1:

String tension (N)	Depth (cm)	Buoyant force (N)
5.2	0	
4.9	1	
4.2	3	
3.7	5	
2.9	8	
2.3	10	
1.7	12	
0.7	15	
0.3	16	
0	17	

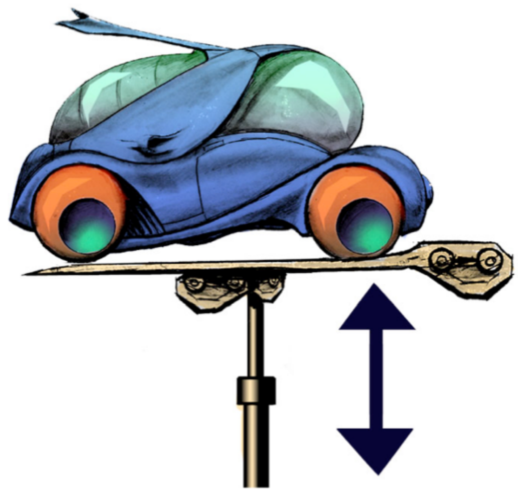
(a) Complete the chart by calculating the buoyant force acting on the cup at each depth.

(b) Make a graph of buoyant force vs. depth, find a best-fit line for the data points, and calculate its slope.

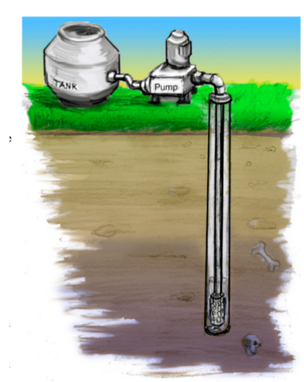
(c) What does this slope physically represent? (That is, what would a *greater* slope mean?)

(d) With this slope, and the value for the density of water, calculate the area of the circular cup's bottom and its radius.

(e) Design an experiment using this apparatus to measure the density of an unknown fluid.



9. A 1500 kg car is being lifted by a hydraulic jack attached to a flat plate. Underneath the plate is a pipe with radius 24 cm.
   a. If there is no net force on the car, calculate the pressure in the pipe.
   b. The other end of the pipe has a radius of 2.00 cm. How much force must be exerted at this end?
   c. To generate an upward acceleration for the car of $1.0\ \mathrm{m/s^2}$, how much force must be applied to the small end of the pipe?
10. A SCUBA diver descends deep into the ocean. Calculate the water pressure at each of the following depths.
   a. 15 m.
   b. 50 m.
   c. 100 m.
11. What happens to the gravitational potential energy density of water when it is siphoned out of a lower main ditch on your farm and put into a higher row ditch? How is this consistent with Bernoulli's principle?
12. Water flows through a horizontal water pipe 10.0 cm in diameter into a smaller 3.00 cm pipe. What is the ratio in water pressure between the larger and the smaller water pipes?



13. A pump is required to pipe water from a well 7.0 m in depth to an open-topped water tank at ground level. The pipe at the top of the pump, where the water pours into the water tank, is 2.00 cm in diameter. The water flow in the pipe is $5.00\ \mathrm{m/s}$.
   a. What is the kinetic energy density of the water flow?
   b. What pressure is required at the bottom of the well? (Assume no energy is lost – i.e., that the fluid is traveling smoothly.)
   c. What power is being delivered to the water by the pump? (Hint: For the next part, refer to Chapter 12)
   d. If the pump has an efficiency of 45%, what is the pump's electrical power consumption?

e. If the pump is operating on a 220 V power supply (typical for large pieces of equipment like this), how much electrical current does the pump draw?
f. At 13.5 cents per kilowatt-hour, how much does it cost to operate this pump for a month if it is running 5% of the time?

14. Ouch! You stepped on my foot! That is, you put a force of 550 N in an area of 9 cm^2 on the tops of my feet!
    a. What was the pressure on my feet?
    b. What is the ratio of this pressure to atmospheric pressure?
15. A submarine is moving directly upwards in the water at constant speed. The weight of the submarine is 500,000 N. The submarine's motors are off.
    a. Draw a sketch of the situation and a free body diagram for the submarine.
    b. What is the magnitude of the buoyant force acting on the submarine?
16. You dive into a deep pool in the river from a high cliff. When you hit the water, your speed was 20 m/s. About 0.75 seconds after hitting the water surface, you come to a stop before beginning to rise up towards the surface. Take your mass to be 60 kg.
    a. What was your average acceleration during this time period?
    b. What was the average net force acting on you during this time period?
    c. What was the buoyant force acting on you during this time period?

17. A glass of water with weight 10 N is sitting on a scale, which reads 10 N. An antique coin with weight 1 N is placed in the water. At first, the coin accelerates as it falls with an acceleration of $g/2$. About half-way down the glass, the coin reaches terminal velocity and continues at constant speed. Eventually, the coin rests on the bottom of the glass. What was the scale reading when…
    a. … the coin had not yet been released into the water?
    b. … the coin was first accelerating?
    c. … the coin reached terminal velocity?
    d. … the coin came to rest on the bottom?
18. You are planning a trip to the bottom of the Mariana Trench, located in the western Pacific Ocean. The trench has a maximum depth of 11,000 m, deeper than Mt. Everest is tall! You plan to use your bathysphere to descend to the bottom, and you want to make sure you design it to withstand the pressure. A bathysphere is a

spherical capsule used for ocean descent – a cable is attached to the top, and this cable is attached to a winch on your boat on the surface.

a. Name and sketch your bathysphere.
b. What is the radius of your bathysphere in meters? (You choose – estimate from your picture.)
c. What is the volume of your bathysphere in m^3?
d. What is the pressure acting on your bathysphere at a depth of $11,000$ m? The density of sea water is 1027 kg/m^3.
e. If you had a circular porthole of radius $0.10 \text{ m} (10 \text{ cm})$ on your bathysphere, what would the inward force on the porthole be?
f. If the density of your bathysphere is 1400 kg/m^3, what is the magnitude of the buoyant force acting on it when it is at the deepest point in the trench?
g. In order to stop at this depth, what must the tension in the cable be? (Draw an FBD!)

Answers to Selected Problems

1. 0.84
2. 1.4×10^5 kg
3. a. 90% of the berg is underwater b. 57%
4. b. 5.06×10^{-4} N c. 7.05 m/s^2
5. 4.14 m/s
6. 6. 40 coins
7. b. upward c. 4.5 m/s^2 d. Cooler air outside, so more initial buoyant force e. Thin air at high altitudes weighs almost nothing, so little weight displaced.
8. a. At a depth of 10 cm, the buoyant force is 2.9 N d. The bottom of the cup is 3 cm in radius
9. a. $83,000$ Pa b. 104 N c. 110 N
10. a. 248 kPa b. 591 kPa c. 1081 kPa
11. .
12. .0081
13. a. 12500 J/m^3 b. 184 kPa c. 1.16 kW d. 2.56 kW e. 11.8 A f. \$12.60
14. a. 611 kPa b. 6 atm
15. b. $500,000$ N
16. a. $27 \text{ m/s}^2, (2.7 \text{ g})$ upward b. 1600 N c. 2200 N
17. a. 10 N b. 10.5 N c. 11 N d. 11 N
18. a. "The Thunder Road" b. 2.0 m (note: here and below, you may choose differently) c. 33.5 m^3 e. 3.5 million N f. 111 MPa

CHAPTER 19 Thermodynamics and Heat Engines Version 2

Chapter Outline

19.1 The Big Ideas

Heat is a form of energy transfer. It can change the kinetic energy of a substance. For example, the average molecular kinetic energy of gas molecules is related to temperature. A heat engine turns a portion of the input heat (thermal energy) into mechanical work. A second portion of the input heat must be exhausted in order for the engine to have repetitive motion. Therefore, in a practical engine it is impossible for all the input heat to be converted to work.

Entropy is a measure of disorder, or the variety of ways in which a system can organize itself with the same total energy. The entropy of any isolated system always tends to disorder (i.e. entropy is always increasing). In the universe, the entropy of a subset (like evolution on Earth) can decrease (i.e. more order) but the total entropy of the universe is increasing (i.e. more disorder).

Thermodynamics is the study of heat engines. Any engine or power plant obeys the laws of thermodynamics. The *first law of thermodynamics* is a statement of conservation of energy. Total energy, including heat, is conserved in any process and in the complete cycle of a heat engine. The *second law of thermodynamics* as it applies to heat engines gives an absolute limit on the efficiency of any heat engine that goes through repetitious cycles.

19.2 Molecular Kinetic Theory of a Monatomic Ideal Gas

The empirical combined gas law is simply a generalization of observed relationships. Using kinetic theory, it is possible to derive it from the principles of Newtonian mechanics. Previously, we thought of an ideal gas as one that obeys the combined gas law exactly. Within the current model, however, we can give a specific definition. **We treat a monatomic ideal gas as a system of an extremely large number of very small particles in random motion that collide elastically between themselves and the walls of their container, where there are no interaction between particles other than collisions.**

Consider some amount (n atoms) of such a gas in a cubical container with side length L. Let's trace the path of a single gas atom as it collides with the walls:

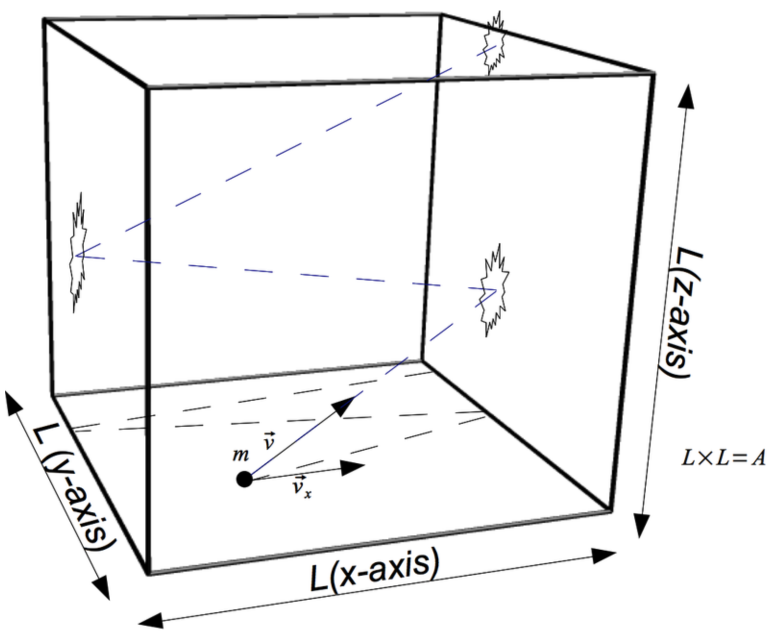


FIGURE 19.1

The path of a single gas atom as it undergoes collisions with the walls of its container

Further, let's restrict ourselves to considering the motion of the particle along the x axis, and its collisions with the right $y-z$ wall, as shown in the picture. Therefore, we only consider v_x the component of the velocity vector perpendicular to the $y-z$ wall.

If the particle's mass is m, in one collision, the particle's momentum in the x direction changes by

$$\Delta p = 2mv_x$$

Also, since it has to travel a distance $2L$ (back and forth, basically) in the x direction between collisions with the right $y-z$ wall, the time δt between collisions will be

$$\Delta t = \frac{2L}{v_x}$$

.

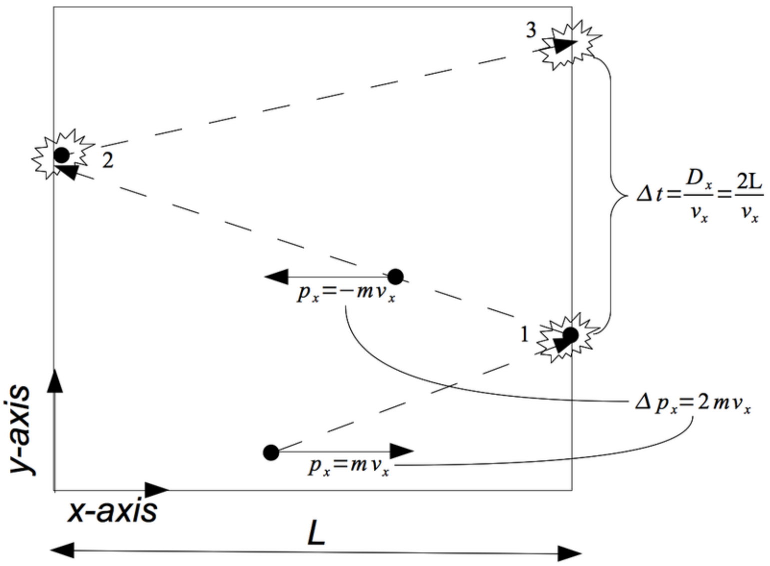


FIGURE 19.2

Illustration x-direction atom from above.

According to Newton's second law, the force imparted by the single particle on the wall is

$$F = \frac{\Delta p}{\Delta t} = 2mv_x \times \frac{v_x}{2L} = \frac{m{v_x}^2}{L}$$

Now, since there are n (a very large number) atoms present, the net force imparted on the wall will be

$$F_{\text{net}} = n \times \frac{m({v_x}^2)_{\text{avg}}}{L}$$

Where the${v_x}^2$ is averaged over all n atoms.

Now let us attempt to relate this to the state variables we considered last chapter. Recall that pressure is defined as force per unit area:

$$P = \frac{F}{A}$$

Since the area of the wall in question is L^2, the **pressure** exerted by the gas atoms on it will equal:

$$P_{\text{net}} = \frac{F_{\text{net}}}{A} = n\frac{m({v_x}^2)_{\text{avg}}}{L \times L^2} = n\frac{m({v_x}^2)_{\text{avg}}}{L^3}$$

Since, for a cubical box, volume $V = L^3$, the formula above can be reduced to:

$$P_{\text{net}} = n\frac{m({v_x}^2)_{\text{avg}}}{V} \qquad \text{or,}$$

$$P_{\text{net}}V = nm({v_x}^2)_{\text{avg}} \qquad [1]$$

By the Pythagorean theorem, any three-dimensional velocity vector has the following property:

$$v^2 = {v_x}^2 + {v_y}^2 + {v_z}^2$$

Averaging this for the *n* particles in the box, we get

$$(v^2)_{\text{avg}} = ({v_x}^2)_{\text{avg}} + ({v_y}^2)_{\text{avg}} + ({v_z}^2)_{\text{avg}}$$

Since the motions of the particles are completely random (as stated in our assumptions), it follows that the averages of the squares of the velocity components should be equal: there is no reason the gas particles would prefer to travel in the *x* direction over any other. In other words,

$$({v_x}^2)_{\text{avg}} = ({v_y}^2)_{\text{avg}} = ({v_z}^2)_{\text{avg}}$$

Plugging this into the average equation above, we find:

$$(v^2)_{\text{avg}} = 3 \times ({v_x}^2)_{\text{avg}} = 3 \times ({v_y}^2)_{\text{avg}} = 3 \times ({v_z}^2)_{\text{avg}}$$

and

$$(v^2)_{\text{avg}}/3 = ({v_x}^2)_{\text{avg}}$$

Plugging this into equation [1], we get:

$$P_{\text{net}}V = \frac{nm(v^2)_{\text{avg}}}{3} \qquad [2]$$

The left side of the equation should look familiar; this quantity is proportional to the average **kinetic energy** of the molecules in the gas, since

$$KE_{\text{avg}} = \frac{1}{2}m(v^2)_{\text{avg}}$$

Therefore, we have:

$$P_{\text{net}}V = \frac{2}{3}n(KE)_{\text{avg}} \qquad [3]$$

This is a very important result in kinetic theory, since it expresses the product of two **state variables**, or system parameters, pressure and volume, in terms of an average over the microscopic constituents of the system. Recall the empirical ideal gas law from last chapter:

$$PV = nkT$$

The left side of this is identical to the left side of equation [3], whereas the only variable on the right side is temperature. By setting the left sides equal, we find:

$$\frac{2}{3}n(KE)_{\text{avg}} = nkT$$

or

$$T = \frac{2}{3k}(KE)_{\text{avg}}$$

Therefore, according to the kinetic theory of an monoatomic ideal gas, the quantity we called temperature is — up to a constant coefficient — a direct measure of the average kinetic energy of the atoms in the gas. This definition of temperature is much more specific than the one from the previous chapter, since it is based essentially on Newtonian mechanics, rather than a somewhat ambiguous system of ranking.

Temperature, Again

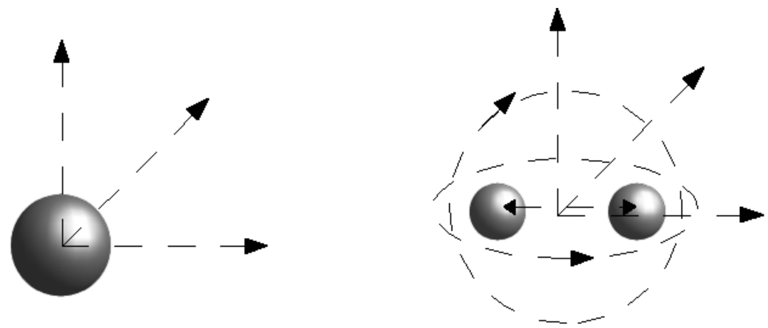
Now that we have defined temperature for a monoatomic gas, a relevant question is: can we extend this definition to other substances? It turns out that yes, we can, but with a significant caveat. In fact, according to classical kinetic theory, temperature is always proportional to the average kinetic energy of molecules in a substance. The constant of proportionality, however, is not always the same.

Consider: the only way to increase the kinetic energies of the atoms in a monoatomic gas is to increase their translational velocities. Accordingly, we assumed above that the kinetic energies of such atoms are stored equally in the three components ($x, y,$ and z) of their velocities.

On the other hand, other gases — called diatomic — consist of two atoms held by a bond. This bond can be modeled as a spring, and the two atoms and bond together as a harmonic oscillator. Now, a single molecule's kinetic energy can be increased *either by increasing its speed, by making it vibrate in simple harmonic motion, or by making it rotate around its center of mass*. This difference is understood in physics through the concept of **degrees of freedom**: each degree of freedom for a molecule or particle corresponds to a possibility of increasing its kinetic energy independently of the kinetic energy in other degrees.

It might seem to you that monatomic gases should have one degree of freedom: their velocity. They have three because their velocity can be altered in one of three mutually perpendicular directions without changing the kinetic energy in other two — just like a centripetal force does not change the kinetic energy of an object, since it is always perpendicular to its velocity. These are called translational degrees of freedom.

Diatomic gas molecules, on the other hand have more: the three translational explained above still exist, but there are now also vibrational and rotational degrees of freedom. Monatomic and diatomic degrees of freedom can be illustrated like this:



Temperature is an *average of kinetic energy over degrees of freedom, not a sum.* Let's try to understand why this is in reference to our monoatomic ideal gas. In the derivation above, volume was constant; so, temperature was essentially proportional to pressure, which in turn was proportional to the kinetic energy **due to translational motion** of the molecules. If the molecules had been able to rotate as well as move around the box, they could have had the same kinetic energy with slower translational velocities, and, therefore, lower temperature. In other words, in that case, or assumption that the kinetic energy of the atoms only depends on their velocities, implied between equations [2] and [3], **would not have held**.Therefore, **the number of degrees of freedom in a substance determines the proportionality between molecular kinetic energy and temperature: the more degrees of freedom, the more difficult it will be to raise its temperature with a given energy input.**

In solids, degrees of freedom are usually entirely vibrational; in liquids, the situation becomes more complicated. We will not attempt to derive results about these substances, however.

A note about the above discussion: since the objects at the basis of our understanding of thermodynamics are atoms and molecules, quantum effects can make certain degrees of freedom inaccessible at specific temperature ranges. Unlike most cases in your current physics class, where these can be ignored, in this case, quantum effects can make an appreciable difference. For instance, the vibrational degrees of freedom of diatomic gas molecules discussed above are, for many gases, inaccessible in very common conditions, although we do not have the means to explain this within our theory. In fact, this was one of the first major failures of classical physics that ushered in the revolutionary discoveries of the early 20th century.

Thermal Energy

In light of the above derivation, it should not surprise you that the kinetic energy from motion of molecules contributes to what is called the **thermal energy** of a substance. This type of energy is called **sensible energy**. In ideal gases, this is the only kind of thermal energy present.

Solids and liquids also have a different type of thermal energy as well, called **Latent Energy**, which is associated with potential energy of their intermolecular bonds in that specific phase — for example the energy it takes to break the bonds between water molecules in melting ice (remember, we assumed molecules do not interact in the ideal gas approximation).

To recap, there are two types of **Thermal Energy**:

- **The kinetic energy from the random motion of the molecules or atoms of the substance, called** Sensible Energy
- The intermolecular potential energy associated with changes in the phase of a system (called **Latent Energy**).

Heat

The term **heat** is formally defined as a transfer of thermal energy between substances. Note that *heat is not the same as thermal energy*. Before the concept of thermal energy, physicists sometimes referred to the 'heat energy' of a substance, that is, the energy it received from actual 'heating' (heating here can be understood as it is defined above, though for these early physicists and chemists it was a more 'common sense' idea of heating: think beaker over Bunsen burner). The idea was then to try to explain thermodynamic phenomena through this concept.

The reason this approach fails is that — as stated in the paragraphs above — it is in fact thermal energy that is most fundamental to the science, and *'heating' is not the only way to change the thermal energy of a substance*. For example, if you rub your palms together, you increase the thermal energy of both palms.

Once heat (a transfer of thermal energy) is absorbed by a substance, it becomes indistinguishable from the thermal energy already present: what methods achieved that level of thermal energy is no longer relevant. In other words, 'to heat' is a well defined concept as a verb: its use automatically implies some kind of transfer. When heat using as a noun, one needs to be realize that it must refer to this transfer also, not something that can exist independently.

Specific Heat Capacity and Specific Latent Heat

The ideas in the paragraphs above can be understood better through the concept of **specific heat capacity** (or specific heat for short), which relates an increase in temperature of some mass of a substance to the amount of heat required to achieve it. In other words, for any substance, it **relates thermal energy transfers to changes in temperature.** It has units of Joules per kilogram Kelvin. Here is how we can define and apply specific heat (Q refers to heat supplied, m to the mass of the substance and c to its specific heat capacity):

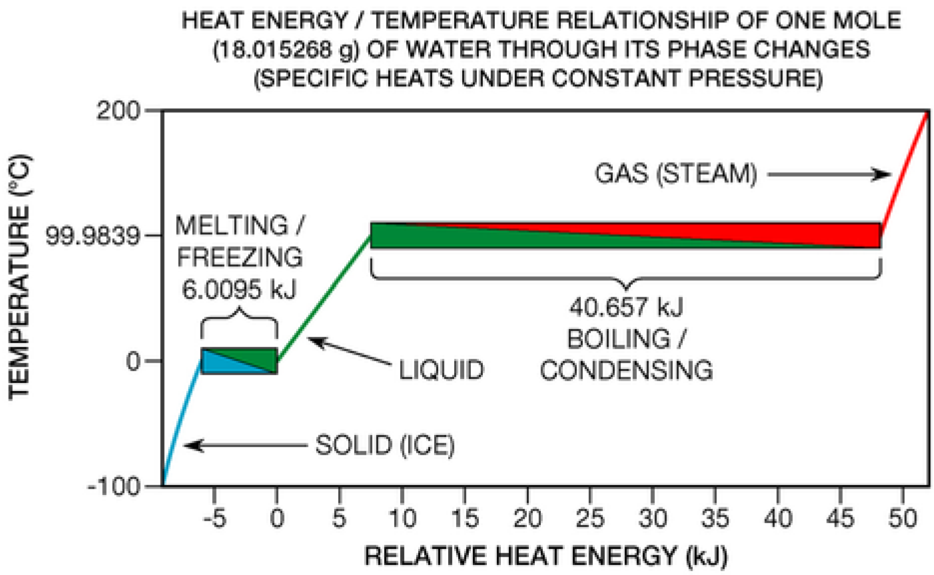
$$Q = cm\Delta T \qquad [2]$$

Heat capacity is largely determined by the number of degrees of freedom of the molecules in a substance (why?). However, it also depends on other parameters, such as pressure. Therefore, the formula above implicitly assumes that these external parameters are held constant (otherwise we wouldn't know if we're measuring a change in specific heat is real or due to a change in pressure).

When a substance undergoes a phase change, its temperature does not change as it absorbs heat. We referred to this as an increase or decrease in latent energy earlier. In this case, the relevant question is how much heat energy does it require to change a unit mass of the substance from one phase to another? This ratio is known as **latent heat**, and is related to heat by the following equation (L refers to the latent heat):

$$Q = Lm \qquad [3]$$

During a phase change, the number of degrees of freedom changes, and so does the specific heat capacity. Heat capacity can also depend on temperature within a given phase, but many substances, under constant pressure, exhibit a constant specific heat over a wide range of temperatures. For instance, here is a graph of temperature vs heat input for a **mole** (6.0221415×10^{23} molecules) of water. Note that the x-axis of the graph is called 'relative heat energy' because it takes a mole of water at 0 degrees Celcius as the reference point.

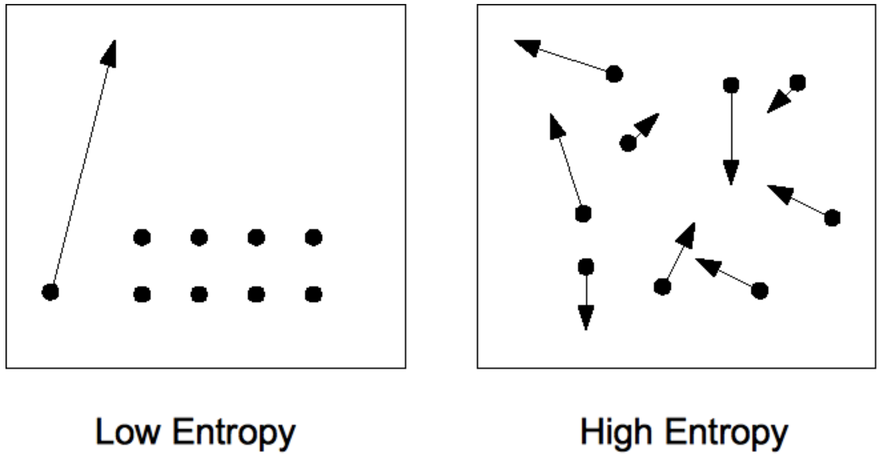


The sloped segments on the graph represent increases in temperature. These are governed by equation [1]. The flat segments represent phase transitions, governed by equation [2]. Notice that the sloped segments have constant, though different, slopes. According to equation [1], the heat capacity at any particular phase would be the slope of the segment that corresponds to that phase on the graph. The fact that the slopes are constant means that, within a particular phase, the heat capacity does not change significantly as a function of temperature.

Entropy

The last major concept we are going to introduce in this chapter is entropy. We noted earlier that temperature is determined not just by how much thermal energy is present in a substance, but also how it *can* be distributed. Substances whose molecules have more degrees of freedom will generally require more thermal energy for an equal temperature increase than those whose molecules have fewer degrees of freedom.

Entropy is very much related to this idea: it quantifies how the energy actually *is* distributed among the degrees of freedom available. In other words, it is a measure of disorder in a system. An example may illustrate this point. Consider a monatomic gas with N atoms (for any appreciable amount of gas, this number will be astronomical). It has $3N$ degrees of freedom. For any given value of thermal energy, there is a plethora of ways to distribute the energy among these. At one extreme, it could all be concentrated in the kinetic energy of a single atom. On the other, it could be distributed among them all. According to the discussion so far, these systems would have the same temperature and thermal energy. Clearly, they are not identical, however. This difference is quantified by entropy: the more evenly distributed the energy, the higher the entropy of the system. Here is an illustration:


Low Entropy
High Entropy

19.3 The Laws of Thermodynamics

Now that we have defined the terms that are important for an understanding of thermodynamics, we can state the laws that govern relevant behavior. These laws, unlike Newton's Laws or Gravity, are *not* based on new empirical observations: they can be derived based on statistics and known principles, such as conservation of energy. By understanding the laws of thermodynamics we can analyze **heat engines**, or machines that use heat energy to perform mechanical work.

The First Law

The **First Law of Thermodynamics** is simply a statement of energy conservation applied to thermodynamics systems: *the change in the internal* — for our purposes, this is the same as thermal — *energy (denoted* ΔU*) of a closed system is equal to the difference of net input heat and performed work*. In other words,

$$\Delta U = Q_{net} - W \qquad \text{[4] First Law}$$

Note that this does not explain how the system will transform input heat to work, it simply enforces the energy balance.

The Second Law

The **Second Law of Thermodynamics** states that *the entropy of an isolated system will always increase until it reaches some maximum value*. Consider it in light of the simplified example in the entropy section: if we allow the low entropy system to evolve, it seems intuitive collisions will eventually somehow distribute the kinetic energy among the atoms.

The Second Law generalizes this intuition to all closed thermodynamic systems. It is based on the idea that in a closed system, energy will be randomly exchanged among constituent particles — like in the simple example above — until the distribution reaches some equilibrium (again, in any macroscopic system there will be an enormous number of of atoms, degrees of freedom, etc). Since energy is conserved in closed systems, this equilibrium has to preserve the original energy total. In this equilibrium, the Second Law — fundamentally a probabilistic statement — posits that the energy will be distributed in the most likely way possible. This typically means that energy will be distributed evenly across degrees of freedom.

This allows us to formulate the **Second Law in another manner**, specifically: *heat will flow spontaneously from a high temperature region to a low temperature region, but not the other way*. This is just applying the thermodynamic vocabulary to the logic of the above paragraph: in fact, this is the reason for the given definition of temperature. When two substances are put in thermal contact (that is, they can exchange thermal energy), heat will flow from the system at the higher temperature (because it has more energy in its degrees of freedom) to the system with lower temperature until their temperatures are the same.

When a single system is out equilibrium, there will be a net transfer of energy from one part of it to another. In equilibrium, energy is still exchanged among the atoms or molecules, but not on a system-wide scale. Therefore, entropy places a limit on how much work a system can perform: the higher the entropy, the more even the distribution of energy, the less energy available for transfer.

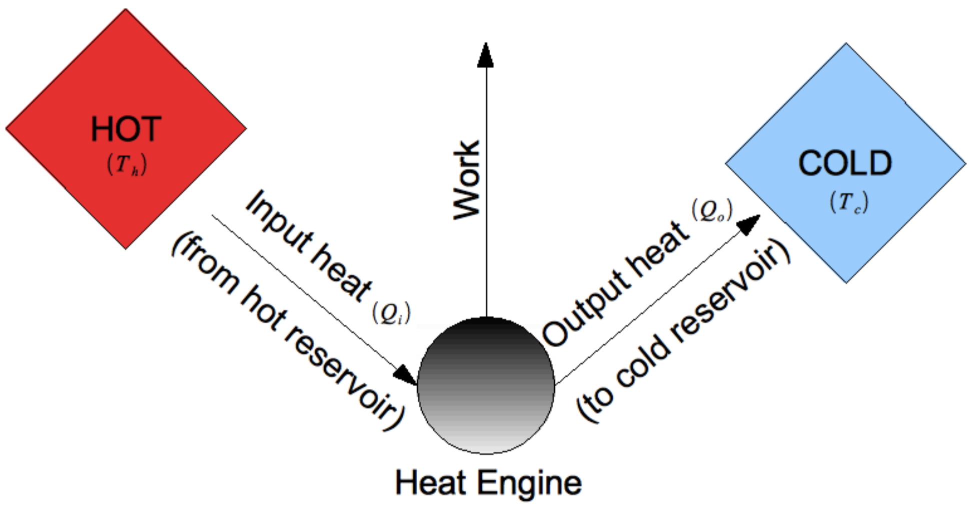
19.4 Heat Engines

Heat engines transform input heat into work in accordance with the laws of thermodynamics. For instance, as we learned in the previous chapter, increasing the temperature of a gas at constant volume will increase its pressure. This pressure can be transformed into a force that moves a piston.

The mechanics of various heat engines differ but their fundamentals are quite similar and involve the following steps:

1. Heat is supplied to the engine from some source at a higher temperature (T_h).
2. Some of this heat is transferred into mechanical energy through work done (W).
3. The rest of the input heat is transferred to some source at a lower temperature (T_c) until the system is in its original state.

A single cycle of such an engine can be illustrated as follows:



In effect, such an engine allows us to 'siphon off' part of the heat flow between the heat source and the heat sink. The efficiency of such an engine is define as the ratio of net work performed to input heat; this is the fraction of heat energy converted to mechanical energy by the engine:

$$e = \frac{W}{Q_i}$$ [5] Efficiency of a heat engine

If the engine does not lose energy to its surroundings (of course, all real engines do), then this efficiency can be rewritten as

$$e = \frac{Q_i - Q_o}{Q_i}$$ [6] Efficiency of a lossless heat engine

A **Carnot Engine**, the most efficient heat engine possible, has an efficiency equal to

$$e_c = 1 - \frac{T_c}{T_h}$$ [7] Efficiency of a Carnot (ideal) heat engine

where T_c and T_h are the temperatures of the hot and cold reservoirs, respectively.

Some Important Points

- In a practical heat engine, the change in internal energy must be zero over a complete cycle. Therefore, over a complete cycle $W = \Delta Q$.
- The work done by a gas during a portion of a cycle = $P\Delta V$, note ΔV can be positive or negative.

Gas Heat Engines

- When gas pressure-forces are used to move an object then work is done on the object by the expanding gas. Work can be done on the gas in order to compress it.
- If you plot pressure on the vertical axis and volume on the horizontal axis (see $P-V$ diagrams in the last chapter), the work done in any complete cycle is the area enclosed by the graph. For a partial process, work is the area underneath the curve, or$P\Delta V$.

Question:A heat engine operates at a temperature of 650K. The work output is used to drive a pile driver, which is a machine that picks things up and drops them. Heat is then exhausted into the atmosphere, which has a temperature of 300K.

a) What is the ideal efficiency of this engine?

b.) The engine drives a 1200kg weight by lifting it 50m in 2.5sec. What is the engine's power output?

c) If the engine is operating at 50% of ideal efficiency, how much power is being consumed?

d) The fuel the engine uses is rated at $2.7 \times 10^6 \text{J}/\text{kg}$. How many kg of fuel are used in one hour?

Answer:

a) We will plug the known values into the formula to get the ideal efficiency.

$$\eta = 1 - \frac{T_{\text{cold}}}{T}$$

hot=1-300K$\overline{\text{650K=54\%}}$

b) To find the power of the engine, we will use the power equation and plug in the known values.

$$P = \frac{W}{t} = \frac{Fd}{t} = \frac{mad}{t} = \frac{1200\text{kg} \times 9.8\text{m/s}^2 \times 50\text{m}}{2.5\text{sec}} = 240\text{kW}$$

c) First, we know that it is operating at 50% of ideal efficiency. We also know that the max efficiency of this engine is 54%. So the engine is actually operating at

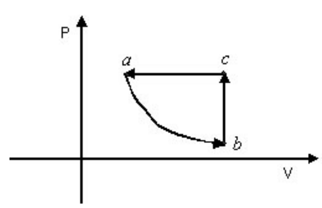
$$.5 \times 54\% = 27\%$$

of 100% efficiency. So 240kW is 27% of what?

$$.27x = 240\text{kW} \Rightarrow x = \frac{240\text{kW}}{.27} = 890\text{kW}$$

1. Consider a molecule in a closed box. If the molecule collides with the side of the box, how is the force exerted by the molecule on the box related to the momentum of the molecule? Explain conceptually, in words rather than with equations.
2. If the number of molecules is increased, how is the pressure on a particular area of the box affected? Explain conceptually, in words rather than with equations.
3. The temperature of the box is related to the average speed of the molecules. Use momentum principles to relate temperature to pressure. Explain conceptually, in words rather than with equations.
4. What would happen to the number of collisions if temperature and the number of molecules remained fixed, but the volume of the box increased? Explain conceptually, in words rather than with equations.
5. Use the reasoning in the previous four questions to qualitatively derive the ideal gas law.
6. Typical room temperature is about 300 K. As you know, the air in the room contains both O_2 and N_2 gases, with nitrogen the lower mass of the two. If the average kinetic energies of the oxygen and nitrogen gases are the same (since they are at the same temperature), which gas has a higher average speed?
7. Use the formula $P = F/A$ to argue why it is easier to pop a balloon with a needle than with a finger (pretend you don't have long fingernails).
8. Take an empty plastic water bottle and suck all the air out of it with your mouth. The bottle crumples. Why, exactly, does it do this?
9. You will notice that if you buy a large drink in a plastic cup, there will often be a small hole in the top of the cup, in addition to the hole that your straw fits through. Why is this small hole necessary for drinking?
10. Suppose you were swimming in a lake of liquid water on a planet with a lower gravitational constant g than Earth. Would the pressure 10 meters under the surface be the same, higher, or lower, than for the equivalent depth under water on Earth? (You may assume that the density of the water is the same as for Earth.)
11. Why is it a good idea for Noreen to open her bag of chips before she drives to the top of a high mountain?
12. Explain, using basic physics conservation laws, why the following conditions would cause the ideal gas law to be violated:
    a. There are strong intermolecular forces in the gas.
    b. The collisions between molecules in the gas are inelastic.
    c. The molecules are not spherical and can spin about their axes.
    d. The molecules have non-zero volume.

    To the right is a graph of the pressure and volume of a gas in a container that has an adjustable volume. The lid of the container can be raised or lowered, and various manipulations of the container change the properties of the gas within. The points a, b, and c represent different stages of the gas as the container undergoes changes (for instance, the lid is raised or lowered, heat is added or taken away, etc.) The arrows represent the flow of time. Use the graph to answer the following questions.



13. Consider the change the gas undergoes as it transitions from point b to point c. What type of process is this?
    a. adiabatic
    b. isothermal
    c. isobaric
    d. isochoric
    e. entropic
14. Consider the change the gas undergoes as it transitions from point c to point a. What type of process is this?
    a. adiabatic
    b. isothermal

c. isobaric
d. isochoric
e. none of the above

15. Consider the change the gas undergoes as it transitions from point *a* to point *b*. Which of the following *best* describes the type of process shown?

a. isothermal
b. isobaric
c. isochoric

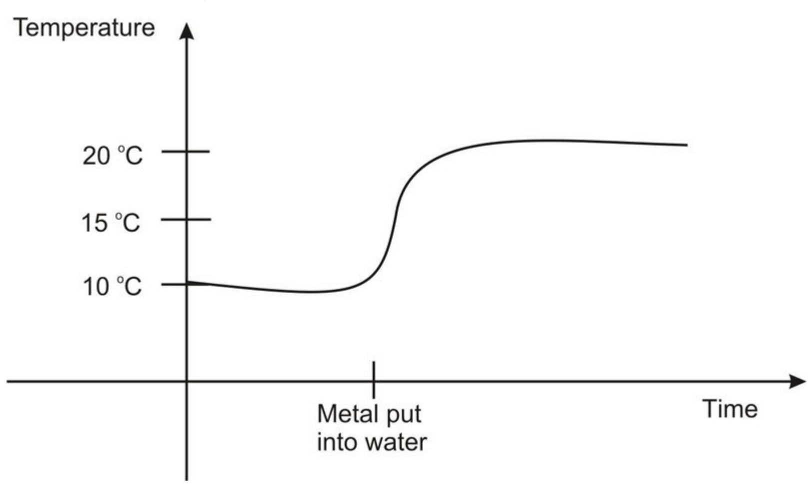
16. How would an isothermal process be graphed on $aP-V$ diagram?
17. Write a scenario for what you would do to the container to make the gas within undergo the cycle described above.
18. Why is it so cold when you get out of the shower wet, but not as cold if you dry off first before getting out of the shower? ____
19. Antonio is heating water on the stove to boil eggs for a picnic. How much heat is required to raise the temperature of his 10.0-kg vat of water from 20°C to 100°C?
20. Amy wishes to measure the specific heat capacity of a piece of metal. She places the 75-g piece of metal in a pan of boiling water, then drops it into a styrofoam cup holding 50 g of water at 22°C. The metal and water come to an equilibrium temperature of 25°C. Calculate:

a. The heat gained by the water
b. The heat lost by the metal
c. The specific heat of the metal

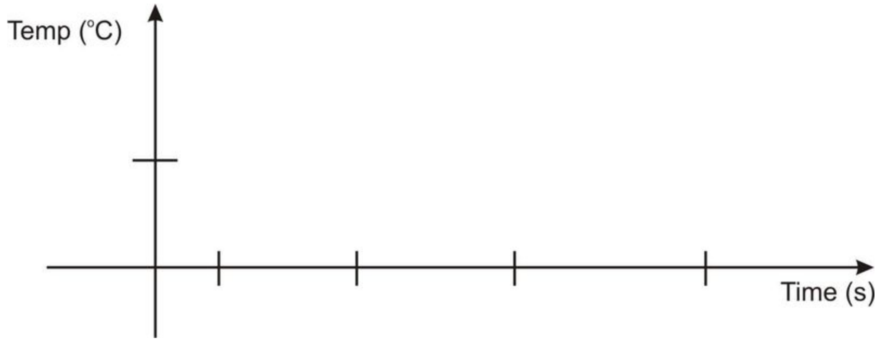
21. John wishes to heat a cup of water to make some ramen for lunch. His insulated cup holds 200 g of water at 20°C. He has an immersion heater rated at 1000 W (1000 J/s) to heat the water.

a. How many JOULES of heat are required to heat the water to 100°C?
b. How long will it take to do this with a 1000-W heater?
c. Convert your answer in part b to minutes.

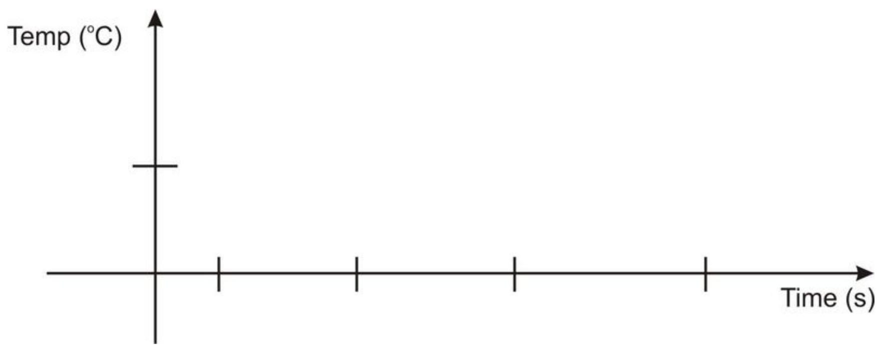
22. You put a 20g cylinder of aluminum $(c = 0.2\ cal/g/°C)$ in the freezer $(T = -10°C)$. You then drop the aluminum cylinder into a cup of water at 20°C. After some time they come to a common temperature of 12°C. How much water was in the cup?
23. Emily is testing her baby's bath water and finds that it is too cold, so she adds some hot water from a kettle on the stove. If Emily adds 2.00 kg of water at 80.0°C to 20.0 kg of bath water at 27.0°C, what is the final temperature of the bath water?
24. You are trying to find the specific heat of a metal. You heated a metal in an oven to 250°C. Then you dropped the hot metal immediately into a cup of cold water. To the right is a graph of the temperature of the water versus time that you took in the lab. The mass of the metal is 10g and the mass of the water is 100g. Recall that water has a specific heat of 1 $cal/g°C$.



25. How much heat is required to melt a 20 g cube of ice if
    a. the ice cube is initially at $0°C$
    b. the ice cube is initially at $-20°C$ (be sure to use the specific heat of ice)
26. A certain alcohol has a specific heat of 0.57 $cal/g°C$ and a melting point of $-114°C$. You have a 150 g cup of liquid alcohol at $22°C$ and then you drop a 10 g frozen piece of alcohol at $-114°C$ into it. After some time the alcohol cube has melted and the cup has come to a common temperature of $7°C$. (a) What is the latent heat of fusion (i.e. the 'L' in the $Q = mL$ equation) for this alcohol? (b) Make a sketch of the graph of the alcohol's temperature vs. time



(c) Make a sketch of the graph of the water's temperature vs. time



27. Calculate the average speed of N_2 molecules at room temperature (300 K). (You remember from your chemistry class how to calculate the mass (in kg) of an N_2 molecule, right?)
28. How high would the temperature of a sample of O_2 gas molecules have to be so that the average speed of the molecules would be 10% the speed of light?
29. How much pressure are you exerting on the floor when you stand on one foot? (You will need to estimate the area of your foot in square meters.)
30. Calculate the amount of force exerted on a 2 cm $\times$ 2 cm patch of your skin due to atmospheric pressure ($P_0 = 101,000$ Pa). Why doesn't your skin burst under this force?
31. Use the ideal gas law to estimate the number of gas molecules that fit in a typical classroom.
32. Assuming that the pressure of the atmosphere decreases exponentially as you rise in elevation according to the formula $P = P_0 e$

-h a , *where* P_0 *istheatmosphericpressureatsealevel* (101,000 Pa) , h *isthealtitudeinkmandaisthescale heightoftheatmosphere* (a ≈ 8.4 km).

    a. Use this formula to determine the change in pressure as you go from San Francisco to Lake Tahoe, which is at an elevation approximately 2 km above sea level.
    b. If you rise to half the scale height of Earth's atmosphere, by how much does the pressure decrease?
    c. If the pressure is half as much as on sea level, what is your elevation?

33. At Noah's Ark University the following experiment was conducted by a professor of Intelligent Design (formerly Creation Science). A rock was dropped from the roof of the Creation Science lab and, with expensive equipment, was observed to gain 100 J of internal energy. Dr. Dumb explained to his students that the law of conservation of

energy required that if he put 100 J of heat into the rock, the rock would then rise to the top of the building. When this did *not* occur, the professor declared the law of conservation of energy invalid.

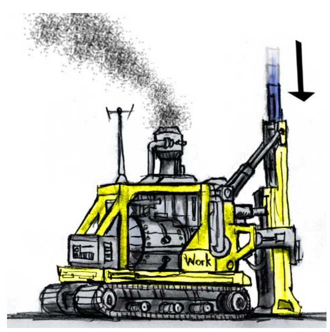
a. Was the law of conservation of energy violated in this experiment, as was suggested? Explain.
b. If the law wasn't violated, then why didn't the rock rise?

34. An instructor has an ideal monatomic helium gas sample in a closed container with a volume of 0.01 m^3, a temperature of 412 K, and a pressure of 474 kPa.

a. Approximately how many gas atoms are there in the container?
b. Calculate the mass of the individual gas atoms.
c. Calculate the speed of a typical gas atom in the container.
d. The container is heated to 647 K. What is the new gas pressure?
e. While keeping the sample at constant temperature, enough gas is allowed to escape to decrease the pressure by half. How many gas atoms are there now?
f. Is this number half the number from part (a)? Why or why not?
g. The closed container is now compressed isothermally so that the pressure rises to its original pressure. What is the new volume of the container?
h. Sketch this process on a P-V diagram.
i. Sketch cubes with volumes corresponding to the old and new volumes.

35. A famous and picturesque dam, 80 m high, releases 24,000 kg of water a second. The water turns a turbine that generates electricity.

a. What is the dam's maximum power output? Assume that all the gravitational potential energy of the water is converted into electrical energy.
b. If the turbine only operates at 30% efficiency, what is the power output?
c. How many Joules of heat are exhausted into the atmosphere due to the plant's inefficiency?



36. A heat engine operates at a temperature of 650 K. The work output is used to drive a pile driver, which is a machine that picks things up and drops them. Heat is then exhausted into the atmosphere, which has a temperature of 300 K.

a. What is the ideal efficiency of this engine?
b. The engine drives a 1200 kg weight by lifting it 50 m in 2.5 sec. What is the engine's power output?
c. If the engine is operating at 50% of ideal efficiency, how much power is being consumed?
d. How much power is exhausted?
e. The fuel the engine uses is rated at 2.7×10^6 J/kg. How many kg of fuel are used in one hour?

37. Calculate the ideal efficiencies of the following sci-fi heat engines:

a. A nuclear power plant on the moon. The ambient temperature on the moon is 15 K. Heat input from radioactive decay heats the working steam to a temperature of 975 K.
b. A heat exchanger in a secret underground lake. The exchanger operates between the bottom of a lake, where the temperature is 4 C, and the top, where the temperature is 13 C.
c. A refrigerator in your dorm room at Mars University. The interior temperature is 282 K; the back of the fridge heats up to 320 K.

38. How much external work can be done by a gas when it expands from 0.003 m^3 to 0.04 m^3 in volume under a constant pressure of 400 kPa? Can you give a practical example of such work?
39. In the above problem, recalculate the work done if the pressure linearly decreases from 400 kPa to 250 kPa under the same expansion. Hint: use a *PV* diagram and find the area under the line.
40. One mole ($N = 6.02 \times 10^{23}$) of an ideal gas is moved through the following states as part of a heat engine. The engine moves from state A to state B to state C, and then back again. Use the **Table** (19.1) to answer the following questions:
    a. Draw a P-V diagram.
    b. Determine the temperatures in states A, B, and C and then fill out the table.
    c. Determine the type of process the system undergoes when transitioning from A to B and from B to C. (That is, decide for each if it is isobaric, isochoric, isothermal, or adiabatic.)
    d. During which transitions, if any, is the gas doing work on the outside world? During which transitions, if any, is work being done on the gas?
    e. What is the amount of net work being done by this gas?

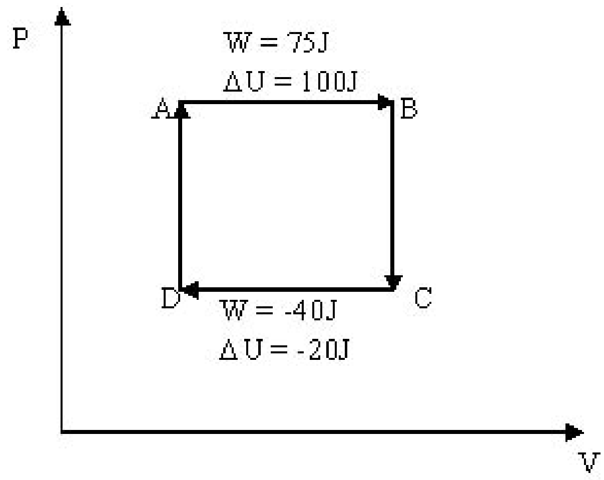
TABLE 19.1:

State	Volume $(m3)$	Pressure (atm)	Temperature (K)
A	0.01	0.60	
B	0.01	0.25	
C	0.02	0.25	

41. A sample of gas is used to drive a piston and do work. Here's how it works:
    - The gas starts out at standard atmospheric pressure and temperature. The lid of the gas container is locked by a pin.
    - The gas pressure is increased isochorically through a spigot to twice that of atmospheric pressure.
    - The locking pin is removed and the gas is allowed to expand isobarically to twice its volume, lifting up a weight. The spigot continues to add gas to the cylinder during this process to keep the pressure constant.
    - Once the expansion has finished, the spigot is released, the high-pressure gas is allowed to escape, and the sample settles back to 1 atm.
    - Finally, the lid of the container is pushed back down. As the volume decreases, gas is allowed to escape through the spigot, maintaining a pressure of 1 atm. At the end, the pin is locked again and the process restarts.

    a. Draw the above steps on a $P-V$ diagram.
    b. Calculate the highest and lowest temperatures of the gas.

42. A heat engine operates through 4 cycles according to the *PV* diagram sketched below. Starting at the top left vertex they are labeled clockwise as follows: a, b, c, and d.
    a. From $a-b$ the work is 75 J and the change in internal energy is 100 J; find the net heat.
    b. From the a-c the change in internal energy is -20 J. Find the net heat from b-c.
    c. From c-d the work is -40 J. Find the net heat from c-d-a.
    d. Find the net work over the complete 4 cycles.
    e. The change in internal energy from b-c-d is -180 J. Find:
        a. the net heat from c-d
        b. the change in internal energy from d-a
        c. the net heat from d-a



43. A 0.1 sample mole of an ideal gas is taken from state A by an isochoric process to state B then to state C by an isobaric process. It goes from state C to D by a process that is linear on a *PV* diagram, and then it goes back to state A by an isobaric process. The volumes and pressures of the states are given in the **Table** 19.2); use this data to complete the following:

  a. Find the temperature of the 4 states
  b. Draw a *PV* diagram of the process
  c. Find the work done in each of the four processes
  d. Find the net work of the engine through a complete cycle
  e. If 75 J of heat is exhausted in D-A and A-B and C-D are adiabatic, how much heat is inputted in B-C?
  f. What is the efficiency of the engine?

TABLE 19.2:

state	Volume in $m^3 \times 10^{-3}$	Pressure in $N/m^2 \times 10^5$
A	1.04	2.50
B	1.04	4.00
C	1.25	4.00
D	1.50	2.50

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. .
12. .

13. .
14. .
15. .
16. .
17. .
18. .
19. 800,000 cal or 3360 kJ

    1. 150 cal (630 J)
    2. same as a!
    3. 0.027 $cal/g°C$ $(0.11\ J/g°C)$

    1. 67,000 J
    2. 67.2 s
    3. 1.1 min

20. 11.0 g
21. 31.8$°C$
22. 0.44 $cal/g°C$

    1. 1600 cal (6720 J)
    2. 1800 cal (7560 J)

23. 59.3 cal/g
24. 517 m/s
25. 1.15×10^{12} K
26. .
27. 40 N
28. $\approx$ 10^{28} molecules

    1. $21,000$ Pa
    2. Decreases to $61,000$ Pa
    3. 5.8 km

    1. No
    2. allowed by highly improbable state. More likely states are more disordered.

    1. 8.34×10^{23}
    2. 6.64×10^{-27} kg
    3. 1600 m/s
    4. 744 kPa
    5. 4.2×10^{20} or 0.0007 moles
    6. 0.00785 m^3

    1. 1.9 MW
    2. 0.56 MW
    3. 1.3 Mw

    1. 54%
    2. 240 kW
    3. 890 kW
    4. 590 kW
    5. 630 kg

    1. 98%
    2. 4.0%
    3. 12%

29. 14800 J
30. 12,000 J
31. (b) 720 K, 300 K, 600 K (c) isochoric; isobaric (d) C to A; B – C (e) 0.018 J
32. b. 300 K, 1200 K

    1. 1753 J
    2. −120 J
    3. 80 J
    4. 35 J
    5. −100 J, 80 J, 80 J

19.5 References

1. Alex Zaliznyak. Ideal Gas Theory. CC-BY-SA 3.0
2. Alex Zaliznyak. Ideal Gas Theory 2. CC-BY-SA 3.0

CHAPTER 20 Gas Laws

Chapter Outline

20.1 Thermodynamics

This chapter is a short introduction to the basics of thermodynamics: state variables and their measurements, as well as the empirical gas laws. Thermodynamics is a rich and complicated science, and the chapters that follow attempt to only outline its tenets. This chapter presents some thermodynamic phenomena in a manner that reflects how they were first discovered — through observation of experiments on various gases. The next chapter links this empirical understanding with statistical mechanics and kinetic theory and discusses the practical applications of thermodynamics.

The Thermodynamic Approach to Describing Systems

In kinematics, once the initial conditions of a system are set — we are given the masses and positions of objects in question as well as the forces acting on them, we can theoretically obtain all future information about the system. By applying Newton's Laws, we can determine the positions and velocities of the objects at any point in time.

However, once we are talking about systems that consist of trillions of individual particles in constant motion, such a description becomes inadequate. In this case, instead of tracking the velocities and positions of each individual particle, we track several **parameters**, or **state variables**: aggregate quantities that sufficiently describe the system in question. The state variables we will use in our study of thermodynamics include pressure, denoted by the letter P, volume (V), and temperature (T).

For example, it can be shown that the temperature of a substance (whether gas, solid, or liquid) is related to the internal motion (and therefore, kinetic energy) of the molecules or atoms that constitute it. In a gas, the molecules or atoms might be flying around freely, while in a solid they can be thought of as trillions of masses connected by springs. Keeping track of such complicated motions on such a large scale is impossible, so we use the concept of temperature to obtain a significant amount of information about these motions in a single number.

Two Roads to Thermodynamics

Upon formulating his law of universal gravitation, Newton remarked that:

I have not as yet been able to discover the reason for these properties of gravity from phenomena, and I do not feign hypotheses.

In other words, Newton realized that his theory of gravity was descriptive: it could predict when and where an object would fall, but could not explain why this happened.

Thermodynamics started in the same manner. Specifically, early physicists and chemists performed experiments on various gases and found certain empirical relationships between the parameters defined above (pressure, temperature, volume) that seemed to hold universally. The values of such parameters were measured using items such as pressure gauges and thermometers. This is the basis for the **empirical gas laws**, which are the main substance of this chapter. In reality, the laws they discovered were not always exact for all gases, but always seemed close to simple mathematical representations. To account for this discrepancy, scientists created the theoretical construct known as an **ideal gas**. We will define this concept later, but for now an ideal gas can be thought of one that always exactly obeys the laws listed later in the chapter.

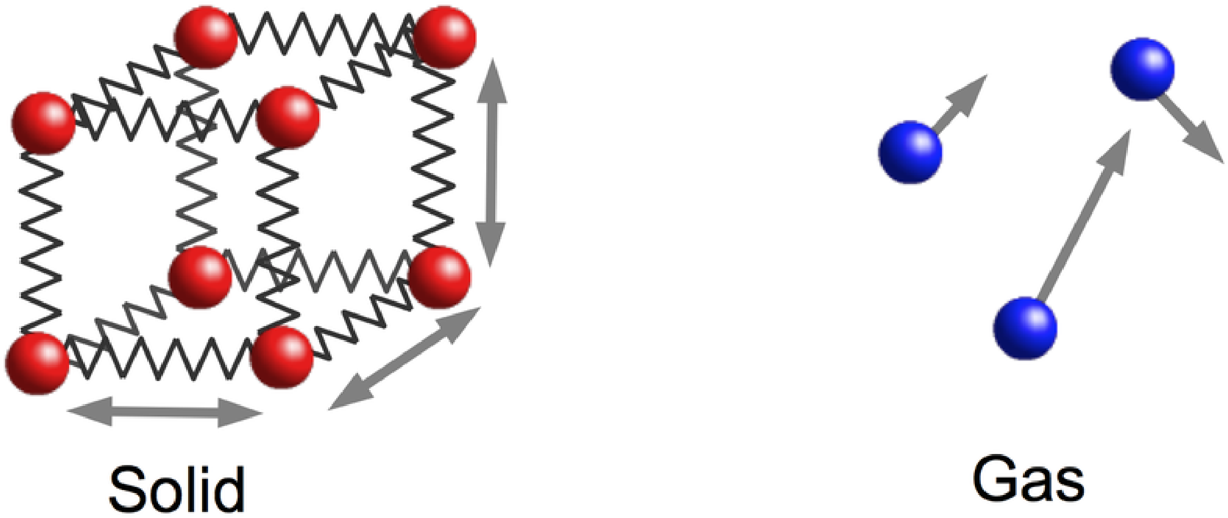


FIGURE 20.1
Internal motion of molecules in a solid and gas. The concept of temperature allows us to obtain information about such motions without keeping precise track of them.

However, it was later understood that the results of this chapter are not, in themselves, irreducible, like (for our purposes) Newton's laws were. Using other approaches (statistical mechanics and molecular kinetic theory) it was shown that the results we study can be considered statistical in their nature. To do this, various substances are represented through models, based on their microscopic constituents, that accurately model their behavior. The macroscopic parameters defined above are then found by 'averaging' some quantity over all the particles that make up a system. As hinted in the figure above, for instance, temperature is related to the kinetic energy of molecules in a substance.

For the purposes of this chapter, however, we look at thermodynamics as an empirical science based on observation.

Pressure and Volume

Before we go further into the empirical gas laws, let's consider the parameters described above and how exactly they relate to gases. Since gases generally occupy all the volume available to them, **the volume of a gas is simply the volume of the container that holds it**.

As you may remember, pressure is defined as force per unit area. When talking about gases, we refer to the **pressure that a gas exerts on the walls of its container**. You should remember from the pressure chapter that atmospheric pressure exerts a force of about $100,000\frac{\text{N}}{\text{m}^2}$, or Pascals. Often, when actually measuring gas pressure, we really measure the difference between the pressure exerted by the gas and the air pressure, since most common pressure gauges do not take air pressure into account. It is important to keep this fact in mind when performing experiments.

For instance, consider the example below: an inflatable balloon. The gas pressure inside is greater than the air pressure outside, since the elastic force wants to contract the balloon and essentially pushes in the same direction as air pressure. To measure the difference between the air pressure and the gas pressure, we can use a simple gauge: a piston connected to a spring. If we connect it to the inside of the balloon (without any gas leaks), the piston will contract the spring until the spring force is equal to the pressure of the gas times the area of the piston.

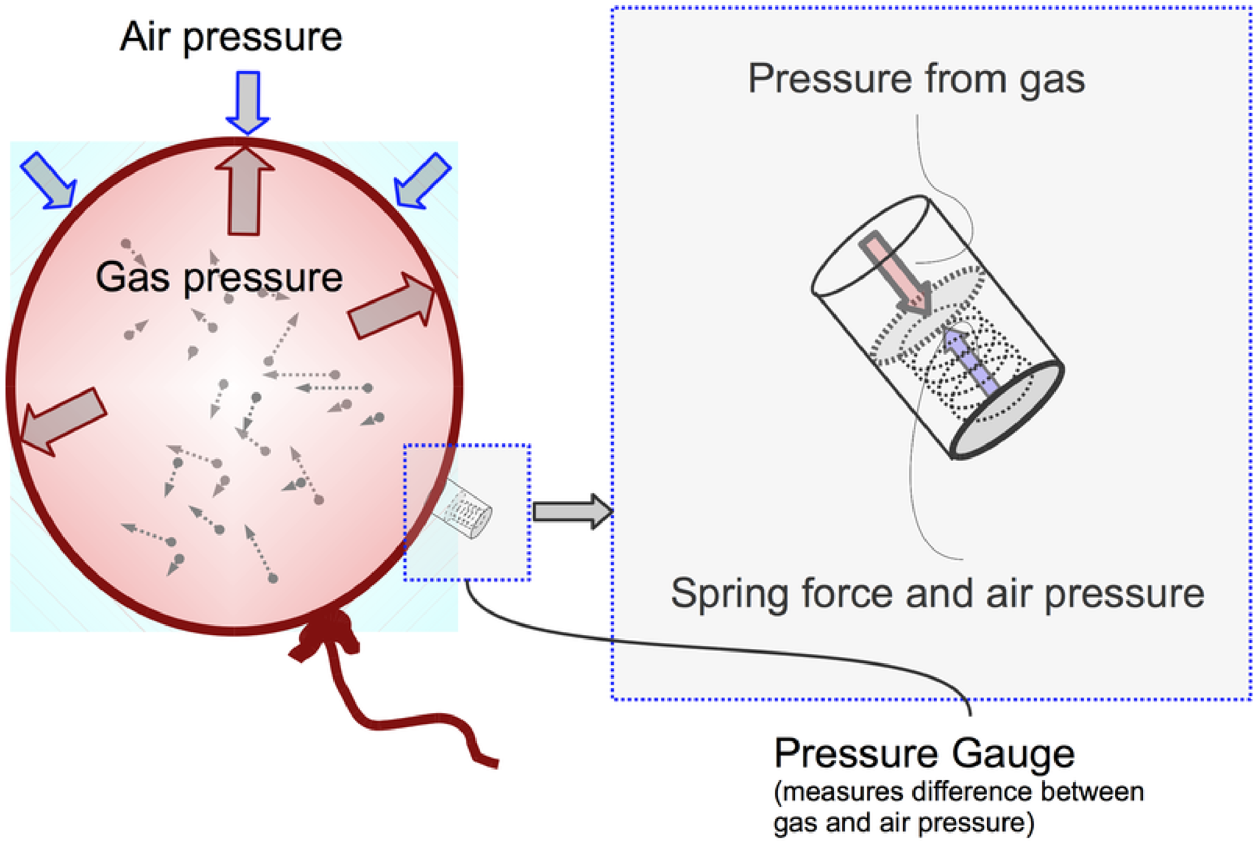


FIGURE 20.2
An illustration of gas pressure and its measurement

Temperature

Defining temperature rigorously is a difficult task and we will attempt to refine our understanding through the following chapters. As a first approximation, temperature can be defined as a quantifiable measure of the 'hotness' of an object. It's important to note that temperature is rigorously defined only for systems in **thermodynamic equilibrium**:

Thermodynamic Equilibrium
A closed system is in thermodynamic equilibrium if the macroscopic parameters associated with it (such as pressure, temperature, and volume) will remain constant indefinitely.

In mechanics, an object was in equilibrium if the forces on it were balanced; when this was true the quantities associated with the object (velocity, position, acceleration) would not change. Likewise, a system in thermodynamic equilibrium is in a kind of balance: if left alone, the relevant macroscopic parameters will not change.

The usefulness of the concept of temperature becomes apparent with the following experimentally verifiable fact: when two objects or systems **at different temperatures** are brought into thermal contact, the hotter one will cool, while the cooler one will heat up, until their temperatures are equal. As we will see more clearly later, this is a corollary of the famous **Second Law of Thermodynamics**.

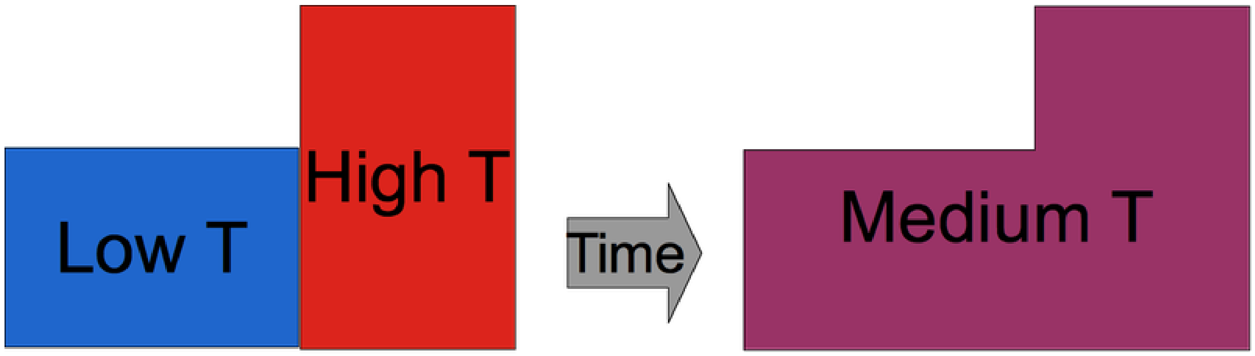


FIGURE 20.3
When two objects at different temperatures are brought into contact, their temperatures will get closer until the objects are in thermal equilibrium.

With these definitions, we can refine our understanding of temperature in terms of the following property.

Temperature
Ranks substances according to their 'hotness'; systems at different temperatures that are brought into thermal contact will eventually reach thermal equilibrium and have equal temperatures.

A Note on Temperature Scales and Measurement

To measure temperature, we have to use a device called a thermometer, which can be anything that exhibits **quantifiable physical changes with changes in temperature**. An example of this is the familiar mercury-based thermometer. This type of thermometer, which resembles the model below, works because the length of the mercury strip (in reality, its volume) increases linearly with temperature (this relationship breaks down under certain conditions, but we can ignore this). As an aside, it should be noted that this discussion of temperature is far from complete. However, for the sake of brevity, we leave this definition as is and will return to a more formal approach in the next chapter (which should clear up some of the confusion that may arise).

A thermometer is brought into thermal contact with a system, and once it is in thermal equilibrium with the system — exactly like in the illustration above — its particular physical property (for example, the length of the mercury strip) is translated into a number that is referred to as the temperature of the system. Of course, the thermometer alters the temperature of the system as well, but generally this change is small and can be ignored (for instance, a human being is much much bigger than a mercury thermometer).

There are different temperature scales: defining the units of this measure is up to us. The most frequently used ones are called Fahrenheit, Celsius, and Kelvin, after their respective popularizers. So, does any particular choice of units for temperature matter? Not really; any consistent scale will work. Consider the SI unit of length, the meter: it was actually originally defined as one ten-millionth of the distance from the Equator to the North Pole through Paris.

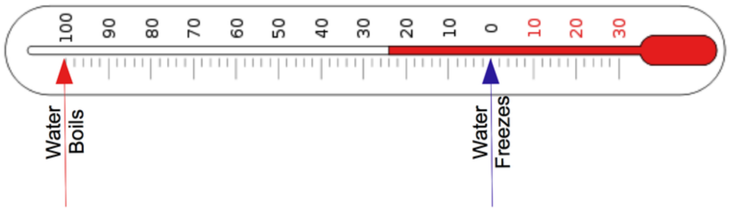


FIGURE 20.4
A Celsius thermometer.

This choice was entirely arbitrary in any universal sense, but it served physics completely adequately. Any other unit of distance would have worked as well.

The slightly tricky aspect to scales of measure is that — to completely define one — in addition to picking the unit size in terms of physical quantities (which is clear from above) one needs to set a zero level. When dealing with length scales, the zero level is apparent: the complete absence of length. Because of this, 0 miles is the same as 0 feet and 0 meters and conversion between lengths is a matter of multiplication alone (why?). This seems obvious, but let's ask a more subtle question: would a length scale where a length of zero corresponds to, say 1 meter on the SI scale, work consistently? The answer is that yes, it would, but it would be cumbersome in mathematical analysis; there would be negative lengths: a rather counter-intuitive concept.

As we will later see, **the properties of temperature also suggest an *absolute* zero for temperature**. If all our scales were set with that zero, conversion between temperature scales would be as easy as between length scales: simple multiplication (miles to kilometers, meters to centimeters, etc). Unfortunately, the Celsius and Fahrenheit scales were created before temperature was this well defined, so they assign the value of '0 degrees' to arbitrary points, and, therefore, have negative temperatures and are cumbersome in mathematical analysis. Still, any two temperature scales can be related through a linear relationship.

The Celsius scale — used throughout most of the world — establishes its unit, or degree Celsius, by defining the temperature difference between the freezing and boiling point of water as 100 degrees Celsius. This is analogous to the definition of meter above. However, it assigns a temperature of 0 to the freezing point of water; this temperature is considerably higher than absolute zero.

Question

Find the conversion relationship between Celsius and Fahrenheit temperature scales. Explain how this is different from converting miles to kilometers.

Thus, scientists generally use the Kelvin temperature scale, which has degree increments equal to the Celsius scale's (and so is pretty easy to recognize and interpret — at least for people outside the U.S.), but sets the value of zero temperature to the **absolute zero** — the point at which all molecular motion ceases. On the Celsius scale, this temperature is -273.15, so to convert between from degrees Celsius to Kelvins (frustratingly, while we call the Celsius Scale units degrees Celsius, the Kelvin scale units are conventionally referred to simply as Kelvins) we us the following:

$$T_k = T_c - 273.15 \qquad [1]$$

For the rest of this chapter, temperature will is assumed to be measured in Kelvins.

Empirical Gas Laws

Early experiments with various gases showed the following relationships to hold **for a fixed amount (mass or number of molecules) of gas**:

Boyle's Law (at constant temperature)

$$PV = k_1 \text{ if } T = \text{ Constant} \qquad [2]$$

Gay-Lussac's Law (at constant volume)

$$\frac{P}{T} = k_2 \text{ if } V = \text{ Constant} \qquad [3]$$

Charle's Law (at constant pressure)

$$\frac{V}{T} = k_3 \text{ if } P = \text{ Constant} \qquad [4]$$

It should be first noted that these laws only hold for **quasistatic processes**, which are defined as processes where the system is always at (or very near) thermal equilibrium. Informally, we may think of these as 'slow', in the sense that at any point in its path (see diagrams below), the system has time to reach thermal equilibrium. An important fact about such processes is that they are **reversible**. All the processes we consider in this book are quasistratic.

Boyle's law states that at constant temperature, the pressure and volume of a given amount of gas are inversely related. If you squeeze a balloon, for instance, while keeping its temperature constant, its volume will decrease and the gas will exert a greater amount of pressure than before to counterbalance the force you apply in addition to atmospheric pressure.

Gay-Lussac's law states that at constant volume, the pressure exerted by a gas is proportional to its temperature. For instance, if the balloon pictures above were perfectly rigid (could not stretch beyond its current volume), by increasing the temperature of the gas, you would increase the pressure inside, and the balloon would eventually burst.

Charles' Law states that at constant pressure, the volume of a gas is proportional to its temperature.

Avogadro's Law

Finally, the Italian scientist Amedeo Avogadro determined experimentally that at constant temperature and pressure, **equal volumes of different gases contain equal numbers of molecules**; in other words (if n is the number of molecules of a gas):

$$\frac{V}{n} = k_4 \text{ if } P = \text{ constant and } T = \text{ constant} \qquad [5]$$

Combined Ideal Gas Law

By combining any two of the empirical gas laws above with Avogadro's Law, we can derive the following relationship, called the **Ideal Gas Law** and states that for n molecules of any ideal gas,

$$PV = nkT \qquad [6]$$

In this case, the constant k (a combination of the constants above) can be empirically measured, since, **because of the addition of Avogadro's Law, it is identical for all ideal gases**. Note that the constants in the empirical gas laws are **not** necessarily identical for all ideal gases (why?). Its value is $k = 1.38 \times 10^{-23}$ J/K and it is known as the Boltzmann Constant.

A different, though completely equivalent form of the ideal gas law is:

$$PV = NRT \qquad [7]$$

V is the volume, N is the number of moles of the gas (R is the universal gas constant $= 8.315$ J/K mol); this form is often more useful for thermodynamics.

The relationship between n and N above, that is the number of molecules per mole of a substance, is called Avogadro's number. By comparing the two equations above, you can find the ratio is

$$\frac{n}{N} = N_A = 6.0 \times 10^{23} \qquad [8]$$

.

Question

Assuming the atmosphere is isothermal (at constant temperature), what will happen to a perfectly elastic (no elastic force, **unlike** the example above) gas balloon as it floats higher and higher?

Answer

A perfectly elastic balloon is in equilibrium when the gas pressure inside matches the air pressure outside. As $T =$ const, we can use Boyle's law, that is, $PV =$ const. As the balloon rises higher, the air pressure outside it drops, and therefore the gas pressure inside must drop as well. By Boyle's law, we determine that its volume must increase.

Question

Explain why this result also holds for a balloon that is not perfectly elastic.

Diagrams of Quasistatic Gas Processes

Using the formulas above, it is possible to graphically display processes suggested by the laws above. For instance, consider the diagram on the left below, which shows an ideal gas being heated and compressed on $T-V$ axes. As we will see later, it is often useful to graph such processes on a different set of axes, say $P-V$, as shown on the right.

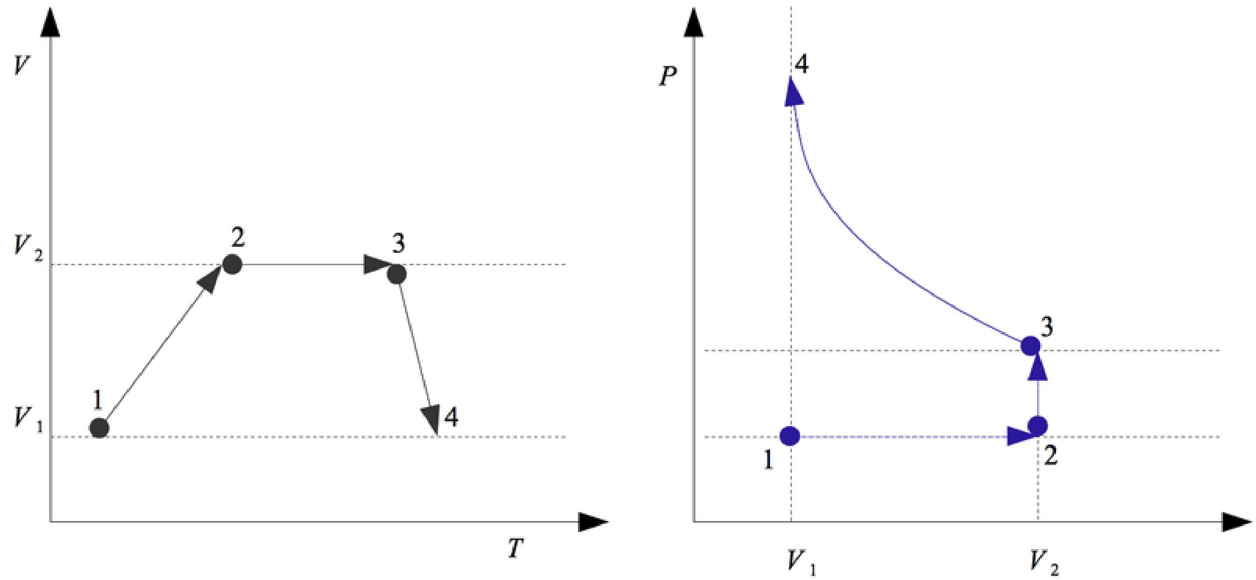


FIGURE 20.5
Diagrams of the same process on different axes

Question

explain how we can draw the graph of the processes on the left on a $P-V$ diagram.

Answer

During steps 1-2, the gas's temperature is linearly proportional to its volume; therefore, by the combined ideal gas law, the pressure must remain constant while the volume increases. During steps 2-3, the volume remains constant, so by Gay-Lussac's law, pressure must increase linearly with temperature. Finally, during steps 3-4, volume falls linearly with increases in temperature. In other words, $V = a - bT$ for some constants a, b. Plugging in for T from the combined ideal gas law, we find that pressure is inversely proportional to volume (derive the formula); that is, as volume decreases linearly, pressure grows along a hyperbola.

An Ideal Gas Thermometer, Absolute Zero

Finally, let's look at the ideal gas law [7] for 1 mole of some gas that doesn't deviate from the law, like helium gas, for instance:

$$PV = RT \qquad \text{1 mole of ideal gas}$$

This formula (alternatively, we could have used Charles' Law), combined with the idea of the perfectly elastic balloon above, suggests a way to measure temperature: at constant pressure, the volume of the balloon should be directly proportional to its temperature, with the slope of the line equal to the ratio of the ideal gas constant and the constant pressure. That is,

$$V = T\frac{R}{P_0} \qquad R, P_0 \text{ constants}$$

In standard conditions on earth, $P_0 \approx 100,000\text{Pa}$, so

$$V = T \times R \times 10^{-5}$$

Equivalently, any two temperature-volume pairs can be related by the formula (why?):

$$T_2 = T_1\frac{V_2}{V_1}$$

.

Such a setup might look like this:

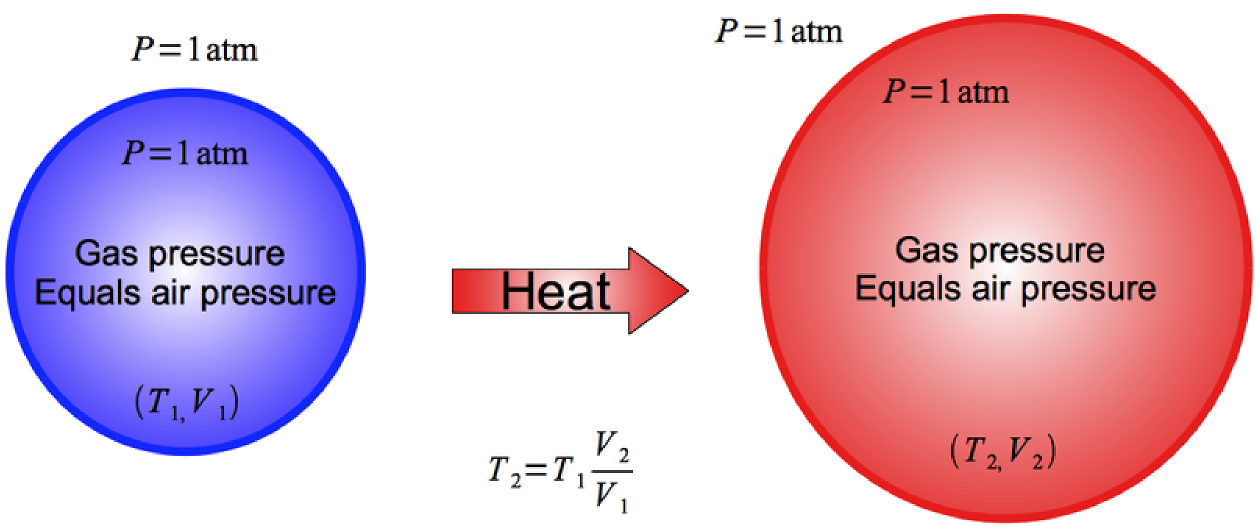


FIGURE 20.6
What an ideal gas thermometer setup might look like

Therefore, a Kelvin scale ideal gas thermometer would function according to the following principle: bring the balloon into thermal contact with a substance, wait for it to reach thermal equilibrium, measure its volume, and convert that into temperature.

Alternatively, by finding the volume of the balloon at a temperature that corresponds to some easily observable physical event (the melting point of water, for instance) we can now **find any other temperature** by multiplying the original (in Kelvins) by the ratio of the volumes.

We can illustrate how one might use the relationship between volume and temperature for a Kelvin scale ideal thermometer in the following manner:

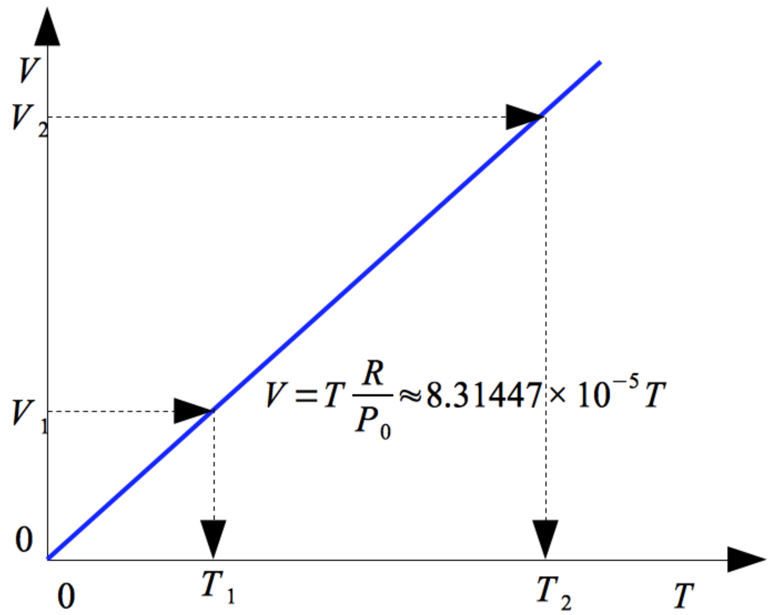


FIGURE 20.7
An ideal gas thermometer can be used to easily find temperatures in Kelvins.

Absolute Zero

Before the advent of the Kelvin scale, scientists generally used the Celsius scale. They also used gas thermometers which operated according to the same principles as the one described above. As shown below, a Celsius scale gas thermometer also replicates the linear relationship between volume and temperature.

Because of practical limitations, they could not create temperatures near what we called absolute zero above, but by extrapolating from observed regions, they found that hypothetically, an ideal gas would have zero volume at a temperature of about -273 degrees Celsius. This is how the concept of **absolute zero** was first identified:

Questions

Since the gas laws as we introduced them use Kelvins, they could not have been used before the Kelvin scale was around. Using formula [1], show how each of the gas laws and the combined law would be modified for Celsius units. Indeed, this is how the empirical gas laws were first formulated. Explain the graph above and how and why the switch to Kelvins might have occurred in terms of your answer to the question above.

Answer

The gas laws still hold in their general relationships; Boyle's law, being temperature-independent, remain unchanged. The other two laws are still linear relationships, but now there is an x -intercept (work out the algebra). The Kelvin scale can be explained as a way to eliminate the x-intercept found in various temperature-dependent phenomena, such as the gas laws. Also, note that — as suggested by the discussion on scales above — converting between any two temperatures scales with zeroes calibrated to absolute zero will be as simple as converting between length or time scales.

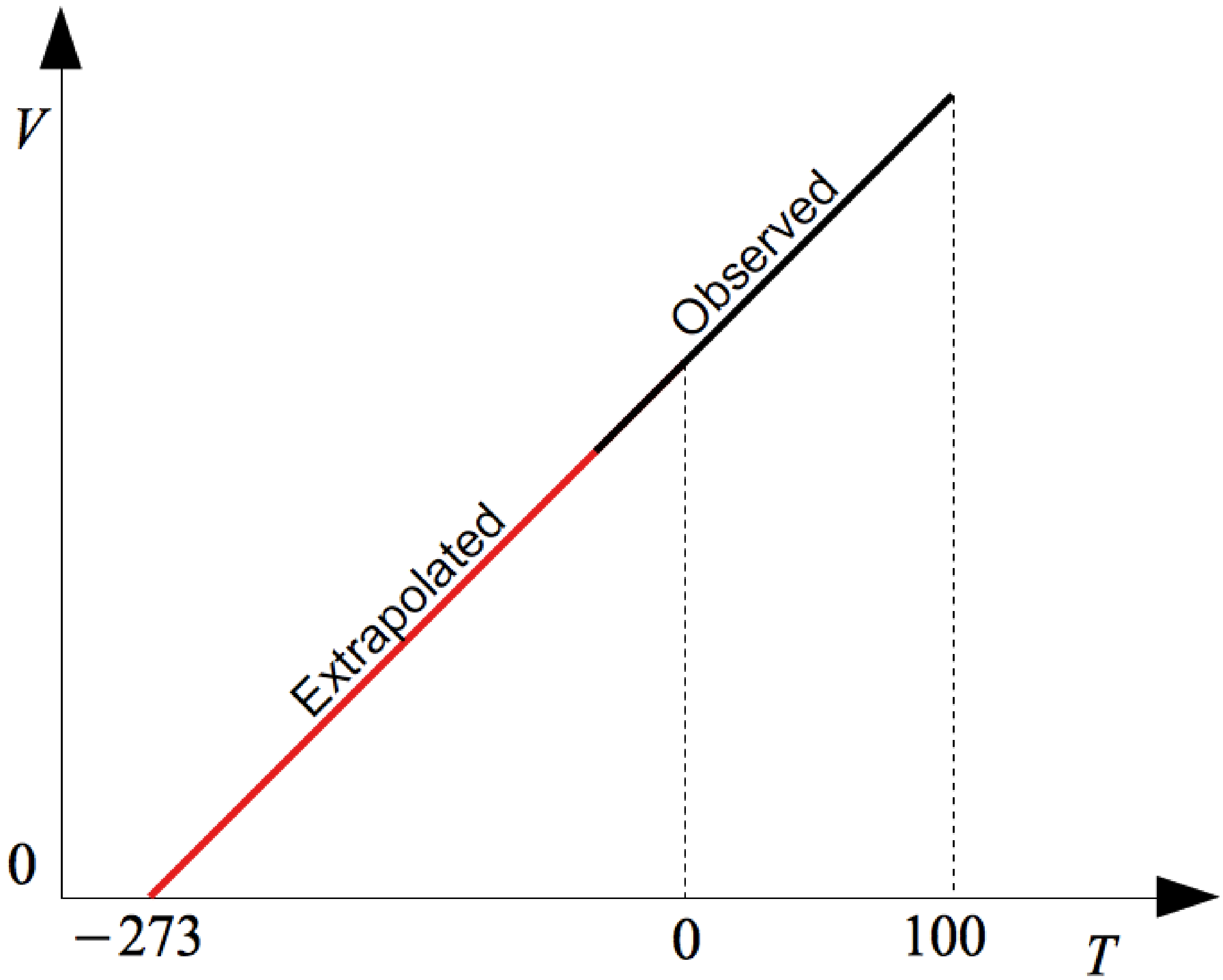


FIGURE 20.8

A Celsius scale graph of volume vs temperature for an ideal gas.

20.2 References

1. Alex Zaliznyak. Molecules in a Solid and Gas. CC-BY-SA 3.0
2. Alex Zaliznyak. Gas and Air Pressure. CC-BY-SA 3.0
3. Alex Zaliznyak. Thermal Equilibrium. CC-BY-SA 3.0
4. Alex Zaliznyak, unknown author at http://en.wikipedia.org/wiki/File:Thermometer_CF.svg. Thermometer. CC-BY-SA 3.0
5. Alex Zaliznyak, adapted from MFTI materials. Thermometer. CC-BY-SA 3.0
6. Alex Zaliznyak. Gas Thermometer. CC-BY-SA 3.0
7. Alex Zaliznyak. Gas Thermometer Graph. CC-BY-SA 3.0
8. Alex Zaliznyak. Gas Thermometer. CC-BY-SA 3.0

CHAPTER 21 Heat Engines and The Laws of Thermodynamics

Chapter Outline

21.1 The Laws of Thermodynamics

Now that we have defined the terms that are important for an understanding of thermodynamics, we can state the laws that govern relevant behavior. These laws, unlike Newton's Laws or Gravity, are *not* based on new empirical observations: they can be derived based on statistics and known principles, such as conservation of energy. By understanding the laws of thermodynamics we can analyze **heat engines**, or machines that use heat energy to perform mechanical work.

The First Law

The **First Law of Thermodynamics** is simply a statement of energy conservation applied to thermodynamics systems: *the change in the internal* — for our purposes, this is the same as thermal — *energy (denoted ΔU) of a closed system is equal to the difference of net input heat and performed work*. In other words,

$$\Delta U = Q_{net} - W \qquad \text{[4] First Law}$$

Note that this does not explain how the system will transform input heat to work, it simply enforces the energy balance.

The Second Law

The **Second Law of Thermodynamics** states that *the entropy of an isolated system will always increase until it reaches some maximum value*. Consider it in light of the simplified example in the entropy section: if we allow the low entropy system to evolve, it seems intuitive collisions will eventually somehow distribute the kinetic energy among the atoms.

The Second Law generalizes this intuition to all closed thermodynamic systems. It is based on the idea that in a closed system, energy will be randomly exchanged among constituent particles — like in the simple example above — until the distribution reaches some equilibrium (again, in any macroscopic system there will be an enormous number of of atoms, degrees of freedom, etc). Since energy is conserved in closed systems, this equilibrium has to preserve the original energy total. In this equilibrium, the Second Law — fundamentally a probabilistic statement — posits that the energy will be distributed in the most likely way possible. This typically means that energy will be distributed evenly across degrees of freedom.

This allows us to formulate the **Second Law in another manner**, specifically: *heat will flow spontaneously from a high temperature region to a low temperature region, but not the other way*. This is just applying the thermodynamic vocabulary to the logic of the above paragraph: in fact, this is the reason for the given definition of temperature. When two substances are put in thermal contact (that is, they can exchange thermal energy), heat will flow from the system at the higher temperature (because it has more energy in its degrees of freedom) to the system with lower temperature until their temperatures are the same.

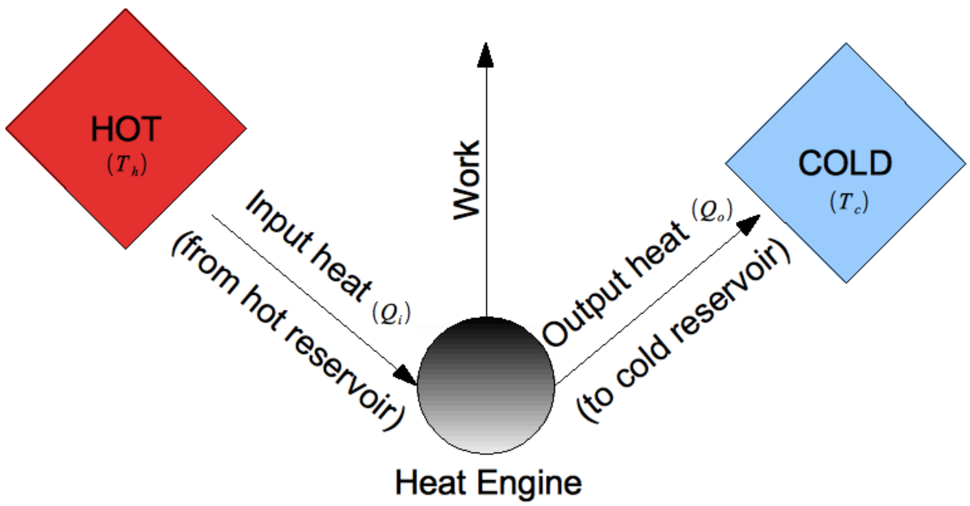
When a single system is out equilibrium, there will be a net transfer of energy from one part of it to another. In equilibrium, energy is still exchanged among the atoms or molecules, but not on a system-wide scale. Therefore, entropy places a limit on how much work a system can perform: the higher the entropy, the more even the distribution of energy, the less energy available for transfer.

21.2 Heat Engines

Heat engines transform input heat into work in accordance with the laws of thermodynamics. The mechanics of various heat engines differ but their fundamentals are quite similar and involve the following steps:

1. Heat is supplied to the engine from some source at a higher temperature (T_h).
2. Some of this heat is transferred into mechanical energy through work done (W).
3. The rest of the input heat is transferred to some source at a lower temperature (T_c) until the system is in its original state.

A single cycle of such an engine can be illustrated as follows:



In effect, such an engine allows us to 'siphon off' part of the heat flow between the heat source and the heat sink. The efficiency of such an engine is define as the ratio of net work performed to input heat; this is the fraction of heat energy converted to mechanical energy by the engine:

$$e = \frac{W}{Q_i}$$ [5] Efficiency of a heat engine

If the engine does not lose energy to its surroundings (of course, all real engines do), then this efficiency can be rewritten as

$$e = \frac{Q_i - Q_o}{Q_i}$$ [6] Efficiency of a lossless heat engine

A **Carnot Engine**, the most efficient heat engine possible, has an efficiency equal to

$$e_c = 1 - \frac{T_c}{T_h}$$ [7] Efficiency of a Carnot (ideal) heat engine

where T_c and T_h are the temperatures of the hot and cold reservoirs, respectively.

Application to Gases

- The pressure of a gas is the force the gas exerts on a certain area. For a gas in a container, the amount of pressure is directly related to the number and intensity of atomic collisions on a container wall.
- An *ideal* gas is a gas for which interactions between molecules are negligible, and for which the gas atoms or molecules themselves store no potential energy. For an "ideal" gas, the pressure, temperature, and volume are simply related by the ideal gas law.
- Atmospheric pressure (1 atm $= 101,000$ Pascals) is the pressure we feel at sea level due to the weight of the atmosphere above us. As we rise in elevation, there is less of an atmosphere to push down on us and thus less pressure.
- When gas pressure-forces are used to move an object then work is done on the object by the expanding gas. Work can be done on the gas in order to compress it.
- Adiabatic process: a process that occurs with no heat gain or loss to the system in question.
- Isothermal: a process that occurs at constant temperature (i.e. the temperature does not change during the process).
- Isobaric: a process that occurs at constant pressure.
- Isochoric: a process that occurs at constant volume.
- If you plot pressure on the vertical axis and volume on the horizontal axis, the work done in any complete cycle is the area enclosed by the graph. For a partial process, work is the area underneath the curve, or$P\Delta V$.
- In a practical heat engine, the change in internal energy must be zero over a complete cycle. Therefore, over a complete cycle $W = \Delta Q$.
- The work done by a gas during a portion of a cycle = $P\Delta V$, note ΔVcan be positive or negative.

Key Equations

Temperature and kinetic energy:

$$(\frac{1}{2}mv^2)_{\text{avg}} = \frac{3}{2}kT \qquad [8]$$

The average kinetic energy of atoms (each of mass m and average speed v) in a gas is proportional to the temperature T of the gas, measured in Kelvin. This is just a restatement of the definition of temperature above. The Boltzmann constant k is a constant of nature, equal to 1.38×10^{-23} J/K.

Definition of pressure:

$$P = \frac{F}{A} \qquad [9]$$

The pressure on an object is equal to the force pushing on the object divided by the area over which the force is exerted. Unit for pressure are N/m^2 (called Pascals)

The Ideal Gas Law:

$$PV = NkT \qquad [10]$$

An ideal gas is a gas where the atoms are treated as point-particles and assumed to never collide or interact with each other. If you have N molecules of such a gas at temperature T and volume V, the pressure can be calculated from this formula. Note that $k = 1.38 \times 10^{-23}$ J/K.

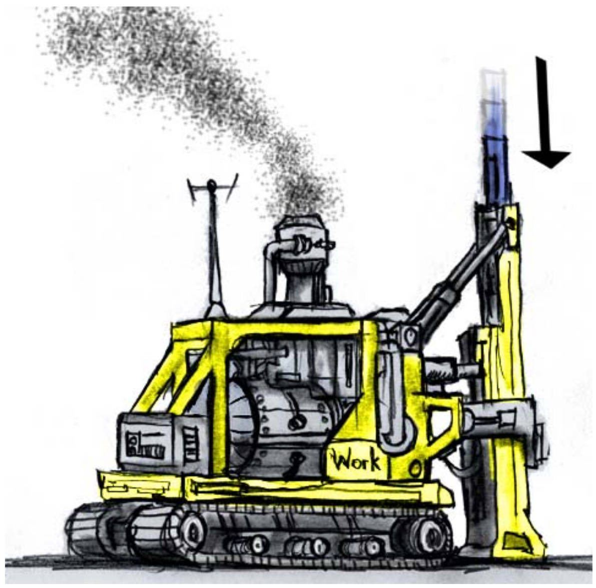
Different form of the Ideal Gas Law:

$$PV = nRT \qquad [11]$$

V is the volume, n is the number of moles; R is the universal gas constant $= 8.315\ \mathrm{J/K-n}$; this is the most useful form of the gas law for thermodynamics.

21.3 Examples

Example 1



Question:A heat engine operates at a temperature of 650K. The work output is used to drive a pile driver, which is a machine that picks things up and drops them. Heat is then exhausted into the atmosphere, which has a temperature of 300K.

a) What is the ideal efficiency of this engine?

b.) The engine drives a 1200kg weight by lifting it 50*m* in 2.5*sec*. What is the engine's power output?

c) If the engine is operating at 50% of ideal efficiency, how much power is being consumed?

d) The fuel the engine uses is rated at $2.7 \times 10^6 \mathrm{J/kg}$. How many kg of fuel are used in one hour?

Answer:

a) We will plug the known values into the formula to get the ideal efficiency.

$$\eta = 1 - \frac{T_{\text{cold}}}{T}$$

hot=1-300K$_{\overline{650K=54\%}}$

b) To find the power of the engine, we will use the power equation and plug in the known values.

$$P = \frac{W}{t} = \frac{Fd}{t} = \frac{mad}{t} = \frac{1200\text{kg} \times 9.8\text{m/s}^2 \times 50\text{m}}{2.5\text{sec}} = 240\text{kW}$$

c) First, we know that it is operating at 50% of ideal efficiency. We also know that the max efficiency of this engine is 54%. So the engine is actually operating at

$$.5 \times 54\% = 27\%$$

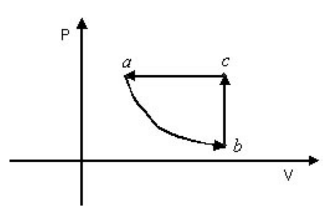
of 100% efficiency. So 240kW is 27% of what?

$$.27x = 240\text{kW} \Rightarrow x = \frac{240\text{kW}}{.27} = 890\text{kW}$$

21.4 Thermodynamics and Heat Engines Problem Set

1. Consider a molecule in a closed box. If the molecule collides with the side of the box, how is the force exerted by the molecule on the box related to the momentum of the molecule? Explain conceptually, in words rather than with equations.
2. If the number of molecules is increased, how is the pressure on a particular area of the box affected? Explain conceptually, in words rather than with equations.
3. The temperature of the box is related to the average speed of the molecules. Use momentum principles to relate temperature to pressure. Explain conceptually, in words rather than with equations.
4. What would happen to the number of collisions if temperature and the number of molecules remained fixed, but the volume of the box increased? Explain conceptually, in words rather than with equations.
5. Use the reasoning in the previous four questions to qualitatively derive the ideal gas law.
6. Typical room temperature is about 300 K. As you know, the air in the room contains both O_2 and N_2 gases, with nitrogen the lower mass of the two. If the average kinetic energies of the oxygen and nitrogen gases are the same (since they are at the same temperature), which gas has a higher average speed?
7. Use the formula $P = F/A$ to argue why it is easier to pop a balloon with a needle than with a finger (pretend you don't have long fingernails).
8. Take an empty plastic water bottle and suck all the air out of it with your mouth. The bottle crumples. Why, exactly, does it do this?
9. You will notice that if you buy a large drink in a plastic cup, there will often be a small hole in the top of the cup, in addition to the hole that your straw fits through. Why is this small hole necessary for drinking?
10. Suppose you were swimming in a lake of liquid water on a planet with a lower gravitational constant g than Earth. Would the pressure 10 meters under the surface be the same, higher, or lower, than for the equivalent depth under water on Earth? (You may assume that the density of the water is the same as for Earth.)
11. Why is it a good idea for Noreen to open her bag of chips before she drives to the top of a high mountain?
12. Explain, using basic physics conservation laws, why the following conditions would cause the ideal gas law to be violated:
    a. There are strong intermolecular forces in the gas.
    b. The collisions between molecules in the gas are inelastic.
    c. The molecules are not spherical and can spin about their axes.
    d. The molecules have non-zero volume.

    To the right is a graph of the pressure and volume of a gas in a container that has an adjustable volume. The lid of the container can be raised or lowered, and various manipulations of the container change the properties of the gas within. The points a, b, and c represent different stages of the gas as the container undergoes changes (for instance, the lid is raised or lowered, heat is added or taken away, etc.) The arrows represent the flow of time. Use the graph to answer the following questions.



13. Consider the change the gas undergoes as it transitions from point b to point c. What type of process is this?
    a. adiabatic
    b. isothermal

c. isobaric
d. isochoric
e. entropic

14. Consider the change the gas undergoes as it transitions from point c to point a. What type of process is this?

a. adiabatic
b. isothermal
c. isobaric
d. isochoric
e. none of the above

15. Consider the change the gas undergoes as it transitions from point a to point b. Which of the following *best* describes the type of process shown?

a. isothermal
b. isobaric
c. isochoric

16. How would an isothermal process be graphed on $aP - V$ diagram?
17. Write a scenario for what you would do to the container to make the gas within undergo the cycle described above. ____
18. Calculate the average speed of N_2 molecules at room temperature (300 K). (You remember from your chemistry class how to calculate the mass (in kg) of an N_2 molecule, right?)
19. How high would the temperature of a sample of O_2 gas molecules have to be so that the average speed of the molecules would be 10% the speed of light?
20. How much pressure are you exerting on the floor when you stand on one foot? (You will need to estimate the area of your foot in square meters.)
21. Calculate the amount of force exerted on a 2 cm $\times$ 2 cm patch of your skin due to atmospheric pressure $(P_0 = 101,000\text{ Pa})$. Why doesn't your skin burst under this force?
22. Use the ideal gas law to estimate the number of gas molecules that fit in a typical classroom.
23. Assuming that the pressure of the atmosphere decreases exponentially as you rise in elevation according to the formula $P = P_0 e$

-h a , *where* P_0 *istheatmosphericpressureatsealevel* (101,000 Pa) , h *isthealtitudeinkmandaisthescale heightoftheatmosphere* (a $\approx$ 8.4 km).

a. Use this formula to determine the change in pressure as you go from San Francisco to Lake Tahoe, which is at an elevation approximately 2 km above sea level.
b. If you rise to half the scale height of Earth's atmosphere, by how much does the pressure decrease?
c. If the pressure is half as much as on sea level, what is your elevation?

24. At Noah's Ark University the following experiment was conducted by a professor of Intelligent Design (formerly Creation Science). A rock was dropped from the roof of the Creation Science lab and, with expensive equipment, was observed to gain 100 J of internal energy. Dr. Dumb explained to his students that the law of conservation of energy required that if he put 100 J of heat into the rock, the rock would then rise to the top of the building. When this did *not* occur, the professor declared the law of conservation of energy invalid.

a. Was the law of conservation of energy violated in this experiment, as was suggested? Explain.
b. If the law wasn't violated, then why didn't the rock rise?

25. An instructor has an ideal monatomic helium gas sample in a closed container with a volume of 0.01 m^3, a temperature of 412 K, and a pressure of 474 kPa.

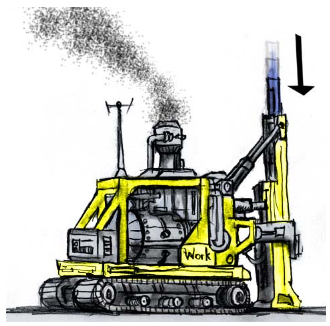
a. Approximately how many gas atoms are there in the container?
b. Calculate the mass of the individual gas atoms.
c. Calculate the speed of a typical gas atom in the container.
d. The container is heated to 647 K. What is the new gas pressure?

e. While keeping the sample at constant temperature, enough gas is allowed to escape to decrease the pressure by half. How many gas atoms are there now?
f. Is this number half the number from part (a)? Why or why not?
g. The closed container is now compressed isothermally so that the pressure rises to its original pressure. What is the new volume of the container?
h. Sketch this process on a P-V diagram.

i. Sketch cubes with volumes corresponding to the old and new volumes.

26. A famous and picturesque dam, 80 m high, releases 24,000 kg of water a second. The water turns a turbine that generates electricity.

    a. What is the dam's maximum power output? Assume that all the gravitational potential energy of the water is converted into electrical energy.
    b. If the turbine only operates at 30% efficiency, what is the power output?
    c. How many Joules of heat are exhausted into the atmosphere due to the plant's inefficiency?



27. A heat engine operates at a temperature of 650 K. The work output is used to drive a pile driver, which is a machine that picks things up and drops them. Heat is then exhausted into the atmosphere, which has a temperature of 300 K.

    a. What is the ideal efficiency of this engine?
    b. The engine drives a 1200 kg weight by lifting it 50 m in 2.5 sec. What is the engine's power output?
    c. If the engine is operating at 50% of ideal efficiency, how much power is being consumed?
    d. How much power is exhausted?
    e. The fuel the engine uses is rated at 2.7×10^6 J/kg. How many kg of fuel are used in one hour?

28. Calculate the ideal efficiencies of the following sci-fi heat engines:

    a. A nuclear power plant on the moon. The ambient temperature on the moon is 15 K. Heat input from radioactive decay heats the working steam to a temperature of 975 K.
    b. A heat exchanger in a secret underground lake. The exchanger operates between the bottom of a lake, where the temperature is 4 C, and the top, where the temperature is 13 C.
    c. A refrigerator in your dorm room at Mars University. The interior temperature is 282 K; the back of the fridge heats up to 320 K.

29. How much external work can be done by a gas when it expands from 0.003 m^3 to 0.04 m^3 in volume under a constant pressure of 400 kPa? Can you give a practical example of such work?
30. In the above problem, recalculate the work done if the pressure linearly decreases from 400 kPa to 250 kPa under the same expansion. Hint: use a *PV* diagram and find the area under the line.
31. One mole ($N = 6.02 \times 10^{23}$) of an ideal gas is moved through the following states as part of a heat engine. The engine moves from state A to state B to state C, and then back again.

TABLE 21.1:

State	Volume ($m3$)	Pressure (atm)	Temperature (K)
A	0.01	0.60	
B	0.01	0.25	
C	0.02	0.25	

(a) Draw a P-V diagram.

(b) Determine the temperatures in states A, B, and C and then fill out the table.

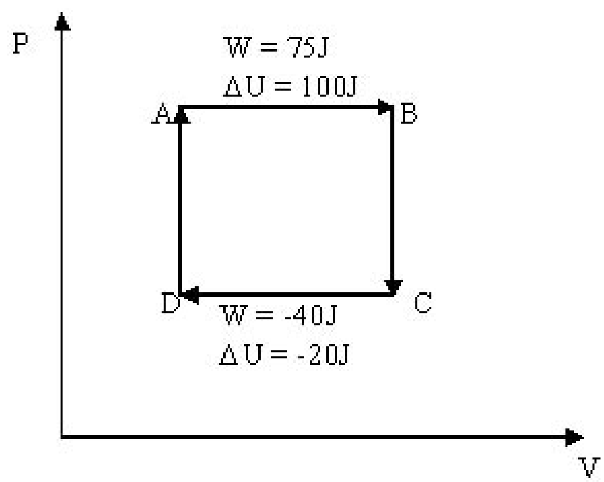
(c) Determine the type of process the system undergoes when transitioning from A to B and from B to C. (That is, decide for each if it is isobaric, isochoric, isothermal, or adiabatic.)

(d) During which transitions, if any, is the gas doing work on the outside world? During which transitions, if any, is work being done on the gas?

(e) What is the amount of net work being done by this gas?

32. A sample of gas is used to drive a piston and do work. Here's how it works: The gas starts out at standard atmospheric pressure and temperature. The lid of the gas container is locked by a pin. The gas pressure is increased isochorically through a spigot to twice that of atmospheric pressure. The locking pin is removed and the gas is allowed to expand isobarically to twice its volume, lifting up a weight. The spigot continues to add gas to the cylinder during this process to keep the pressure constant. Once the expansion has finished, the spigot is released, the high-pressure gas is allowed to escape, and the sample settles back to 1 atm. Finally, the lid of the container is pushed back down. As the volume decreases, gas is allowed to escape through the spigot, maintaining a pressure of 1 atm. At the end, the pin is locked again and the process restarts.
    a. Draw the above steps on a $P-V$ diagram.
    b. Calculate the highest and lowest temperatures of the gas.
33. A heat engine operates through 4 cycles according to the PV diagram sketched below. Starting at the top left vertex they are labeled clockwise as follows: a, b, c, and d.
    a. From $a-b$ the work is 75 J and the change in internal energy is 100 J; find the net heat.
    b. From the a-c the change in internal energy is -20 J. Find the net heat from b-c.
    c. From c-d the work is -40 J. Find the net heat from c-d-a.
    d. Find the net work over the complete 4 cycles.
    e. The change in internal energy from b-c-d is -180 J. Find:

    i. the net heat from c-d ii. the change in internal energy from d-a iii. the net heat from d-a



34. A 0.1 sample mole of an ideal gas is taken from state A by an isochoric process to state B then to state C by an isobaric process. It goes from state C to D by a process that is linear on a PV diagram, and then it goes back to state A by an isobaric process. The volumes and pressures of the states are given below:

TABLE 21.2:

state	Volume in $m^3 \times 10^{-3}$	Pressure in $N/m^2 \times 10^5$
A	1.04	2.50
B	1.04	4.00
C	1.25	4.00
D	1.50	2.50

(a) Find the temperature of the 4 states

(b) Draw a PV diagram of the process

(c) Find the work done in each of the four processes

(d) Find the net work of the engine through a complete cycle

(e) If 75 J of heat is exhausted in D-A and A-B and C-D are adiabatic, how much heat is inputted in B-C?

(f) What is the efficiency of the engine?

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. .
12. .
13. .
14. .
15. .
16. .
17. .
18. 517 m/s
19. 1.15×10^{12} K
20. .
21. 40 N
22. $\approx 10^{28}$ molecules
    1. 21,000 Pa
    2. Decreases to 61,000 Pa
    3. 5.8 km

    1. No
    2. allowed by highly improbable state. More likely states are more disordered.

    1. 8.34×10^{23}
    2. 6.64×10^{-27} kg

3. 1600 m/s
4. 744 kPa
5. 4.2×10^{20} or 0.0007 moles
6. 0.00785 m^3

1. 1.9 MW
2. 0.56 MW
3. 1.3 Mw

1. 54%
2. 240 kW
3. 890 kW
4. 590 kW
5. 630 kg

1. 98%
2. 4.0%
3. 12%

23. 14800 J
24. 12,000 J
25. (b) 720 K, 300 K, 600 K (c) isochoric; isobaric (d) C to A; B – C (e) 0.018 J
26. (b) 300 K, 1200 K

1. 1753 J
2. −120 J
3. 80 J
4. 35 J
5. −100 J, 80 J, 80 J

CHAPTER 22 BCTherm

Chapter Outline

22.1 Microscopic Description of an Ideal Gas

Evidence for the kinetic theory

Why does matter have the thermal properties it does? The basic answer must come from the fact that matter is made of atoms. How, then, do the atoms give rise to the bulk properties we observe? Gases, whose thermal properties are so simple, offer the best chance for us to construct a simple connection between the microscopic and macroscopic worlds.

A crucial observation is that although solids and liquids are nearly incompressible, gases can be compressed, as when we increase the amount of air in a car's tire while hardly increasing its volume at all. This makes us suspect that the atoms in a solid are packed shoulder to shoulder, while a gas is mostly vacuum, with large spaces between molecules. Most liquids and solids have densities about 1000 times greater than most gases, so evidently each molecule in a gas is separated from its nearest neighbors by a space something like 10 times the size of the molecules themselves.

If gas molecules have nothing but empty space between them, why don't the molecules in the room around you just fall to the floor? The only possible answer is that they are in rapid motion, continually rebounding from the walls, floor and ceiling. In chapter 2, we have already seen some of the evidence for the kinetic theory of heat, which states that heat is the kinetic energy of randomly moving molecules. This theory was proposed by Daniel Bernoulli in 1738, and met with considerable opposition, because there was no precedent for this kind of perpetual motion. No rubber ball, however elastic, rebounds from a wall with exactly as much energy as it originally had, nor do we ever observe a collision between balls in which none of the kinetic energy at all is converted to heat and sound. The analogy is a false one, however. A rubber ball consists of atoms, and when it is heated in a collision, the heat is a form of motion of those atoms. An individual molecule, however, cannot possess heat. Likewise sound is a form of bulk motion of molecules, so colliding molecules in a gas cannot convert their kinetic energy to sound. Molecules can indeed induce vibrations such as sound waves when they strike the walls of a container, but the vibrations of the walls are just as likely to impart energy to a gas molecule as to take energy from it. Indeed, this kind of exchange of energy is the mechanism by which the temperatures of the gas and its container become equilibrated.

Pressure, volume, and temperature

A gas exerts pressure on the walls of its container, and in the kinetic theory we interpret this apparently constant pressure as the averaged-out result of vast numbers of collisions occurring every second between the gas molecules and the walls. The empirical facts about gases can be summarized by the relation

$PV \propto nT$, [ideal gas]

which really only holds exactly for an ideal gas. Here n is the number of molecules in the sample of gas.

The proportionality of volume to temperature at fixed pressure was the basis for our definition of temperature.

Pressure is proportional to temperature when volume is held constant. An example is the increase in pressure in a car's tires when the car has been driven on the freeway for a while and the tires and air have become hot.

We now connect these empirical facts to the kinetic theory of a classical ideal gas. For simplicity, we assume that the gas is monoatomic (i.e., each molecule has only one atom), and that it is confined to a cubical box of volume V , with L being the length of each edge and A the area of any wall. An atom whose velocity has an x component v_x will collide regularly with the left-hand wall, traveling a distance $2L$ parallel to the x axis between collisions with that wall. The time between collisions is $\Delta t = 2L/v_x$, and in each collision the x component of the atom's momentum is reversed from $-mv_x$ to mv_x. The total force on the wall is

$F =$

$$\frac{\Delta p_{x,1}}{\Delta t_1}$$

+

$$\frac{\Delta p_{x,2}}{\Delta t_2}$$

+. . . [monoatomic ideal gas] ,

where the indices 1, 2, . . . refer to the individual atoms. Substituting $\Delta p_{x,i} = 2mv_x,i$ and $\Delta t_i = 2L/v_{x,i}$, we have

$F =$

$$\frac{mv_{x,1}^2}{L}$$

+

$$\frac{mv_{x,2}^2}{L}$$

[monoatomic ideal gas] .

The quantity $mv_{x,i}{}^2$ is twice the contribution to the kinetic energy from the part of the atom's center of mass motion that is parallel to the x axis. Since we're assuming a monoatomic gas, center of mass motion is the only type of motion that gives rise to kinetic energy. (A more complex molecule could rotate and vibrate as well.) If the quantity inside the sum included the y and z components, it would be twice the total kinetic energy of all the molecules. By symmetry, it must therefore equal 2/3 of the total kinetic energy, so

$F =$

$$\frac{2KE_{total}}{3L}$$

[monoatomic ideal gas] .

Dividing by A and using $AL = V$, we have

$P =$

$$\frac{2KE_{total}}{3V}$$

[monoatomic ideal gas] .

This can be connected to the empirical relation $PV \propto nT$ if we multiply by V on both sides and rewrite KE_{total} as nKE_{av}, where KE_{av} is the average kinetic energy per molecule:

$PV =$

$$\frac{2}{3}$$

nKE_{av} monoatomic ideal gas

For the first time we have an interpretation for the temperature based on a microscopic description of matter: in a monoatomic ideal gas, the temperature is a measure of the average kinetic energy per molecule. The proportionality between the two is KE_{av}= (3/2)kT, where the constant of proportionality k, known as Boltzmann's constant, has a numerical value of 1.38 x 10^{-23} J/K. In terms of Boltzmann's constant, the relationship among the bulk quantities for an ideal gas becomes

$PV = nkT$, [ideal gas]

which is known as the ideal gas law. Although I won't prove it here, this equation applies to all ideal gases, even though the derivation assumed a monoatomic ideal gas in a cubical box. (You may have seen it written elsewhere as $PV = NRT$, where $N = U/N_A$ is the number of moles of atoms, $R = kN_A$, and N_A = 6.0 X 10^{23}, called Avogadro's number, is essentially the number of hydrogen atoms in 1 g of hydrogen.)

CHAPTER 23 Special and General Relativity Version 2

Chapter Outline

23.1 The Big Ideas

Einstein believed that the laws of physics do not depend on the how fast you are moving through space: every *reference frame* sees the same world of physics. In other words, if you are on a moving train and drop a ball or if you are standing on a farm and drop a ball, the physics that describe the motion of that ball will be the same. Einstein realized that the speed of light, c, should depend only on the laws of physics that describe light as electromagnetic radiation. Therefore, Einstein made the bold assertion that light always travels at the same speed, *no matter how fast you are moving with respect to the source of light*. Consider for a moment how counterintuitive this concept really is. This is the theoretical underpinning of Einstein's theory of Special Relativity, one of the most successfully predictive theories of physics ever formulated.

The most important consequence of this new understanding is that our intuition that time moves at the same rate for everyone (whether standing still or moving at a fast speed) is WRONG. In fact, the rate at which time passes depends on your speed. Since Einstein's work in the early part of the 20^{th} century, this fact has been demonstrated many times by experiments in particle accelerators and through the use of atomic clocks aboard fast moving jet airplanes. The effect is only noticeable at extremely fast speeds, thus the normal laws of motion apply in all but the most extreme cases.

Einstein was finally led to believe that the very fabric of space and time must have a more active and influential role in the laws of physics than had previously been believed. Eventually, Einstein became convinced that gravity itself amounted to no more than a curvature in *spacetime*. This theory is called General Relativity.

Key Concepts

- The speed of light will always be measured to be the same (about 3×10^8 m/s) *regardless* of your motion towards or away from the source of light.
- In order for this bizarre fact to be true, we must reconsider what we mean by 'space,' 'time,' and related concepts, such as the concept of 'simultaneous' events. (Events which are seen as simultaneous by one observer might appear to occur at different times to an observer moving with a different velocity. Note that both observers see the same laws of physics, just a different sequence of events.)
- Clocks moving towards or away from you run more slowly, and objects moving towards or away from you shrink in length. These are known as Lorentz time dilation and length contraction; both are real, measured properties of the universe we live in.
- If matter is compressed highly enough, the curvature of spacetime becomes so intense that a black hole forms. Within a certain distance of a black hole, called an *event horizon*, nothing can escape the intense curvature, not even light. No events which occur within the horizon can have any effect, ever, on events which occur outside the horizon.

Key Equations

$$\beta = \frac{v}{c}$$

An object moving with speed v has a dimensionless speed β calculated by dividing the speed v by the speed of light ($c = 3 \times 10^8$ m/s). $0 \leq \beta \leq 1$.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

The dimensionless Lorentz "gamma" factor γ can be calculated from the speed, and tells you how much time dilation or length contraction there is. $1 \leq \gamma \leq \infty$.

TABLE 23.1:

Object	Speed (km/sec)	β	Lorentz γ Factor
Commercial Airplane	0.25	8×10^{-7}	1.00000000000
Space Shuttle	7.8	3×10^{-5}	1.00000000034
UFO ?	$150,000$	0.5	1.15
Electron at the Stanford Linear Accelerator	$\sim 300,000$	0.9999999995	$\sim 100,000$

$$L' = \frac{L}{\gamma}$$

If you see an object of length$L$ moving towards you at a Lorentz gamma factor γ, it will appear shortened (contracted) in the direction of motion to new length L .

$$T' = \gamma T$$

If a moving object experiences some event which takes a period of time T (say, the amount of time between two heart beats), and the object is moving towards or away from you with Lorentz gamma factor γ, the period of time T' measured by you will appear longer.

$$R_s = \frac{2Gm}{c^2}$$

The radius of the spherical event horizon of a black hole is determined by the mass of the black hole and fundamental constants. A typical black hole radius is about 3 km.

$$m_r = m_0\gamma$$

The mass of an object moving at relativistic speeds increases by a factor of γ.

$$E = mc^2$$

The potential energy of mass is equal to mass times the speed of light squared.

23.2 Relativity Example

Question: The muon particle ($\mu-$) has a half-life of 2.20×10^{-6}s. Most of these particles are produced in the atmosphere, a good 5-20km above Earth, yet we see them all the time in our detectors here on Earth. In this problem you will find out how it is possible that these particles make it all the way to Earth with such a short lifetime.

a) Calculate how far muons could travel before half decayed, without using relativity and assuming a speed of $0.999c$ (i.e. 99.9% of the speed of light)

b) Now calculate γ for this muon.

c) Calculate its 'relativistic' half-life.

d) Now calculate the distance before half decayed using relativistic half-life and express it in kilometers. (This has been observed experimentally. This first experimental verification of time dilation was performed by Bruno Rossi at Mt. Evans, Colorado in 1939.)

Answer:

a) To calculate the distance that the muon particle could travel we will use the equation for distance and then plug in the known values to get the answer.

$$d = v \times t = (.999 \times 3 \times 10^8 \text{m/s}) \times (2.20 \times 10^{-6} \text{s}) = 659.34m$$

b) To solve for γ, we must first solve for β.

$$\beta = \frac{v}{c} = \frac{.999 \times 3 \times 10^8 \text{m/s}}{3 \times 10^8 \text{m/s}} = .999$$

Now we can solve for γ.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-.999^2}} = 22.4$$

c) To calculate the muon particle's relativistic half-life, we will use the γ value we calculated in part b) and the equation for determining relativistic half-life.

$$T' = \gamma T = 22.4 \times 2.20 \times 10^{-6} \text{s} = 4.92 \times 10^{-5}$$

d) To calculate the distance the muon particle can travel we will use the same distance equation but we will use the new half-life instead of the non-relativistic half-life.

$$d = vt = (.999 \times 3 \times 10^8 \text{m/s})(4.92 \times 10^{-5} \text{s}) = 14,700 \text{m}$$

Now we will convert this into km.

$$14700 \text{m} \times \frac{1 \text{km}}{1000 \text{m}} = 14.7 \text{km}$$

23.3 Relativity Problem Set

1. Suppose you discover a speedy subatomic particle that exists for a nanosecond before disintegrating. This subatomic particle moves at a speed close to the speed of light. Do you think the lifetime of this particle would be *longer* or *shorter* than if the particle were at rest?
2. What would be the Lorentz gamma factor γ for a space ship traveling at the speed of light c? If you were in this space ship, how wide would the universe look to you?
3. Suppose your identical twin blasted into space in a space ship and is traveling at a speed of 0.100 c. Your twin performs an experiment which he clocks at 76.0 minutes. You observe this experiment through a powerful telescope; what duration does the experiment have according to your clock? Now the opposite happens and you do the 76.0 minute experiment. How long does the traveling twin think the experiment lasted?
4. An electron is moving to the east at a speed of 1.800×10^7 m/s. What is its dimensionless speed β? What is the Lorentz gamma factor γ?
5. What is the speed v of a particle that has a Lorentz gamma factor $\gamma = 1.05$?
6. How fast would you have to drive in your car in order to make the road 50% shorter through Lorentz contraction?
7. The muon particle $(\mu-)$ has a half-life of 2.20×10^{-6} s. Most of these particles are produced in the atmosphere, a good $5-20$ km above Earth, yet we see them all the time in our detectors here on Earth. In this problem you will find out how it is possible that these particles make it all the way to Earth with such a short lifetime.

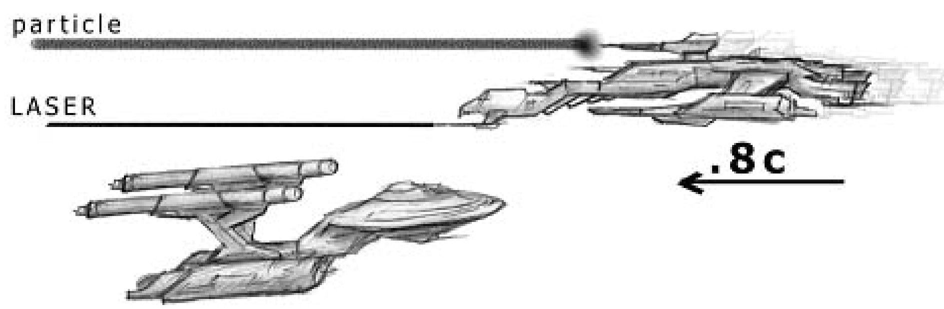
   a. Calculate how far muons could travel before half decayed, without using relativity and assuming a speed of 0.999 c (i.e. 99.9% of the speed of light)
   b. Now calculate γ, for this muon.
   c. Calculate its 'relativistic' half-life.

   a. Now calculate the distance before half decayed using relativistic half-life and express it in kilometers. (This has been observed experimentally. This first experimental verification of time dilation was performed by Bruno Rossi at Mt. Evans, Colorado in 1939.)

8. Calculate the radius of the event horizon of a super-massive black hole (SMBH) with a mass $200,000,000$ times the mass of our Sun. (Unless you have it memorized, you will have to look up the mass of the Sun in kg.)
9. If an electron were "really" a black hole, what would the radius of its event horizon be? Is this a measurable size?
10. An alien spaceship moves past Earth at a speed of .15 c with respect to Earth. The alien clock ticks off 0.30 seconds between two events on the spaceship. What will earthbound observers determine the time interval to be?
11. In 1987 light reached our telescopes from a supernova that occurred in a near-by galaxy $160,000$ light years away. A huge burst of neutrinos preceded the light emission and reached Earth almost two hours ahead of the light. It was calculated that the neutrinos in that journey lost only 13 minutes of their lead time over the light.

   a. What was the ratio of the speed of the neutrinos to that of light?
   b. Calculate how much space was Lorentz-contracted form the point of view of the neutrino.
   c. Suppose you could travel in a spaceship at that speed to that galaxy and back. It that were to occur the Earth would be $320,000$ years older. How much would you have aged?

12. An electron moves in an accelerator at 95% the speed of light. Calculate the relativistic mass of the electron.



13. Enterprise crew members notice that a passing Klingon ship moving 0.8 c with respect to them is engaged in weapons testing on board. At the closest point of contact the Klingons are testing two weapons: one is a laser, which in their frame moves at c; the other is a particle gun, which shoots particles at 0.6 c in the Klingon frame. Both weapons are pointed in the same line as the Klingon ship is moving. Answer the following two questions choosing one of the following options: A. $V < 0.6$ c B. 0.6 c $<$ $V < 0.8$ c C. 0.8 c $<$ $V <$ c D. c $<$ $V < 1.4$ c E. $V > 1.4$ c F. $V =$ c
    a. Question 1: What speed, V, does the Enterprise measure the laser gun to achieve with respect to the Enterprise?
    b. Question 2: What speed, V, does the Enterprise measure the particle gun to achieve with respect to the Enterprise?
14. How much energy is produced by a .5 kilogram softball?
15. The isotope of silicon Si^{31} has an atomic mass of 30.975362 amu. It can go through beta radioactivity, producing P^{31} with a mass of 30.973762 amu.
    a. Calculate the total energy of the beta particle emitted, assuming the P^{31} nucleus remains at rest relative to the Si^{31} nucleus after emission.
    b. Another possibility for this isotope is the emission of a gamma ray of energy 1.2662 Mev. How much kinetic energy would the P^{31} nucleus gain?
    c. What is the frequency and wavelength of the gamma ray?

    a. What is the rebound velocity of the P^{31} nucleus in the case of gamma ray emission?

Answers to Selected Problems

1. longer
2. $\gamma = \infty$, the universe would be a dot
3. .
4. .
5. $\gamma = 1.002$
6. 9.15×10^7 m/s
7. a. 0.659 km b. 22.4 c. 4.92×10^{-5} m/s d. 14.7 km
8. 2900 m
9. 1.34×10^{-57} m
10. 0.303 s
11. .
12. 2.9×10^{-30}kg, yes harder to accelerate
13. a. f b. c
14. 4.5×10^{16} J; 1.8×10^{13} softballs

15. a. 1.568×10^{-13} J b. 3.04×10^{6} J

CHAPTER 24 Radioactivity and Nuclear Physics Version 2

Chapter Outline

24.1 The Big Idea

The nuclei of atoms are affected by three forces: the strong nuclear force, which causes protons and neutrons to bind together, the electric force, which is manifested by repulsion of the protons and tends to rip the nucleus apart, and the weak nuclear force, which causes neutrons to change into protons and vice versa.

The strong force predominates and can cause nuclei of complex atoms with many protons to be stable. The electric force of repulsion is responsible for fission, the breaking apart of nuclei, and therefore also for atom bombs and nuclear power. A form of fission where a helium nucleus is a product, is called *alpha radiation.* The actions of the weak force give rise to *beta radiation.* A change in nuclear energy can also give rise to *gamma radiation*, high energy electromagnetic waves. Particles that emit alpha radiation, beta radiation, and gamma radiation go through the process of *radioactive decay,* which causes the heating of the molten core of the earth, and has even played a role in the mutations in our evolutionary history. Fission and fusion, the latter occurring when light nuclei combine to form new elements, are accompanied by copious amounts of gamma radiation. These processes often produce radioactive nuclei; in nature these radioactive nuclei were forged in the explosive deaths of ancient stars.

24.2 Key Concepts

- Atomic symbols like $^{A}_{Z}X$ are interpreted in the following way: X is the symbol for the element involved. For example, U is the symbol for the element uranium. Z is the atomic number. A is the atomic mass number, the total number of nucleons (protons and neutrons). A would be 235 for uranium.
- Some of the matter on Earth is unstable and undergoing nuclear decay.
- When mass is lost during radioactive decay, the energy released is given by Einsten's famous formula: $E = \Delta mc^2$
- Alpha decay is the emission of a helium nucleus and causes the product to have an atomic number 2 lower than the original and an atomic mass number 4 lower than the original.
- Beta minus decay is the emission of an electron, causing the product to have an atomic number 1 greater than the original
- Beta plus decay is the emission of a positron, causing the product to have an atomic number 1 lower than the original.
- When an atomic nucleus decays, it does so by releasing one or more particles. The atom often (but not always) turns into a different element during the decay process. The amount of radiation given off by a certain sample of radioactive material depends on the amount of material, how quickly it decays, and the nature of the decay product. Big, rapidly decaying samples are most dangerous.
- The measure of how quickly a nucleus decays is given by the *half-life* of the nucleus. One half-life is the amount of time it will take for half of the radioactive material to decay.
- The type of atom is determined by the atomic number (i.e. the number of protons). The atomic mass of an atom is approximately the number of protons plus the number of neutrons. Typically, the atomic mass listed in a periodic table is an average, weighted by the natural abundances of different isotopes.
- The atomic mass number in a nuclear decay process is conserved. This means that you will have the same *total* atomic mass number on both sides of the equation. Charge is also conserved in a nuclear process.
- It is impossible to predict when an individual atom will decay; one can only predict the probability. However, it is possible to predict when a portion of a macroscopic sample will decay extremely accurately because the sample contains a vast number of atoms.
- The nuclear process is largely random in direction. Therefore, radiation strength decreases with distance by the inverse square of the distance (the $1/r^2$ law, which also holds for gravity, electric fields, light intensity, sound intensity, and so on.)

24.3 Decay Equations

Nuclear decay is often measured in terms of half lives. During the span of one half life, the amount of a decaying substance decreases by half. Therefore, after k half lives, the amount of a substance starting at N_0 left is

$$N(k) = N_0 \times \frac{1}{2}^k$$

If we need to know the amount left after some time t, we first need to see find many half lives transpired (this will be given by $\frac{t}{t_H}$, then use the formula above:

$$N(t) = N_0 \times \frac{1}{2}^{\frac{t}{t_H}}$$

If on the other hand, we know how much of a substance is left and would like to find how much time has transpired, we can solve the equation above for t (left to reader):

$$t = t_H \frac{\ln \frac{N}{N_0}}{\ln \frac{1}{2}}$$

This equation is used in radioactive dating:

Question: The half-life of ^{239}Pu is $24,119$ years. You have 31.25 micrograms left, and the sample you are studying started with 2000 micrograms. How long has this rock been decaying?

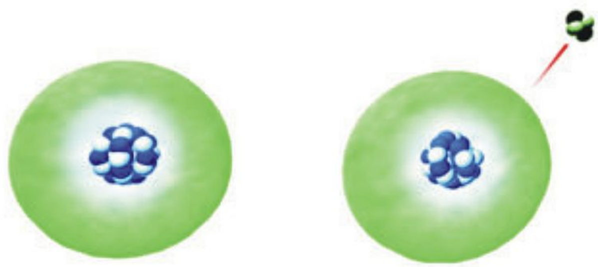
Answer: We will use the equation for time and simply plug in the known values.

$$t = t_H \frac{\ln \frac{N}{N_0}}{\ln \frac{1}{2}} = 24119\text{y} \frac{\ln \frac{31.25\mu\text{g}}{2000\mu\text{g}}}{\ln \frac{1}{2}} = 144,700\text{years}$$

Radioactive carbon dating is a technique that allows scientists to determine the era in which a sample of biological material died. A small portion of the carbon we ingest every day is actually the radioactive isotope ^{14}C rather than the usual ^{12}C. Since we ingest carbon every day until we die (we do this by eating plants; the plants do it through photosynthesis), the amount of ^{14}C in us should begin to decrease from the moment we die. By analyzing the ratio of the number of ^{14}C to ^{12}C atoms in dead carbon-based life forms (including cloth made from plants!) and using the technique illustrated above, we can determine how long ago the life form died.

24.4 Key Applications

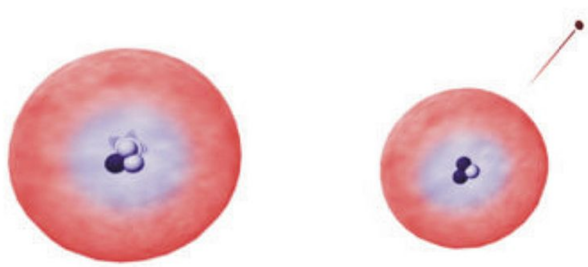
Alpha Decay



- *Alpha decay* is the process in which an isotope releases a helium nucleus (2 protons and 2 neutrons, ${}^{4}_{2}\text{He}$) and thus decays into an atom with two less protons.

Example: ${}^{232}_{90}\text{Th} \rightarrow {}^{228}_{88}\text{Ra} + {}^{4}_{2}\text{He}$

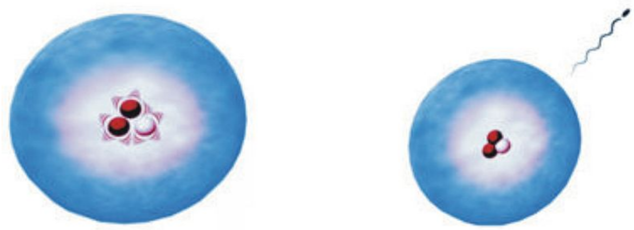
Beta Decay



- *Beta decay* is the process in which one of the neutrons in an isotope decays, leaving a proton, electron and anti-neutrino. As a result, the nucleus decays into an atom that has the same number of nucleons, with one neutron replaced by a proton. (Beta positive decay is the reverse process, in which a proton decays into a neutron, anti-electron and neutrino.)

Example: ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}^{-} + \text{v}$

Gamma Decay



- *Gamma decay* is the process in which an excited atomic nucleus kicks out a photon and releases some of its energy. The makeup of the nucleus doesn't change, it just loses energy. (It can be useful to think of this as energy of motion – think of a shuddering nucleus that only relaxes after emitting some light.)

  Example: ${}^{137}_{56}\text{Ba}^* \rightarrow {}^{137}_{56}\text{Ba} + \text{y}$

Fission and Fusion

- *Fission* is the process in which an atomic nucleus breaks apart into two less massive nuclei. Energy is released in the process in many forms, heat, gamma rays and the kinetic energy of neutrons. If these neutrons collide with nuclei and induce more fission, then a runaway *chain reaction* can take place. Fission is responsible for nuclear energy and atom-bomb explosions: the fission of uranium acts as a heat source for the Earth's molten interior.

  Example: ${}^{1}\text{n} + {}^{235}\text{U} \rightarrow {}^{141}\text{Ba} + {}^{92}\text{Kr} + 3{}^{1}\text{n}$

- *Fusion* is the process in which two atomic nuclei fuse together to make a single nucleus. Energy is released in the form of nuclear particles, neutrons, and gamma-rays.

  Example: ${}^{3}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n} + \text{y}$

24.5 Radioactivity and Nuclear Physics Problem Set

1. After 6 seconds, the mass of a sample of radioactive material has reduced from 100 grams to 25 grams. Its half-life must be
   a. 1 s
   b. 2 s
   c. 3 s

   a. 4 s
   b. 6 s
2. Which of the following is true for the following reaction? $^{236}\text{U} \rightarrow {}^{90}\text{Sr} + {}^{143}\text{Xe} + 3^{1}\text{n}$
   a. This is a fission reaction.
   b. This is a fusion reaction.
   c. This is not a valid reaction, because the equations don't balance.
3. For any radioactive material, its half-life...
   a. ...first decreases and then increases.
   b. ...first increases and then decreases.
   c. ...increases with time.

   a. ...decreases with time.
   b. ...stays the same.
4. If the half-life of a substance is 5 seconds, it ceases to be radioactive (i.e. it ceases emitting particles), ...
   a. ... after 5 seconds.
   b. ... after 10 seconds
   c. ... after 20 seconds.

   a. ... after a very long time.
5. You detect a high number of alpha particles every second when standing a certain distance from a radioactive material. If you triple your distance from the source, the number of alpha particles you detect will decrease. By what factor will it decrease?
   a. $\sqrt{3}$
   b. 3
   c. 9

   a. 27
   b. It will stay the same.
6. You have 5 grams of radioactive substance A and 5 grams of radioactive substance B. Both decay by emitting alpha-radiation, and you know that the higher the number of alpha-particles emitted in a given amount of time, the more dangerous the sample is. Substance A has a short half-life (around 4 days or so) and substance B has a longer half-life (around 10 months or so).
   a. Which substance is more dangerous right now? Explain.
   b. Which substance will be more dangerous in two years? Explain.
7. Write the nuclear equations $A \rightarrow B + C$ for the following reactions.
   a. The alpha decay of ^{219}Ra.

b. The beta decay of ^{158}Eu.
c. The beta decay of ^{53}Ti.

a. The alpha decay of ^{211}Bi.

8. A certain radioactive material has a half-life of 8 minutes. Suppose you have a large sample of this material, containing 10^{25} atoms.

a. How many atoms decay in the first 8 minutes?
b. Does this strike you as a dangerous release of radiation? Explain.
c. How many atoms decay in the second 8 minutes?

a. What is the ratio of the number of atoms that decay in the first 8 minutes to the number of atoms that decay in the second 8 minutes?
b. How long would you have to wait until the decay *rate* drops to 1% of its value in the first 8 minutes?

9. There are two equal amounts of radioactive material. One has a short half-life and the other has a very long half-life. If you measured the decay rates coming from each sample, which would you expect to have a higher decay *rate*? Why?
10. Hidden in your devious secret laboratory are 5.0 grams of radioactive substance A and 5.0 grams of radioactive substance B. Both emit alpha-radiation. Quick tests determine that substance A has a half-life of 4.2 days and substance B has a half-life of 310 days.

a. How many grams of substance A and how many grams of substance B will you have after waiting 30 days?
b. Which sample (A or B) is more dangerous at this point (i.e., after the 30 days have passed)?

11. The half-life of a certain radioactive material is 4 years. After 24 years, how much of a 75 g sample of this material will remain?
12. The half life of ^{53}Ti is 33.0 seconds. You begin with 1000 g of ^{53}Ti. How much is left after 99.0 seconds?
13. You want to determine the half-life of a radioactive substance. At the moment you start your stopwatch, the radioactive substance has a mass of 10 g. After 2.0 minutes, the radioactive substance has 0.5 grams left. What is its half-life?
14. The half-life of ^{239}Pu is $24,119$ years. You have 31.25 micrograms left, and the sample you are studying started with 2000 micrograms. How long has this rock been decaying?
15. A certain fossilized plant is $23,000$ years old. Anthropologist Hwi Kim determines that when the plant died, it contained 0.250 g of radioactive $^{14}C(t_H = 5730 \text{ years})$. How much should be left now?
16. Jaya unearths a guinea pig skeleton from the backyard. She runs a few tests and determines that 99.7946% of the original ^{14}C is still present in the guinea pig's bones. The half-life of ^{14}C is 5730 years. When did the guinea pig die?
17. You use the carbon dating technique to determine the age of an old skeleton you found in the woods. From the total mass of the skeleton and the knowledge of its molecular makeup you determine that the amount of ^{14}C it began with was 0.021 grams. After some hard work, you measure the current amount of ^{14}C in the skeleton to be 0.000054 grams. How old is this skeleton? Are you famous?
18. Micol had in her lab two samples of radioactive isotopes: ^{151}Pm with a half-life of 1.183 days and ^{134} Ce with a half-life of 3.15 days. She initially had 100 mg of the former and 50 mg of the latter.

a. Do a graph of quantity remaining (vertical axis) vs. time for both isotopes on the same graph.
b. *Using the graph* determine at what time the quantities remaining of both isotopes are exactly equal and what that quantity is.
c. Micol can detect no quantities less than 3.00 mg. Again, *using the graph*, determine how long she will wait until each of the original isotopes will become undetectable.

a. The *Pm* goes through $\beta-$ decay and the *Ce* decays by means of electron capture. What are the two immediate products of the radioactivity?

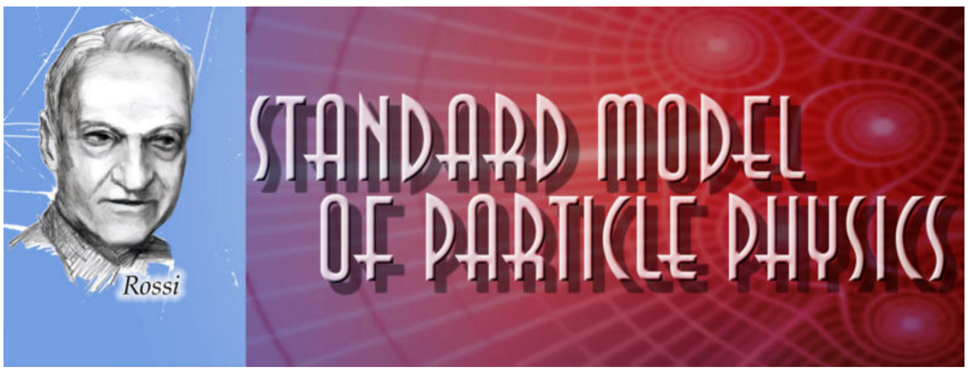
b. It turns out both of these products are themselves radioactive; the *Pm* product goes through $\beta-$ decay before it becomes stable and the *Ce* product goes through $\beta+$ decay before it reaches a stable isotope. When all is said and done, what will Micol have left in her lab?

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. .
6. a. Substance A decays faster than B b. Substance B because there is more material left to decay.
7. a. ${}^{219}{}_{88}\text{Ra} \rightarrow {}^{215}{}_{86}\text{Rn} + {}^{4}{}_{2}\text{He}$ b. ${}^{158}{}_{63}\text{Eu} \rightarrow {}^{158}{}_{64}\text{Gd} + {}^{0}{}_{-1}e^{-}$ c. ${}^{53}{}_{22}\text{Ti} \rightarrow {}^{53}{}_{23}\text{Va} + {}^{0}{}_{-1}e^{-}$ d. ${}^{211}{}_{83}\text{Bi} \rightarrow {}^{207}{}_{81}\text{Tl} + {}^{4}{}_{2}\text{He}$
8. a. 5×10^{24} atoms b. Decay of a lot of atoms in a short period of time c. 2.5×10^{24} atoms d. $\frac{1}{2}$ e. 26.6 minutes
9. The one with the short half life, because half life is the rate of decay.
10. a. Substance $B = 4.6$ g and substance $A = 0.035$ g b. substance B
11. 1.2 g
12. 125 g
13. 0.46 minutes
14. $t = 144,700$ years
15. 0.0155 g
16. 17 years
17. $49,000$ years

CHAPTER 25 Standard Model of Particle Physics Version 2

Chapter Outline



25.1 The Big Idea

All matter is composed of fundamental building blocks, called the elementary particles. These building blocks are much smaller than an atom, and so are sometimes referred to as *subatomic* particles. Particles interact with one another according to a set of laws. There are two types of particles: force particles (fermions) and matter particles (bosons). What sets them apart is an intrinsic property called 'spin'. The set of particles and the laws that govern their interactions are called the Standard Model. The Standard Model is very powerful and can predict particle interactions to amazing accuracy.

The fifth of the five conservation laws is called CPT symmetry. CPT is a symmetry between matter and anti-matter. The law states that if you reverse the spatial coordinates of a particle, change it from matter to anti-matter, and reverse it in time the new object is now indistinguishable from the original. More on the fifth conservation law in the Feynman Diagram's chapter.

25.2 Matter

- Particles can be grouped into two camps: *fermions* and *bosons.* Typically matter is made up of fermions, while interactions (which lead to forces of nature such as gravity and electromagnetism) occur through the exchange of particles called bosons. (There are exceptions to this.) Electrons and protons are fermions, while photons (light particles) are bosons.
- Fermions (matter particles) can be broken into two groups: *leptons* and *quarks.* Each of these groups comes in three families.
- The first family of leptons consists of the *electron* and the *electron neutrino.* The second family consists of the *muon* and the *muon neutrino.* The third consists of the *tau* and the *tau neutrino.* Particles in each successive family are more massive than the family before it.
- The first family of quarks consists of the up and down quark. The second family consists of the charm and strange quarks. The third family consists of the top and bottom quarks.
- Up and down quarks combine (via the strong force) to form nucleons. Two ups and a down quark make a proton, while an up quark and two down quarks make a neutron. Different combinations of quarks are called *mesons.* In reality, most of the mass of a proton, neutron, etc. is made up of binding energy and virtual particles.
- Particles differ in their mass, their electric charge, their family (in the case of leptons), and their "spin." Spin is a quantum mechanical concept that is best explained as a magnetic moment intrinsic to the particle and manifested as angular momentum.

25.3 Interactions

- There are four fundamental forces in nature. From weakest to strongest, these are the gravitational force, the weak nuclear force, the electromagnetic force, and the strong nuclear force.
- Each fundamental force is transmitted by its own boson(s): for gravity, they are called gravitons; for the weak nuclear force, they are called W^-, W^+, and Z° bosons; for the electromagnetic force, they are called photons; and for the strong nuclear force, they are called gluons.
- In summary, the building blocks of matter and the interactions between matter consist of the following fundamental particles :

TABLE 25.1:

Fermions	Fermions
Leptons	*Quarks*
electron	up
electron neutrino	down
muon	strange
muon neutrino	charm
tau	top
tau neutrino	bottom

TABLE 25.2:

Bosons	Bosons
Force Transmitted	*Associated Boson*
gravity	graviton
electromagnetic	photon
weak	W^-, W^+, and Z°
strong	gluons

25.4 Rules

- For any interaction between particles, the five conservation laws (energy, momentum, angular momentum, charge, and CPT) must be followed. For instance, the total electric charge must always be the same before and after an interaction.
- Electron lepton number is conserved. This means that the total number of electrons *plus* electron neutrinos must be the same before and after an interaction. Similarly, muon lepton number and tau lepton number are also (separately) conserved. Note that matter gets lepton number of $+1$ and antimatter has lepton number of -1.
- Total quark number is conserved. Unlike leptons, however, this total includes *all* families. Again matter particles get quark number of $+1$ and antimatter -1.
- Photons can only interact with objects that have electric charge. This means that particles without charge (such as the electron neutrino) can never interact with photons.
- The strong nuclear force can only act on quarks. This means that gluons (the particle that carries the strong nuclear force) can only interact with quarks, or other gluons.
- The gravitational force can only act on objects with energy, and hence any object with mass.
- The weak nuclear force interacts with both quarks and leptons. However, the weak force is carried by any of three particles, called *intermediate vector bosons:* W^-, W^+, and Z°. Note that the W particles carry electric charge. This means you have to be more careful in making sure that any weak force interaction conserves electric charge.
- Any interaction which obeys all of these rules, and also obeys the usual rules of energy and momentum conservation, is allowed. Due to the randomness of particle interactions, any allowed interaction must eventually happen and thus has a non-zero probability of happening.

Antimatter

- In addition to all of this, there is a further complication: each type of particle that exists (such as an electron or an up quark) has an antiparticle. Antiparticles are strange beasts: they have the same properties as their corresponding particles (mass, size, interactions) but their quantum numbers are exactly reversed electric charge, electron, muon, or tau lepton number, and quark number).
- There are two ways to denote something as an antiparticle. The most common is to draw a horizontal line above the thing. So, for instance, the antiparticle of the up quark is the anti-up quark:

u	$\bar{u}$
up quark	anti-up quark

- For charged leptons, you can merely switch the charge. So, for instance, an electron has negative charge and is written e^- , while its antiparticle, the anti-electron (also called a *positron*) is written e^+.

e^-	e^+
electron	anti-electron (aka positron)

- Particles and antiparticles annihilate each other, and convert their mass directly to energy in the form of gamma rays. Likewise, gamma rays can spontaneously revert to particle-antiparticle pairs. Matter and energy exchange places frequently in this process, with a conversion formula given by the famous equation $E = mc^2$.

25.5 Resources

- Ask your teacher to provide you with a copy of the Standard Model of Particles and Interactions. If there aren't any available, please download and print out a copy of the Standard Model of Particles and Interactions, available at http://particleadventure.org/

25.6 Standard Model of Particle Physics Problem Set

You will need a copy of the Standard Model to do this assignment. See above.

1. Which is more massive, the strange quark or the muon?
2. If you bound an up quark to an anti-strange quark using gluons, would the result be a proton, a neutron, an electron, or some type of meson?
3. Name three particles that do not interact with gluons.
4. Name three particles that do not interact with photons.
5. Which nucleon does not interact with photons? Why?
6. Does the electron neutrino interact with photons? Why or why not?
7. What quarks make up an anti-proton?
8. What rule would be violated if Dr. Shapiro attempted to turn an anti-electron (positron) into a proton?
9. Can any of the intermediate vector bosons (W^-, W^+, and Z°) interact with light? If so, which?
10. What force (of the four) **must** be involved in the process of beta decay, in which a neutron disappears and turns into a proton, an electron, and an electron anti-neutrino?
11. In the world-view provided by the Standard Model, the universe of the very small contains which of the following? (Choose any and all that apply.)
    a. Boson-exchange interactions between different types of quarks and leptons
    b. Annihilation and creation of particle-antiparticle pairs
    c. Electromagnetic interactions between charged objects

    a. Electromagnetic interactions between Z° bosons
    b. Weak interactions involving quarks and leptons

    f. Strong interactions between water molecules Explain.
12. What is string theory? Why isn't string theory mentioned anywhere on the Standard Model? (If you are not already familiar with string theory, you may have to do some research online.)
13. Name three winners of the Nobel Prize who were directly investigating atomic and subatomic particles and interactions. Investigate online.

Answers to Selected Problems

1. strange
2. some type of meson
3. Electron, photon, tau...
4. Neutron, electron neutrino, Z^0
5. Neutron, because it doesn't have electrical charge
6. No, because it doesn't have electrical charge
7. Two anti-up quarks and an anti-down quark
8. Lepton number, and energy/mass conservation
9. Yes, W^+,W^-, because they both have charge
10. The weak force because it can interact with both quarks and leptons
11. Yes; a,b,c,e; no; d,f
12. The standard model makes verifiable predictions, string theory makes few verifiable predictions.

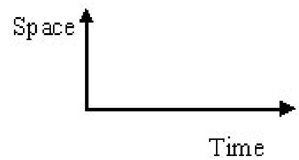
CHAPTER 26 Feynman's Diagrams Version 2

Chapter Outline

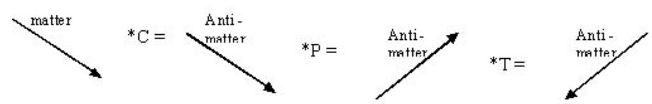
26.1 The Big Idea

The interaction of subatomic particles through the four fundamental forces is the basic foundation of all the physics we have studied so far. There's a relatively simple way to calculate the probability of collisions, annihilations, or decays of particles, invented by physicist Richard Feynman, called Feynman diagrams. Drawing Feynman diagrams is the first step in visualizing and predicting the subatomic world. If a process does not violate a known conservation law, then that process must exist with some probability. All the Standard Model rules of the previous chapter are used here. You are now entering the exciting world of particle physics.

26.2 Key Concepts

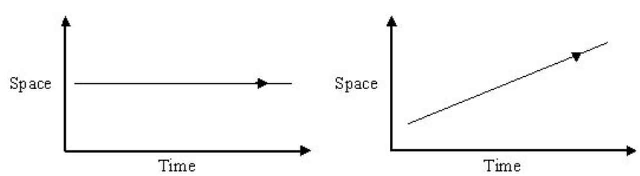


- To make a Feynman diagram, you plot time on the horizontal axis and position on the vertical axis. This is called a space-time diagram.
- The fifth conservation law: CPT symmetry. States that if you charge conjugate (i.e. change matter to anti-matter), Parity reversal (i.e. mirror reflection) and then reverse the flow of time, a matter particle is exactly the same as the anti-matter particle (see below)

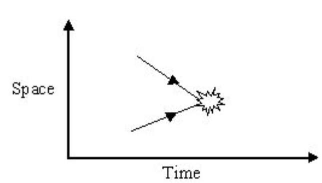


This is why anti-matter has its time arrow pointing backwards. And on collision diagrams, the matter is identical to the anti-matter after a CPT operation.

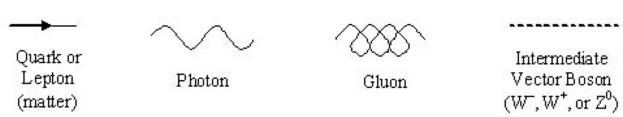
- If a particle is not moving, then we say that its space coordinate is fixed. Of course, if it's just sitting there, then it's moving through *time*. On the diagram below (left), the horizontal line shows the path of motion of a stationary particle. The diagram to the right shows the path of motion of a particle moving away from the origin at some speed.



- Here are two particles colliding! Watch out!

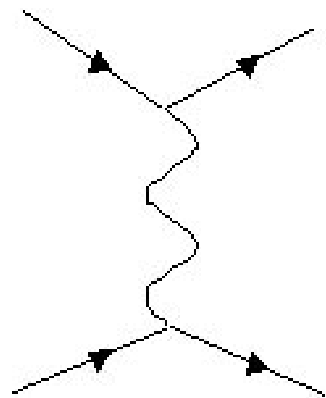


- We use the following symbols in Feynman diagrams:



- *Annihilation Diagram:* When matter and antimatter particles collide, they annihilate, leaving behind pure energy for the example below in the form of electromagnetic radiation (photons!). A different set of matter and antimatter is recreated soon thereafter. The Feynman diagram for that process looks like this:

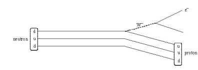
Note that space and time axes have been left out; they are understood to be there. Also note that the arrow on the bottom is supposed to be backwards. We do that any time we have an antiparticle. Most people like to think of antiparticles as traveling backwards in time, and this is roughly explained by CPT symmetry. It is *very* important that you remember that time is the horizontal axis! A lot of people see the drawing above and think of it as two particles coming together at an angle. These two particles are in a *head-on* collision, not hitting at an angle.



- *Scattering Diagram:* Here is the Feynman diagram for two electrons coming towards each other then repelling each other through the electromagnetic force (via *exchange* of a *virtual* photon). Note that the particles are always separated in space (vertical axis) so that they never touch. Hence they are scattering by exchanging virtual photons which cause them to repel. You can think of a virtual photon as existing for an instant of time. Therefore there is no movement in time (horizontal) axis.

26.3 Example

TABLE 26.1:



Question: For the following Feynman diagrams, describe in words the process that is occurring. For instance: (a) what type of interaction: annihilation or scattering (b) what are the incoming articles? (c) which kind of boson mediates the interaction? (d) which fundamental force is involved in the interaction? (e) what are the outgoing particles? Also, is the interaction "allowed"?

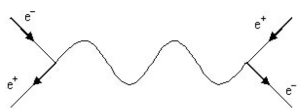
Answer: In this Feynman diagram, one of the down quark from a neutron splits into a upward quark, an electron, and a electron neutrino via a W^- particle. Because this does not break any laws, this interaction is allowed. Infact, this is a interaction that we already know. This is β decay.

26.4 Feynman Diagrams Problem Set

For the following Feynman diagrams, describe in words the process that is occurring. For instance: (a) what type of interaction: annihilation or scattering (b) what are the incoming articles? (c) which kind of boson mediates the interaction? (d) which fundamental force is involved in the interaction? (e) what are the outgoing particles?

Also, for each, decide if the interaction shown is *allowed.* An interaction is allowed if it does not violate any of the rules set out by the Standard Model of physics. If the interactions violate some rule, state which rule it violates. If they do not violate a rule, say that the interactions are allowed.

Hint: the best approach is to verify that the incoming and outgoing particles can interact with the boson (force particle) then to look at each vertex where more than one particle is coming together. Look immediately to the left of the vertex (before) and immediately to the right of the vertex (after). For instance, one rule states that the total electric charge before a vertex must equal the total electric charge after a vertex. Is that true? Check all the conserved quantities from the previous chapter in this way.

1. 

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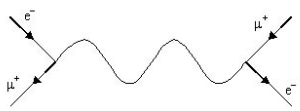
3. 

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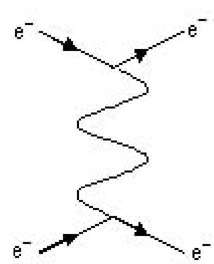
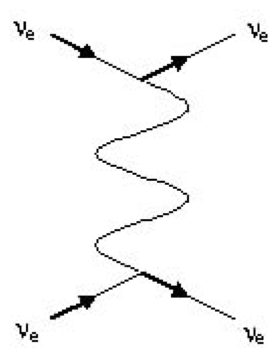
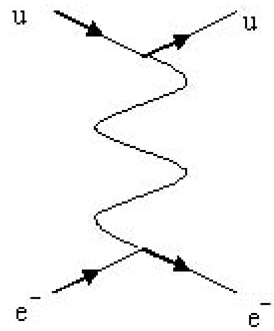
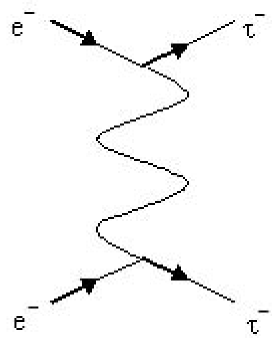




a.

5. #





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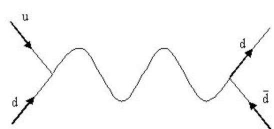
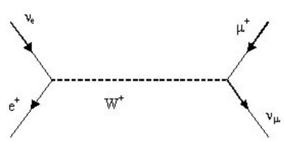
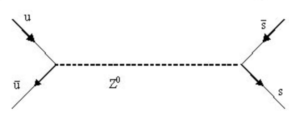
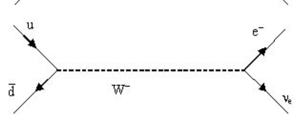
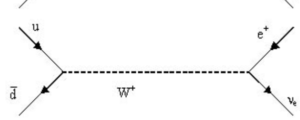
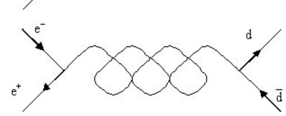
6. 

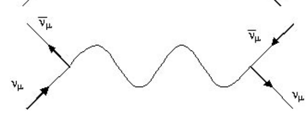
7. 

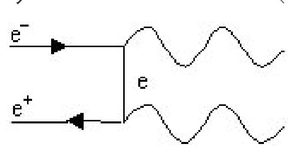
8. 

9. 

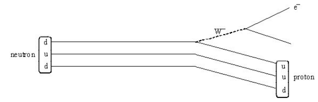
10. 

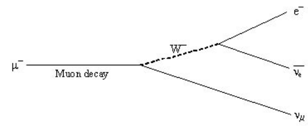
11. 

12. 

13. 

In this case, the electron and positron are exchanging virtual electron/positron pairs.

14. 

15. 

16. Draw all of the possible Feynman diagrams for the annihilation of an electron and positron, followed by motion of an exchange particle, followed by the creation of a new electron and positron.
17. Draw the Feynman diagram for the collision of an up and anti-down quark followed by the production of a positron and electron neutrino.

Answers to Selected Problems

1. Allowed: an electron and anti-electron(positron) annihilate to a photon then become an electron and anti-electron(positron) again.
2. Not allowed: electrons don't go backward though time, and charge is not conserved
3. Not allowed: lepton number is not conserved
4. a. Allowed: two electrons exchange a photon b. Not allowed: neutrinos do not have charge and therefore cannot exchange a photon.
5. a. Allowed: an electron and an up quark exchange a photon b. Not allowed: lepton number not conserved
6. Not allowed: quark number not conserved
7. Allowed: electron neutrino annihilates with a positron becomes a W^+ then splits to muon and muon neutrino.
8. Allowed: up quark annihilates with anti-up quark becomes Z^0, then becomes a strange quark and anti-strange quark
9. Not allowed: charge not conserved
10. Allowed: this is a very rare interaction
11. Not allowed: electrons don't interact with gluons
12. Not allowed: neutrinos don't interact with photons
13. Allowed: the electron and the positron are exchanging virtual electron/positron pairs
14. Allowed: this is beta decay, a down quark splits into an up quark an electron and an electron neutrino via a W^- particle.
15. Allowed: a muon splits into an muon neutrino, an electron and an electron neutrino via a W^- particle.

CHAPTER 27 Quantum Mechanics Version 2

Chapter Outline

27.1 The Big Idea

Quantum Mechanics, discovered early in the 20^{th} century, completely changed the way physicists think. Quantum Mechanics is the description of how the universe works on the very small scale. It turns out that we can't predict what will happen, but only the probabilities of certain outcomes. The uncertainty of quantum events is extremely important at the atomic level (and smaller levels) but not at the macroscopic level. In fact, there is a result called the correspondence principle that states that all results from quantum mechanics must agree with classical physics when quantum numbers are large – that is, for objects with large mass. The foundation of quantum mechanics was developed on the observation of *wave-particle duality*.

Electromagnetic radiation is carried by particles, called photons, which interact with electrons. Depending on the experiment, photons can behave as particles or waves. The reverse is also true; electrons can also behave as particles or waves.

Because the electron has a wavelength, its position and momentum can never be precisely established. This is called the uncertainty principle. (What has been said above about the electron is true for protons or any other particle, but, experimentally, the effects become undetectable with increasing mass.)

The Key Concepts

- The energy of a photon is the product of its frequency and Planck's Constant. This is the exact amount of energy an electron will have if it absorbs a photon.
- A photon, which has neither mass nor volume, carries energy and momentum; the quantity of either energy or momentum in a photon depends on its frequency. The photon travels at the speed of light.
- The five conservation laws hold true at the quantum level. Energy, momentum, angular momentum, charge and CPT are all conserved from the particle level to the astrophysics level.
- If an electron loses energy the photon emitted will have its frequency (and wavelength) determined by the difference in the electron's energy. This obeys the conservation of energy, one of the five conservation laws.
- An electron, which has mass (but probably no volume) has energy and momentum determined by its speed, which is always less than that of light. The electron has a wavelength determined by its momentum.
- If a photon strikes some *photoelectric* material its energy must first go into releasing the electron from the material (This is called the work function of the material.) The remaining energy, if any, goes into kinetic energy of the electron and the voltage of an electric circuit can be calculated from this. The current comes from the number of electrons/second and that corresponds exactly to the number of photons/second.
- Increasing the number of photons will not change the amount of energy an electron will have, but will increase the number of electrons emitted.
- The momentum of photons is equal to Planck's constant divided by the wavelength.
- The wavelength of electrons is equal to Planck's constant divided by the electron's momentum. If an electron is traveling at about .1 c this wavelength is then not much smaller than the size of an atom.
- The size of the electron's wavelength determines the possible energy levels in an atom. These are negative energies since the electron is said to have zero potential energy when it is ionized. The lowest energy level (ground state) for hydrogen is -13.6 eV. The second level is-3.4 eV. Atoms with multiple electrons have multiple sets of energy levels. (And energy levels are different for partially ionized atoms.)
- When an electron absorbs a photon it moves to higher energy level, depending on the energy of the photon. If a 13.6 eV photon hits a hydrogen atom it ionizes that atom. If a 10.2 eV photon strikes hydrogen the electron is moved to the next level.
- Atomic spectra are unique to each element. They are seen when electrons drop from a higher energy level to

a lower one. For example when an electron drops from -3.4 eV to -13.6 eV in the Hydrogen atom a 10.2 eV photon is emitted. The spectra can be in infra-red, visible light, ultra-violet and even $X-$rays. (The 10.2 eV photon is ultra-violet.)

- The wave nature of electrons makes it impossible to determine exactly both its momentum and position. The product of the two uncertainties is on the order of Plank's Constant. (Uncertainty in the electron's energy and time are likewise related.)

The Key Equations

$$E = hf$$

Relates energy of a photon to its frequency.

$$p = \frac{h}{\lambda}$$

Relates the momentum of a photon to its wavelength.

$$\lambda = \frac{h}{p}$$

The Debroglie wavelength of an electron.

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

This is the Heisenberg Uncertainty Principle, (HUP) which relates the uncertainty in the momentum and position of a particle.

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

Relates the uncertainty in measuring the energy of a particle and the time it takes to do the measurement.

$$h = 6.626 \times 10^{-34} \text{ J-sec}$$

Planck's constant.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

The most convenient unit of energy at the atomic scale is the electron volt, defined as the potential energy of the charge of an electron across a potential difference of 1 volt.

$$1240 \text{ nm} \rightarrow 1 \text{ eV}$$

A photon of energy of 1.00 eV has a wavelength of 1240 nm and vice versa. This is a convenient shortcut for determining the wavelengths of photons emitted when electrons change energy levels, or for calculations involving the photoelectric effect.

Problems Set: Quantum Mechanics

1. Calculate the energy and momentum of photons with the following frequency:
   a. From an FM station at 101.9 MHz
   b. Infrared radiation at 0.90×10^{14} Hz
   c. From an AM station at 740 kHz
2. Find the energy and momentum of photons with a wavelength:
   a. red light at 640 nm
   b. ultraviolet light at 98.0 nm
   c. gamma rays at .248 pm
3. Given the energy of the following particles find the wavelength of:
   a. X-ray photons at 15.0 keV
   b. Gamma ray photons from sodium 24 at 2.70 MeV
   c. A 1.70 eV electron
4. The momentum of an electron is measured to an accuracy of 5.10×10^{-15} kg $-$ m/s. What is the corresponding uncertainty in the position of the electron?
5. The four lowest energy levels in electron-volts in a hypothetical atom are respectively: -34 eV, -17 eV, -3.5 eV, $-.27$ eV.
   a. Find the wavelength of the photon that can ionize this atom.
   b. Is this visible light? Why?
   c. If an electron is excited to the fourth level what are the wavelengths of all possible transitions? Which are visible?
6. Light with a wavelength of 620 nm strikes a photoelectric surface with a work function of 1.20 eV. What is the stopping potential for the electron?
7. For the same surface in the previous problem but different frequency light, a stopping potential of 1.40 V is observed. What is the wavelength of the light?
8. An electron is accelerated through 5000 V. It collides with a positron of the same energy. All energy goes to produce a gamma ray.
   a. What is the wavelength of the gamma ray ignoring the rest mass of the electron and positron?
   b. Now calculate the contribution to the wavelength of the gamma ray of the masses of the particles? Recalculate the wavelength.
   c. Was it safe to ignore their masses? Why or why not?
9. An photon of 42.0 eV strikes an electron. What is the increase in speed of the electron assuming all the photon's momentum goes to the electron?
10. A 22.0 keV$X-$ray in the x-direction strikes an electron initially at rest. This time a 0.1 nm$X-$ray is observed moving in the $x-$direction after collision. What is the magnitude and direction of the velocity of the electron after collision?
11. The highly radioactive isotope Polonium 214 has a half-life of 163.7μs and emits a 799 keV gamma ray upon decay. The isotopic mass is 213.99 amu.

a. How much time would it take for 7/8 of this substance to decay?
b. Suppose you had 1.00 g of Po^{214} how much energy would the emitted gamma rays give off while 7/8 decayed?
c. What is the power generated in kilowatts?
d. What is the wavelength of the gamma ray?

12. Ultra-violet light of 110 nm strikes a photoelectric surface and requires a stopping potential of 8.00 volts. What is the work function of the surface?
13. Students doing an experiment to determine the value of Planck's constant shined light from a variety of lasers on a photoelectric surface with an unknown work function and measured the stopping voltage. Their data is summarized below:

a. Construct a graph of energy vs. frequency of emitted electrons.
b. Use the graph to determine the *experimental value of* Planck's constant
c. Use the graph to determine the work function of the surface
d. Use the graph to determine what wavelength of light would require a 6.0 V stopping potential.
e. Use the graph to determine the stopping potential required if 550 nm light were shined on the surface.

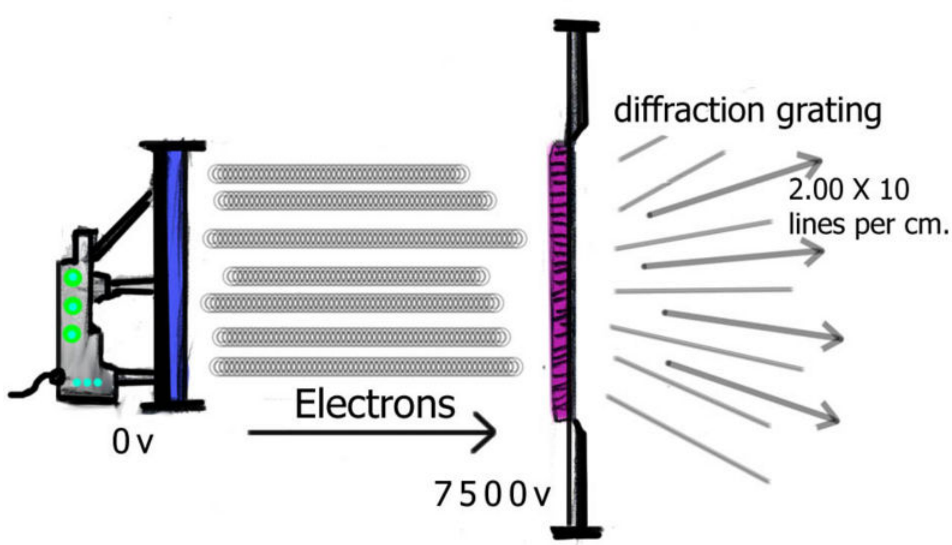
TABLE 27.1:

Laser	Wavelength (nm)	Voltage (V)
Helium-Neon	632.5	.50
Krypton-Flouride	248	3.5
Argon	488	1.1
Europium	612	.60
Gallium arsenide	820	.05

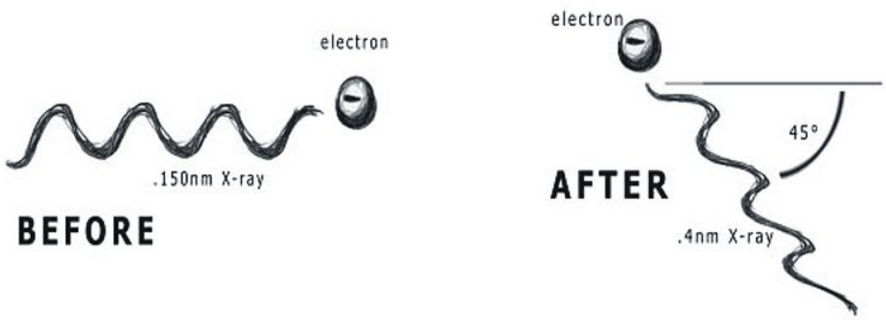
14. An element has the following six lowest energy (in eV) levels for its outermost electron: $-24\text{ eV}, -7.5\text{ eV}, -2.1\text{ eV}, -1.5\text{ eV}$

a. Construct a diagram showing the energy levels for this situation.
b. Show all possible transitions; how many are there?
c. Calculate the wavelengths for transitions to the -7.5 eV level
d. Arrange these to predict which would be seen by infrared, visible and ultraviolet spectroscopes

15. A different element has black absorption lines at 128 nm, 325 nm, 541 nm and677 nm when white light is shined upon it. Use this information to construct an energy level diagram.



16. An electron is accelerated through 7500 V and is beamed through a diffraction grating, which has 2.00×10^7 lines per *cm*.
    a. Calculate the speed of the electron
    b. Calculate the wavelength of the electron
    c. Calculate the angle in which the first order maximum makes with the diffraction grating
    d. If the screen is 2.00 m away from the diffraction grating what is the separation distance of the central maximum to the first order?
17. A light source of 429 nm is used to power a photovoltaic cell with a work function of 0.900 eV. The cell is struck by 1.00×10^{19} photons per second.
    a. What voltage is produced by the cell?
    b. What current is produced by the cell?
    c. What is the cell's internal resistance?



18. A .150 nm$X-$ray moving in the positive $x-$direction strikes an electron, which is at rest. After the collision an $X-$ray of 0.400 nm is observed to move 45 degrees from the positive $x-$axis.
    a. What is the initial momentum of the incident $X-$ray?
    b. What are the x and y components of the secondary $X-$ray?
    c. What must be the x and y components of the electron after collision?
    d. Give the magnitude and direction of the electrons' final velocity.
19. Curium 242 has an isotopic mass of 242.058831 amu and decays by alpha emission; the alpha particle has a mass of 4.002602 amu and has a kinetic energy of 6.1127 Mev.
    a. What is the momentum of the alpha particle?
    b. What is its wavelength?
    c. Write a balanced nuclear equation for the reaction.
    d. Calculate the isotopic mass of the product.
    e. If the alpha particle is placed in a magnetic field of .002 T what is the radius of curvature? (The alpha particle has a double positive charge.)
    f. If the alpha particle is moving in the $x-$direction and the field is in the $z-$direction find the direction of the magnetic force.
    g. Calculate the magnitude and direction of the electric field necessary to make the alpha particle move in a straight line.
20. A student lab group has a laser of unknown wavelength, a laser of known wavelength, a photoelectric cell of unknown work function, a voltmeter and test leads, and access to a supply of resistors.
    a. Design an experiment to measure the work function of the cell, and the wavelength of the unknown laser. Give a complete procedure and draw an appropriate circuit diagram. Give sample equations and graphs if necessary.
    b. Under what circumstances would it be impossible to measure the wavelength of the unknown laser?
    c. How could one using this apparatus also measure the intensity of the laser (number of photons emitted/second)?

21. The momentum of an electron is measured to an accuracy of $\pm 5.1 \times 10^{-24}$ kg $\cdot$ m/s. What is the corresponding uncertainty in the position of the same electron at the same moment? Express your answer in Angstroms ($1 = 10^{-10}$ m, about the size of a typical atom).
22. Thor, a baseball player, passes on a pitch clocked at a speed of 45 ± 2 m/s. The umpire calls a strike, but Thor claims that the uncertainty in the position of the baseball was so high that Heisenberg's uncertainty principle dictates the ball *could* have been out of the strike zone. What is the uncertainty in position for this baseball? A typical baseball has a mass of 0.15 kg. Should the umpire rethink his decision?
23. Consider a box of empty space (vacuum) that contains nothing, and has total energy $E = 0$. Suddenly, in seeming violation of the law of conservation of energy, an electron and a positron (the anti-particle of the electron) burst into existence. Both the electron and positron have the same mass, 9.11×10^{-31} kg.

    a. Use Einstein's formula ($E = mc^2$) to determine how much energy must be used to create these two particles out of *nothing*.
    b. You don't get to violate the law of conservation of energy forever – you can only do so as long as the violation is "hidden" within the HUP. Use the HUP to calculate how long (in seconds) the two particles can exist before they wink out of existence.
    c. Now let's assume they are both traveling at a speed of 0.1 c. (Do a non-relativistic calculation.) How far can they travel in that time? How does this distance compare to the size of an atom?
    d. What if, instead of an electron and a positron pair, you got a proton/anti-proton pair? The mass of a proton is about 2000× higher than the mass of an electron. Will your proton/anti-proton pair last a *longer* or *shorter* amount of time than the electron/positron pair? Why?

Answers to Selected Problems

1.
    1. $6.752 \times 10^{-26} J, 2.253 \times 10^{-34}$ kgm/s
    2. $5.96 \times 10^{-20} J, 1.99 \times 10^{-28}$ kgm/s
    3. $4.90 \times 10^{-28} J, 1.63 \times 10^{-36}$ kgm/s

    1. 1.94 eV, 1.04×10^{-27} kgm/s
    2. 12.7 eV, 6.76×10^{-27} kgm/s
    3. 5.00 eV, 2.67×10^{-21} kgm/s

    1. .0827 nm
    2. 4.59×10^{-4} nm
    3. 730 nm

2. 1.03×10^{-20} m

    1. 36 nm
    2. no
    3. 380 nm, 73 nm, 74 nm, 36 nm, 92 nm, 39 nm

3. .80 V
4. 480 nm

    1. .124 nm
    2. .00120 nm

5. 24,600 m/s
6. 1.84×10^{8} m/s

    1. .491 m/s
    2. 3.1410^{7} J
    3. 64 Mw

    4. 1.55 pm

7. 3.27 eV
8. .
9. (b) 15 (c) 182 nm, 188 nm, 206 nm, 230 nm
10. -10.3 eV, -3.82 eV, -2.29 eV, -1.83 eV

    1. 4.19×10^7 m/s
    2. 1.70×10^{-11} m
    3. 1.95°
    4. .068 m

    1. 1.89 V
    2. 1.60 A
    3. 1.25 Ω

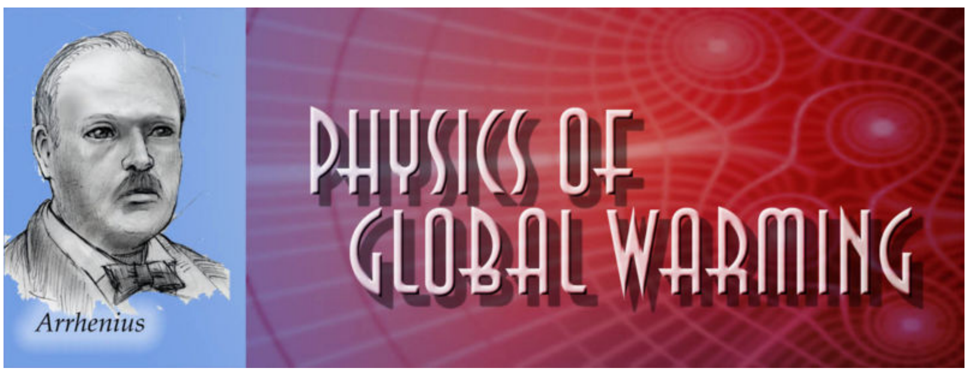
    1. 4.40×10^{-24} kgm/s
    2. 1.17×10^{-24} kgm/s
    3. 3.23×10^{-24} kgm/s
    4. 3.76×10^7 m/s

    1. 1.1365×10^{-22} kgm/s
    2. 5.860 pm
    3. ^{242} Cu $\rightarrow^4$ He $+^2$ 38Pu
    4. 238.0497 amu
    5. 17.7 cm
    6. $-y$
    7. $+y$, 34.2 N/C

11. .
12. 0.10 Angstrom
13. 1.76×10^{-34} eV

CHAPTER 28 The Physics of Global Warming Version 2

Chapter Outline



28.1 The Big Idea

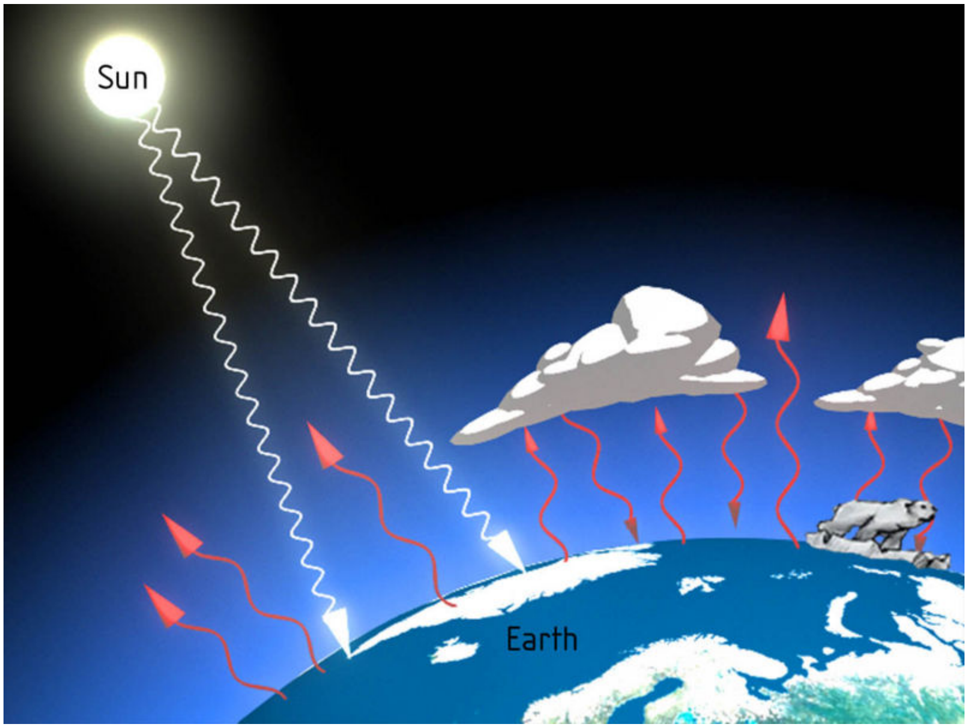
The observed global warming on Earth is a manifestation of the Second Law of Thermodynamics. The Earth operates like any heat engine. Input heat from solar radiation and exhaust heat (terrestrial radiation) largely determine the operating temperature (global surface temperature). Over geological periods this heat exchange reaches equilibrium and the temperature is stable. If the input heat increases or the exhaust heat decreases the temperature rises and vice versa. Natural processes over geologic time have changed the input and affected both output heat and temperature. In the present era the quantity of exhaust heat is being rapidly restricted by the greenhouse effect; consequently, the earth's temperature must rise to reach equilibrium. How much higher it must rise depends entirely on human activity.

The input heat – solar energy received – is a function of solar activity and oscillations in characteristics of the Earth's orbit.

The quantity of exhaust heat, terrestrial radiation, is largely a function of the presence of certain gases in the atmosphere that absorb outgoing infrared radiation. This is known as the greenhouse effect. The greenhouse effect is due to the differential absorption of certain wavelengths of solar as compared to terrestrial radiation.

The solar energy reaching the surface of the Earth is concentrated in short wavelengths, which can easily penetrate the greenhouse gases, such as Carbon Dioxide and Methane. The Earth, however, is cooler than the sun and it radiates its heat in the form of energy in the far infrared range. These longer wavelengths are partially absorbed by the greenhouse gases and some of the solar heat is returned to Earth. At a certain temperature these processes are in equilibrium and the surface temperature of the Earth is stable. However, if more greenhouse gases are put in the atmosphere the amount of trapped terrestrial radiation increases, leading to an increase in global temperature.

Currently the heating effect of extra greenhouse gases (since the start of the industrial revolution) is equal to about $1.0\ \text{W}/\text{m}^2$. Thus the recent period has recorded parallel increases in concentration of carbon dioxide and average global temperature. As more greenhouse gases are put into the atmosphere the temperature will increase further. There are certain effects of a warmer Earth (discussed below) which could accelerate the process, even if no more greenhouse gases are put into the atmosphere (an unlikely prospect for the foreseeable future).

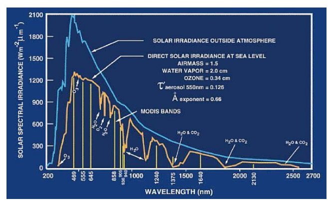

Sun
Earth

28.2 The Key Concepts (Possible Effects That Can Accelerate Global Warming)

1. **Time Lag:** The excess energy warms the ocean very slowly, due to water's high heat capacity. Even in the unlikely event that no more greenhouse gases are added to the atmosphere the temperature increase already measured will be nearly doubled.
2. **The Effect of Water Vapor:** Increasing temperatures will lead to more evaporation and more water vapor in the atmosphere. Water vapor is a greenhouse gas and its increased presence may cause further warming in a positive feedback loop. On the other hand if the water vapor results in more clouds more solar radiation will be reflected, a possible negative feedback.
3. **Albedo** is the amount of light reflected by a surface. Sea ice has an albedo of .85, meaning 85% of light is reflected back from its surface (and leaves the Earth) and 15% is absorbed and stays in the Earth; ice-free water has an albedo of .07.(93% of the solar energy is absorbed.) Thus the observed melting of sea ice could amplify the effect of global warming
4. The **melting of the Arctic Permafrost** also has an amplifying effect by releasing carbon dioxide and methane that is normally trapped in the tundra.
5. Warmer oceans are hostile to **algae and cytoplankton** , which are the most important absorbers of carbon dioxide. The loss of the these two photosynthesizers would remove the most important natural CO_2 sink.
6. **Loss of Rainforests** would have a similar effect. Global warming is likely to lead to desertification of the habitats of rainforests. The rainforest is the second most important CO_2 sink.

28.3 The Key Concepts (Physical Laws and Observations)

1. The relationship between temperature of a body and its radiation wavelength is given by **Wien's Law**: For any body that radiates energy, the wavelength of maximum energy radiated is inversely related to the temperature.
2. The effect of global warming on the solubility of Carbon Dioxide (CO_2) and methane (CH_4) is governed by two laws that have opposing effects. **Henry's Law**: The solubility of a gas is directly proportional to the partial pressure of that gas. The constant of proportionality is Henry's Law Constant. This constant of proportionality is temperature dependent and decreases as temperature increases. Therefore as carbon dioxide increases in the atmosphere the partial pressure of CO_2 increases and more of it tends to dissolve in the oceans, but as the temperature increases the constant decreases and less of it tends to dissolve. The net effect at a given temperature will have to be calculated.



3. The **Solar Radiation** peaks at 610 nm; there is 61.2% of solar radiation is in the visible band (400 – 750 nm) with less than 9% in the uv band and about 30 % in the near infra red. Some 99% is radiated between 275 and 5000 nm. This band largely is unabsorbed by any atmospheric gases. The most significant of the greenhouse gases are H_2O and CO_2. The plot above details the absorbance of various wavelengths of radiation by atmospheric gases in the shortwave region.
4. The **Earth's radiation** peaks at 11,000 nm, with an intensity of .04 W/cm^2 . Some 99% is radiated between 40,000 nm and 3000 nm in the longer infrared regions. This band is unabsorbed by nitrogen, oxygen and argon (99%) of the Earth's current atmosphere), but partially absorbed by carbon dioxide, methane, water vapor, nitrous oxide and some minor gases. The gases that absorb this band of radiation are called *greenhouse gases.*
5. **Earth Orbital Changes:** There are three principal variations in orbit that are collectively known as the Milankovitch Cycles. Atmospheric concentrations of methane closely followed this cycle historically and on a larger time frame so have concentrations of CO_2.
   a. precession of the rotational axis (period: 23,000 years)
   b. variation in tilt of rotational axis from 21.5° to 24.5° (period: 41,000 years)
   c. eccentricity of the elliptical orbit (period: 100,000 years)
6. **Departures from the historical cyclical trend** began 8000 years ago with the development of agriculture. This led to a temperature rise of 0.8 °C above expected trends and concentrations of CO_2 rising 30 ppm above expected trends with the concentration of methane 450 ppb above natural trends. In the last 100 years of industrialization these departures from normal have accelerated with temperature rising an additional 0.8 °C and CO_2 concentrations rising to 370 ppm, which is 90 ppm higher than the recorded CO_2 concentrations at the warmest points in the interglacial periods. Methane concentrations are at 1750 ppb, 1000 ppb above historical highs. Over 70% of the extra greenhouse gases were added after 1950. CO_2 is emitted whenever anything is burned, from wood to coal to gasoline. Methane is produced by animal husbandry, agriculture, and by incomplete combustion or leakage of natural gas. As more greenhouse gases are put into the atmosphere the temperature will increase further. The co-variation of CO_2 concentrations and temperature has been demonstrated not only by recent observation, but by records of the last 700,000 years from Antarctic ice cores. There are many possible effects and feedback mechanisms that are currently being studied and modeled

to better predict possible outcomes of this global trend. Many of these are identified above and in the following sections.

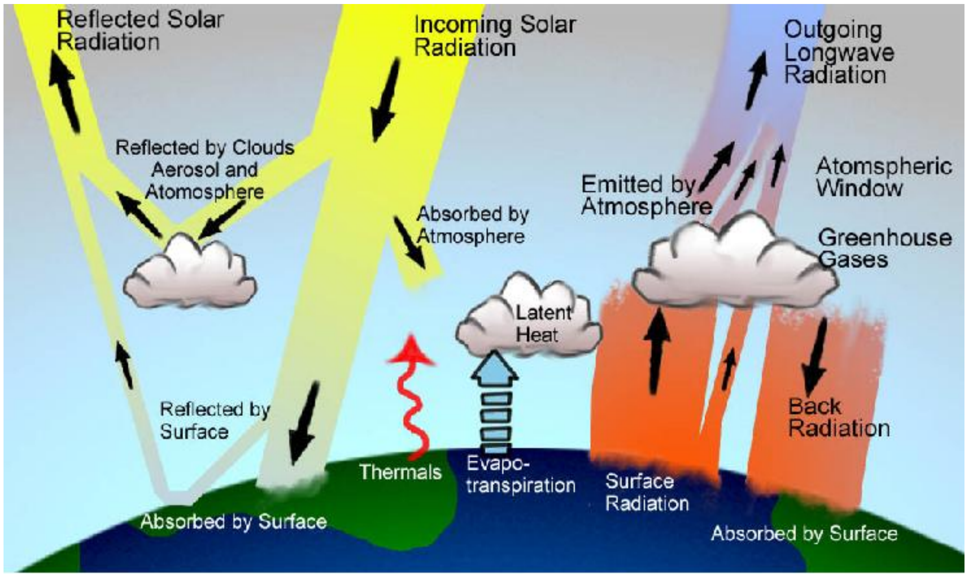
28.4 The Key Applications

1. Changing quantity of CO_2 in oceans will lead to a **change in pH of the oceans**, changing its suitability as a habitat for some species of oceanic life.
2. **Human health problems** are associated with warmer temperatures including a projected 10-fold rise in mosquito populations and the diseases they bring as well as the already documented spread of malaria and dengue fever into areas in which these diseases were hitherto unknown.
3. **Loss of water supply**: A large part of human and other animal water supply is supplied from glaciers or melting snow-packs. This dependable supply will be disrupted or curtailed for many people. Especially vulnerable are Southeast Asia and India, which depend on the Himalayas, and much of South America, which depends on the Andes. In the US, California and the West stand to have a curtailed water supply in the summer months as a result of global warming.
4. **Weather changes:**
   a. Global Warming seems to cause the North Atlantic Oscillation to become stuck in the positive mode. The effect is to have warmer weather in Alaska, Siberia and western Canada, but colder weather in eastern Canada, Europe, and northeast US.
   b. The same effect likely will lead to dry windy conditions in Europe and North America and dry conditions in much of Africa.
   c. Models show global warming leading to droughts in most of the northern hemisphere, particularly in the grain belts of North America, Europe, and Asia.
   d. At the same time, there is predicted to be increased rain overall, but coming in the form of severe storms and consequent flooding.
   e. The conditions that lead to hurricanes and tornadoes are powered by solar energy. More solar energy in the ocean may lead to more severe hurricanes. There is some evidence to support that this has already occurred. The combination of warm Gulf waters and windy plains cause tornadoes. Both of these conditions will be increased by global warming.
5. Melting of the land glaciers will lead to **rising sea levels.** The Greenland ice sheet is moving into irreversible melting, which together with the loss of other land ice raise the ocean levels 8 meters in a century. Thermal expansion of water would add several tens of centimeters to this rising sea level.
6. **Ecosystems under stress:** When temperature changes occur over thousands of years, plants and animals adapt and evolve. When they happen over decades, adaptation is not always possible. The first flowering days of 385 plant species were on average 4.5 days earlier in 1991-2000 than normal. This can lead to lack of pollination and loss of fruiting. A study in the Netherlands showed that weather changes caused oak buds to leaf sooner, causing winter moth caterpillars to peak in biomass earlier. The birds that depend on the caterpillars to feed their chicks did not delay their egg laying. This led to a mismatch of 13 days between food availability and food needs for these birds.

The Key Equations

1. Wien's Law: $T\lambda_{max} = A$; where $A = 2.8978\text{ m} - \text{K}$
2. Henry's Law: $C = kP_{partial}$, where k is temperature dependent and gas dependent; $CO_2@20^\circ = 3.91 \times 10^{-3}$ molal/atm, $CO_2@25^\circ = 3.12 \times 10^{-2}$ molal/atm; $CH_4@20^\circ = 1.52 \times 10^{-3}$ molal/atm. The concentration is given in molals (Molal is moles of solute/kg of solvent) The partial pressure is given in atmospheres.
3. Energy imbalance of 12 watt/m^2-year leads to deglaciation that raises sea levels 1 meter.

4. Climate Sensitivity: Energy imbalance of 1 W/m^2 $\rightarrow .75°C \pm .25°$ C change in average global temperature
5. Present Energy Imbalance = about 1 W/m^2($\pm$.5 W/m^2)



6. The picture above shows the normal energy balance of the Earth. Note that normally the 342 W/m^2 incoming is balanced by 235 W/m^2 outgoing +107 W/m^2 reflected radiation. At present, the atmospheric window allows only 39 W/m^2 out resulting in a total of 234 W/m^2 outgoing and an energy surplus of 1 W/m^2 that results in temperature increases. (These figures are $\pm$.5 W/m^2).
7. 1 kwh = .68 kg CO_2 (EPA estimates)
8. 10,000 kWh = 1.4 cars off the road = 2.9 acres of trees planted (EPA estimates)

28.5 Problem Set Chapter 26

1. One W/m^2 energy imbalance may not seem much. (In the following calculations assume for the sake of significant digits that this is an exact number. It is in fact ± 0.5 W/m^2)
   a. Calculate the total watts received by Earth. Surface area of a sphere is $4\pi r^2$.
   b. Convert to energy in kWh.
   c. How many joules of extra energy are received by Earth in a year?
   d. To estimate the contrasting energy of an atomic bomb, assume 100 kg of U^{235}, isotopic mass of 235.043924, is split into Xe^{142}>, isotopic mass of 141.929630, Sr^{90}, isotopic mass of 89.907738 and 3 neutrons, each with mass of 1.008665. All masses are given in amu's. First, find the mass difference between reactant and products. Then, converting to kilograms and using $E = \Delta mc^2$, find the energy in joules of an atomic bomb.
   e. How many atomic bombs would have to be set off to equal the extra energy the Earth receives in one year from global warming?

2. It is estimated that a 12 W/m^2 energy imbalance leads to sufficient melting of land ice to cause the sea levels to rise one meter.
   a. How many joules is that?
   b. What mass of ice is melted? The heat of fusion of water is 3.33×10^5 J/kg.
   c. What volume of water is that? ($\rho = 1000$ kg/m^3)
   d. From the above result, you should be able to estimate the surface area of the world's oceans and check the given estimate.
3. Given the uncertainty of ± 0.5 W/m^2, give the high and low estimates of global sea level rise in a century. Draw two new world maps using this data. Draw maps of your state, if it is a coastal state, 100 years from now given these estimates. (Perhaps your inland state will become a coastal state.)
4. Given the following table involving the growth in concentration of greenhouse gases:

TABLE 28.1:

year	$[CO_2]$ ppm	$[CH_4]$ ppb
1940	310	1100
1960	315	1250
1980	335	1550
2000	370	1750
2020 (IPCC* projection)	420	2150

* *Intergovernmental Panel on Climate Change*

(a) Graph this data with time on the horizontal axis

(b) Determine the rate of increase in the concentrations of the two gases

i. 1940 - 2000

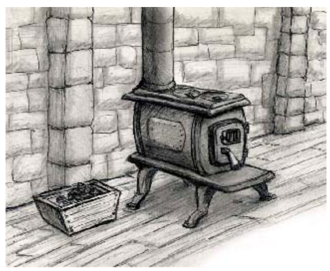
ii. 1960 – 2000

iii. 1980 – 2000

iv. the instantaneous rates of change in 2000

v.the instantaneous rates of change projected for 2020

5. Climate forgings can come from a variety of sources besides methane and carbon dioxide. Determine whether the following are positive feedbacks (contribute to global warming) or negative. You may have to do some research on this.
    a. Black Carbon Soot
    b. Reflective Aerosols
    c. Chlorofluorocarbons
    d. Nitrous Oxide
    e. Ozone
    f. Cloud Droplet Changes



6. An overlooked area of additional global warming is the traditional cook stove. In one Honduran study, the soot smoke produced from one stove absorbed 65% of terrestrial radiation that then went into warming the atmosphere. There are 400 million such cook stoves worldwide, each of which emit 1.5 g of soot per kilogram of wood burned. The average daily use of wood is 7.5 kg per stove. Calculate the mass of soot released through cook stoves per day, per year.

For Problems 7 - 10 use the following tables:

TABLE 28.2: Electricity Emission Rates: (EPA)

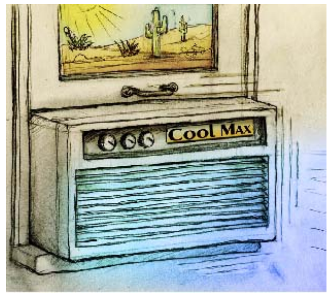
State or region	CO_2 in kg/Mwh	CH_4 in kg/Mwh	N_2O in kg/Mwh
California	364.8	.00304	.00168
Michigan	740.1	.00662	.0133
New York City	494.3	.00367	.00404
Oregon	304.3	.00149	.00154

TABLE 28.3: Global Warming Potential of Gases Compared to Carbon Dioxide (IPCC):

Greenhouse gas	Multiplier
Carbon dioxide CO_2	1
Methane CH_4	23
Nitrous OxideN_2O	296
A/C refrigerant HFC – 143a	4300
Auto A/C refriger HFC – 134a	1300

TABLE 28.3: (continued)

Greenhouse gas	Multiplier
SF_6	22,000
$C_2 F_6$	11,900



7. A typical household air conditioner draws about 20a from a 240 v line.
   a. If used for 8 hours how many kwh does it use?
   b. In the course of a 120 day summer how many Mwh is that?
   c. Calculate the mass of carbon dioxide one summer's use of ac contributes. (Pick a state or region from above.)
   d. Calculate the mass of methane and N_2O emitted.
   e. Using the global warming multipliers for the latter two gases calculate the global warming potential in equivalent kg of CO_2 for all 3 gases.
8. If you "shut down" your computer, but the LED light is still on, it consumes about 4 w of power. Suppose you do that for every weekend (60 hours) every week of the year. Repeat the calculations in problem 7 to find out the global warming potential in kg of CO_2.
9. In 2006 Natomas High School in California used 1692 Mwh of electricity. Repeating the calculations above, find the kg of carbon dioxide emitted.
10. A large car or SUV typically carries 1.0 kg of refrigerant for the a/c.
    a. If this were released into the atmosphere calculate the equivalent of carbon dioxide released.
    b. Repeat this calculation for a residential air conditioner (capacity is 2.8 kg.), using HFC – 143a.
    c. Your school has a commercial chiller maybe (1000 ton) with a refrigerant capacity of 1225 kg. If it uses HFC – 134a calculate the equivalent of CO_2 emitted, if the chiller is decommissioned.

TABLE 28.4: Emissions of Carbon Dioxide for Different Fuels

Fuel	Kg of carbon dioxide emitted/gallon
Gasoline	8.78
California reformulated gasoline, 5.7% ethanol	8.55
Ethanol	6.10
Diesel #2	10.05
biodiesel	9.52
Jet fuel	9.47
propane	5.67
Natural gas/gasoline gallon equivalent	6.86

11. Compare the carbon "footprint" of the following:
    a. a hybrid car (45 mpg) that drives 21,000 mile per year in Calif.
    b. an SUV (17 mpg) that drives 21,000 miles per year also in Calif.

c. a mid-size car (24 mpg) that uses ethanol and drives 21,000 miles per year
d. a commercial flatbed (11 mpg) that drives 21,000 miles per year and uses bio diesel

12. Research some typical mileages, type of fuel used, and miles covered in a year and determine the carbon footprint for:
    a. a tractor-trailer truck
    b. a commercial airliner
    c. a corporate jet
    d. a bus
    e. Amtrak
13. Looking at the above problems another way, suppose you want to travel from California to New York find your carbon footprint for the trip using:
    a. Amtrak
    b. a jet plane
    c. a bus
    d. an SUV
    e. a hybrid

    Assume 90% full loads on the commercial transports and 2 passengers on the cars. You will have to go on-line to find the loads of the commercial transports.
14. China is putting two coal-fired electrical plants in operation each week. These plants do not typically use any scrubbing or pollution controls. Research the typical Mwh output, and, using either the table for problem 7 (Michigan depends more on coal than the other states listed.) or a more direct source for CO_2 emissions for a coal plant, find the gain in greenhouse gas emissions each year from this source alone. Compare to the results in problem 4 and determine if the IPCC is underestimating the problem.

Answers to Selected Problems

1. (a)
2. 5.1×10^{14} W
3. (b)
4. 1.8×10^{15} kWh
5. (c)
6. 1.6×10^{22} J
7. (d)
8. 7.0×10^{15} J
9. (e) About 2.3 million bombs

CHAPTER 29 Answers to Selected Problems Version 2

Chapter Outline

29.1 Appendix A: Answers to Selected Problems (3e)

Ch 1: Units and Problem Solving

1.
    1. A person of height 5 ft. 11 in. is 1.80 m tall
    2. The same person is 180 cm

    1. 3 seconds $= 1/1200$ hours
    2. $3x10^3$ ms
2. 87.5 mi/hr
3. (c) if the person weighs 150 lb. this is equivalent to 668 N
4. Pascals (Pa), which equals N/m^2
5. 168 lb., 76.2 kg
6. 5 mi/hr/s
7. 15.13 m
8. 11.85 m
9. 89,300 mm
10. (f) 2025 mm^2
11. (b) 196 cm^2
12. (c) 250 cm^3
13. 8 : 1, each side goes up by 2 cm, so it will change by 2^3
14. $3.5 \times 10^{51} : 1$
15. 72,000 km/h
16. 0.75 kg/s
17. 8×2^N cm^3/ sec; N is for each second starting with 0 seconds for 8 cm^3
18. About 12 million
19. About $1\frac{1}{2}$ trillion (1.5×10^{12})
20. $[a] = \text{N}/\text{kg} = \text{m}/\text{s}^2$

Ch 2: Energy Conservation

1. d
2. (discuss in class)

    1. 5.0×10^5 J
    2. 3.7×10^5 J
    3. Chemical bonds in the food.
    4. 99 m/s

    1. 5.0×10^5 J
    2. 108 m/s

    1. 450,000 J
    2. 22,500 J

   3. 5,625 J
   4. 21.2 m/s
   5. 9.18 m

3. .
4. (b) $KE = 504,600$ J; $U_g = 1,058,400$ J; $E_{total} = 1,563,000$ J
   1. 34 m/s at B; 28 m/s at D, 40 m/s at E, 49 m/s at C and F; 0 m/s at H
   2. 96 m

   1. 1.7 J
   2. 1.3 m/s
   3. 0.4 J, 0.63 m/s

   1. 1.2 m/s^2
   2. 130 J

   1. 6750 J
   2. 2.25×10^5 J
   3. 1.5×10^5 J/gallon of gas

5. 0.76 m

Ch 3: One-Dimensional Motion

1. .
2. .
3. .
4. .
   1. Zyan
   2. Ashaan is accelerating because the distance he travels every 0.1 seconds is increasing, so the speed must be increasing
   3. Ashaan
   4. Zyan
   5. Ashaan

5. .
6. .
7. 6 minutes
8. (d) 20 meters (e) 40 meters (f) 2.67 m/s (g) 6 m/s (h) Between $t = 15$ s and $t = 20$ sec because your position goes from $x = 30$ m to $x = 20$m. (i) You made some sort of turn
   1. 7.7 m/s^2
   2. 47 m, 150 feet
   3. 34 m/s

   1. 1.22 m
   2. 4.9 m/s
   3. 2.46 m/s
   4. -4.9 m/s

9. (b) 1 second (c) at 2 seconds (d) 4m
   1. 250 m
   2. 13 m/s, -13 m/s

   3. 14 s for round trip

10. Let's say we can jump 20 feet (6.1 m) in the air. ? Then, on the moon, we can jump 36.5 m straight up.
11. $-31 \mathrm{m/s^2}$

   1. 23 m/s
   2. 3.6 seconds
   3. 28 m
   4. 45m

   1. 25 m/s
   2. 30 m
   3. 2.5 $\mathrm{m/s^2}$

12. 2 $\mathrm{m/s^2}$

   1. $v_0 = 0$
   2. 10 $\mathrm{m/s^2}$
   3. $-10 \mathrm{m/s^2}$
   4. 60 m

   1. 0.3 $\mathrm{m/s^2}$
   2. 0.5 m/s

Ch 4: Two-Dimensional and Projectile Motion

1. .
2. .
3. .
4. .
5. .
6. .

   1. 13 m
   2. 41 degrees
   3. $v_y = 26 \mathrm{m/s}; v_x = 45 \mathrm{m/s}$
   4. 56 degrees, 14 m/s

7. .
8. 32 m

   1. 0.5 s
   2. 0.8 m/s

9. 104 m
10. $t = 0.60$ s, 1.8 m below target
11. 28 m.

   1. 3.5 s.
   2. 35 m; 15 m

12. 40 m; 8.5 m
13. 1.3 seconds, 7.1 meters
14. 50 m; $v_{0y} = 30 \mathrm{m/s}; 50^0$; on the way up
15. 4.4 s
16. 19°
17. 0.5 s

18. 2.3 m/s
19. 6 m
20. 1.4 seconds
    1. yes
    2. 14 m/s @ 23 degrees from horizontal
21. 22 m/s @ 62 degrees

Ch 5: Newton's Laws

1. .
2. .
3. .
4. Zero; weight of the hammer minus the air resistance.
5. 2 forces
6. 1 force
7. No
8. The towel's inertia resists the acceleration
    1. Same distance
    2. You go farther
    3. Same amount of force
9. .
    1. 98 N
    2. 98 N
10. .
11. 32 N
12. 5.7 m/s^2
13. .
14. .
15. $F_x = 14$ N, $F_y = 20$ N
16. Left picture: $F = 23$N 98°, right picture: $F = 54$ N 5°
17. 3 m/s^2 east
18. 4 m/s^2; 22.5° NE
19. 0.51
20. 0.2
21. The rope will not break because his weight of 784 N is distributed between the two ropes.
22. Yes, because his weight of 784 N is greater than what the rope can hold.
23. Mass is 51 kg and weight is 82 N
    1. While accelerating down
    2. 686 N
    3. 826 N

    1. 390 N
    2. 490 N
24. 0.33
25. 3.6 kg
26. $g \sin\theta$
27. (b) 20 N (c) 4.9 N (d) 1.63 kg (e) Eraser would slip down the wall

1. 1450 N
2. 5600 N
3. 5700 N
4. Friction between the tires and the ground
5. Fuel, engine, or equal and opposite reaction

28. (b) 210 N (c) no, the box is flat so the normal force doesn't change (d) $2.8\ \mathrm{m/s^2}$ (e) $28\ \mathrm{m/s}$ (f) no (g) 69 N (h) 57 N (i) 40 N (j) 0.33 (k) 0.09
29. .

1. zero
2. $-kx0$

30. (b) $f_1 = \mu_k m_1 g\cos\theta; f_2 = \mu_k m_2 g\cos\theta$ (c) Ma (d) $T_A = (m_1+m_2)(a+\mu\cos\theta)$ and $T_B = m_2 a + \mu m_2\cos\theta$ (e) Solve by using $d = 1/2at^2$ and substituting h for d

1. Yes, because it is static and you know the angle and m_1
2. Yes, T_A and the angle gives you m_1 and the angle and T_C gives you $m_2, m_1 = T_A\cos 25/g$ and $m_2 = T_C\cos 30/g$

31. (a) 3 seconds d. 90 m
32. .
33. .
34. .

1. $1.5\ \mathrm{N}; 2.1\ \mathrm{N}; 0.71$

Ch 6: Centripetal Forces

1. .
2. .
3. .
4. .

1. 100 N
2. $10\ \mathrm{m/s^2}$

1. 25 N towards her
2. 25 N towards you

1. $14.2\ \mathrm{m/s^2}$
2. 7.1×10^3 N
3. friction between the tires and the road

5. .0034g

1. $6.2\times10^5\ \mathrm{m/s^2}$
2. The same as a.

6. 3.56×10^{22}N
7. 4.2×10^{-7} N; very small force
8. $g = 9.8\ \mathrm{m/s^2}$; you'll get close to this number but not exactly due to some other small effects
9. (a) 4×10^{26} N (b) gravity (c) 2×10^{41} kg
10. $.006\ \mathrm{m/s^2}$

1. .765
2. 4880 N

  1. $\sim 10^{-8}$ N very small force
  2. Your pencil does not accelerate toward you because the frictional force on your pencil is much greater than this force.

11. (a) 4.23×10^{7}m (b) 6.6 R_e (d) The same, the radius is independent of mass
12. 1.9×10^{7}m
13. You get two answers for r, one is outside of the two stars one is between them, that's the one you want, 1.32×10^{10}m from the larger star.
14. .
15. .

  1. $v = 28$ m/s
  2. $v-$down, $a-$right
  3. $f-$right
  4. Yes, 640N

Ch 7: Momentum Conservation

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. 37.5 m/s
9. $v_1 = 2v_2$

  1. $24 \frac{kg-m}{5}$
  2. 0.364 m/s
  3. $22 \frac{kg-m}{5}$
  4. 109 N
  5. 109 N due to Newton's third law

10. 2.0 kg, 125 m/s
11. 21 m/s to the left
12. 3250 N

  1. 90 sec
  2. 1.7×10^{5} sec

  1. 60 m/s
  2. .700 sec
  3. yes, 8.16 m

13. 0.13 m/s to the left

  1. 11000 N to the left
  2. tree experienced same average force of 11000 N but to the right
  3. 2500 lb.
  4. about 2.5 "g"s of acceleration

  1. no change
  2. the last two cars

14. (a) 0.00912 s

    1. 0.0058 m/s^2
    2. 3.5 m/s^2

    1. 15 m/s
    2. 49° S of E

15. (b) $4.6 \text{ m/s } 68^\circ$

Ch 8: Energy & Force

1. .
2. .
3. .
4. .
5. .

    1. 7.18×10^9 J
    2. 204 m/s

    1. $34 \text{ m/s} @ B; 28 \text{ m/s} @ D; 40 \text{ m/s} @ E; 49 \text{ m/s} @ C$ and F; $0 \text{ m/s} @ H$
    2. 30 m
    3. Yes, it makes the loop

6. (a) 2.3 m/s (c) No, the baby will not clear the hill.
7. (a) $29,500$ J (b) 7.9 m
8. .

    1. 86 m
    2. 220 m

    1. 48.5 m/s
    2. 128 N

9. 0.32 m/s each

    1. 10 m/s
    2. 52 m

    1. $1.1 \times 10^4 \text{ N/m}$
    2. 2 m above the spring

10. 96%
11. .

    1. .008 m
    2. 5.12°

12. 8 m/s same direction as the cue ball and 0 m/s
13. $v_{golf} = -24.5 \text{ m/s}; \text{vpool} = 17.6 \text{ m/s}$
14. 2.8 m

    1. 0.57 m/s
    2. Leonora's
    3. 617 J

    1. 19.8 m/s

2. 8.8 m/s
3. 39.5 m

1. 89 kW
2. 0.4
3. 15.1 m/s

15. 43.8 m/s
16. .
17. .
18. .

1. 3.15×10^5 J
2. 18.0 m/s
3. 2.41 m
4. 7900 J

1. $v_0/14$
2. $m{v_0}^2/8$
3. $7m{v_0}^2/392$
4. 71%

Ch 9: Rotational Motion

1. .

1. 9.74×10^{37} kg m2
2. 1.33×10^{47} kg m^2
3. 0.5 kg m^2
4. 0.28 kg m^2
5. 0.07 kg m^2

1. True, all rotate 2π for 86,400;sec which is 24 hours,
2. True, $\omega = 2\pi/\text{t}$ and $t = 86,400$ s
3. True, L is the same
4. $L = I\omega$ and $I = 2/5\ \text{mr}^2$
5. True, $K = \frac{1}{2}I\omega^2\ \ I = 2/5\ \text{mr}^2\ \text{sub} - \text{in}\ K = 1/5\ \text{mr}^2\omega^2$
6. True, $K = \frac{1}{2}I\omega^2\ \ I =\ \text{mr}^2\ \text{sub} - \text{in}\ K = \frac{1}{2}mr^2\omega^2$

1. 250 rad
2. 40 rad
3. 25 rad/s
4. Force applied perpendicular to radius allows α
5. 0.27 kg m^2,
6. $K^5 = 84$ J and $K^10 = 340$ J

2. .
3. Moment of inertia at the end $1/3\ \text{ML}^2$ at the center $1/12\ \text{ML}^2$, angular momentum, $L = I\omega$ and torque, $\tau = I\alpha$ change the in the same way
4. .
5. Lower
6. Iron ball

1. 200 N team

2. 40 N
3. 0.02 rad/s^2
4. 25 s

1. Coin with the hole
2. Coin with the hole

1. weight
2. 19.6 N
3. plank's length (0.8m) left of the pivot
4. 15.7 N m,
5. Ba. weight, Bb. 14.7 N, Bc. plank's length (0.3m) left of the pivot, Bd. 4.4 N m, Ca. weight, Cb. 13.6 N, Cc. plank's length (1.00 m) right of the pivot, Cd. 13.6 N m, f) 6.5 N m CC, g) no, net torque doesn't equal zero

1. 7.27×10^{-6} Hz
2. 7.27 Hz

1. 100 Hz
2. 1.25×10^5 J
3. $2500\ \text{J} - \text{s}$
4. $12,500\ \text{m} - \text{N}$

7. 28 rev/sec
8. 2300 N
9. (b) 771 N, 1030 N (c) 554 kgm^2 (d) 4.81rad/sec^2

1. 300 N
2. $240N, -22$ N
3. .092

1. 2280 N
2. 856 n toward beam, 106 N down
3. 425 kgm^2
4. 3.39 rad/sec^2

1. -1.28 Nm
2. CCW

10. (a) 1411 kg (c) 17410 N (d) angular acc goes down as arm moves to vertical

Ch 10: Simple Harmonic Motion

1. 1. Buoyant force and gravity
   2. $T = 6$ s, $f = 1/6$ Hz

   1. 9.8×10^5 N/m
   2. 0.5 mm
   3. 22 Hz, no,

2. 3.2×10^3 N/m
3. (a) 110 N/m (d) $v(t) = (25)\cos(83\text{t})$
4. .
5. .

1. 0.0038 s
2. 0.0038 s

6. .
7. .
8. 4 times
9. 0.04 m

1. 16 Hz
2. 16 complete cycles but 32 times up and down, 315 complete cycles but 630 times up and down
3. 0.063 s

1. $24.8\text{ J}, 165\text{ N}, 413\text{ m/s}^2$
2. $11.1\text{m/s}, 0, 0$
3. $6.2\text{ J}, 18.6\text{ J}, 9.49\text{ m/s}, 82.5\text{ N}, 206\text{ m/s}^2$
4. $.169\text{ sec}, 5.9\text{ Hz}$

10. (b) .245 J (c) 1.40m/s (d) 1.00 m/s (f) 2.82 N (g) 3.10 N

Ch 11: Wave Motion and Sound

1. 390 Hz

1. 4 Hz
2. It was being driven near its resonant frequency.
3. 8 Hz, 12 Hz
4. (Note that earthquakes rarely shake at more than 6 Hz).

2. .
3. .

1. 7 nodes including the 2 at the ends
2. 3.6 Hz

4. 1.7 km

1. 1.7 cm
2. 17 m

1. 4.3×10^{14} Hz
2. 2.3×10^{-15} s– man that electron is moving fast

1. 2.828 m
2. 3.352 m
3. $L = 1/4\,\lambda$ so it would be difficult to receive the longer wavelengths.

5. Very low frequency
6. (b) Same as closed at both ends
7. .
8. 1.9 Hz or 2.1 Hz.
9. 0.53 m
10. 2.2 m, 36 Hz; 1.1 m, 73 Hz; 0.733 m, 110 Hz; 0.55 m, 146 Hz
11. 430 Hz; 1.3×10^3 Hz; 2.1×10^3 Hz; 3.0×10^3 Hz;

1. The tube closed at one end will have a longer fundamental wavelength and a lower frequency.
2. If the temperature increases the wavelength will not change, but the frequency will increase accordingly.

12. struck by bullet first.

13. 80 Hz; 0.6 m
    1. 0.457 m
    2. 0.914 m
    3. 1.37 m
14. 2230 Hz; 2780 Hz; 2970 Hz
15. 498 Hz
16. 150 m/s

Ch 12: Electricity

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. (b) 1350 N (c) 1350 N
    1. 1.1×10^9 N/C
    2. 9000 N
12. $F_g = 1.0 \times 10^{-47}$ N and $F_e = 2.3 \times 10^{-8}$ N. The electric force is 39 orders of magnitudes bigger.
13. $1.0 \times 10^{-4} C$
14. .
15. (a) down (b) Up $16c, 5.5 \times 10^{11}$ m/s^2 (e) 2.9×10^8 m/s^2
    1. Toward the object
    2. 3.6×10^4 N/C to the left with a force of 2.8×10^{-7} N
16. Twice as close to the smaller charge, so 2 m from 12μC charge and 1 m from 3μC charge.
17. 0.293 N and at 42.5°
18. 624 N/C and at an angle of −22.4° from the $+x-$ *axis.*
    1. 7500V
    2. 1.5 m/s

    1. 6.4×10^{-17} N
    2. 1300V
    3. 2.1×10^{-16} J
    4. 2.2×10^7 m/s
19. (b) 0.25m (c) $F_T = 0.022$ N (d) 0.37μC

Ch 13: Electric Circuits – Batteries and Resistors

1.
    1. 4.5C
    2. 2.8×10^{19} electrons

   1. 0.11 A
   2. 1.0 W
   3. 2.5×10^{21} electrons
   4. 3636 W

   1. 192 Ω
   2. 0.42 W

   1. 5.4 mV
   2. 1.4×10^{-8} A
   3. 7.3×10^{-11} W, not a lot
   4. 2.6×10^{-7} J

2. left = brighter, right = longer

   1. 224 V
   2. 448 W
   3. 400 W by 100 Ω and 48 W by 12 Ω

3. (b) 8.3 W
4. 0.5A
5. .
6. 0.8A and the 50 Ω on the left

   1. 0.94 A
   2. 112 W
   3. 0.35 A
   4. 0.94 A
   5. $50, 45, 75$ Ω
   6. both 50 Ω resistors are brightest, then 45 Ω, then 75 Ω

   1. 0.76 A
   2. 7.0 W

7. (b) 1000 W
8. .

   1. 9.1 Ω
   2. 29.1 Ω
   3. 10.8 Ω
   4. 26.8 Ω
   5. 1.8A
   6. 21.5V
   7. 19.4V
   8. 6.1V
   9. 0.24A
   10. 16 kW

   1. 3.66 Ω
   2. 0.36A
   3. 1.32 V

9. .
10. .
11. .
12. .
13. .

14. .
15. (a) 10V

Ch 14: Magnetism

1. No: if $v = 0$ then $F = 0$; yes: $F = qE$
2. .
3. .
    1. Into the page
    2. Down the page
    3. Right
4. Both pointing away from north
5. .
6. .
7. 7.6 T, south
8. Down the page; 60 N
    1. To the right, 1.88×10^4 N
    2. 91.7 m/s
    3. It should be doubled
9. East 1.5×10^4 A
10. 0.00016 T; if CCW motion, B is pointed into the ground.
11. 1.2×105 V, counterclockwise
    1. 15 V
    2. Counter-clockwise

    1. 2×10^{-5} T
    2. Into the page
    3. 2.8 N/m
    4. CW

    1. 2.42×10^8 m/s
    2. 9.69×10^{-12} N
    3. .0055 m
12. E/B
    1. 8×10^{-7} T
    2. 1.3×10^{-6} C

    1. 0.8 V
    2. CCW
    3. .064 N
    4. .16 N/C
    5. .13 w

    1. 1.11×10^8 m/s
    2. 9.1×10^{-30} N $<< 6.4 \times 10^{-14}$ N
    3. .00364 T
    4. .173 m
    5. 7.03×1016 m/s^2

   6. 3.27°

13. 19.2 V

    1. 8.39×10^7 m/s
    2. 2.68×10^{-13} N, $-y$
    3. 2.95×10^17 m/s^2
    4. .00838 m
    5. 1.68×10^6 N/C
    6. 16,800 V

    1. 1.2×10^{-6} T, $+z$
    2. 1.5×10^{-17} N, $-y$
    3. 96 N/C, $-y$

Ch 15: Electric Circuits—Capacitors

1. .
    1. 4×10^7 V
    2. 4×10^9 J
2. .
    1. 100 V
    2. A greater voltage created a stronger electronic field, or because as charges build up they repel each other from the plate.
3. 21 V, V is squared so it doesn't act like problem 4
    1. 3.3 F
    2. 54 Ω

    1. 200 V
    2. 5×10^{-9} F
    3. 2.5×10^{-9} F

    1. 6V
    2. 0.3A
    3. 18V
    4. 3.6×10^{-4}C
    5. 3.2×10^{-3}J
    6. i) 80μF ii) 40μF iii) 120μF

    1. 26.7μF
    2. 166.7μF

    1. 19.0×10^3 N/C
    2. 1.4×10^{-15} N
    3. 1.6×10^{15} m/s^2
    4. 3.3×10^{-11} s
    5. 8.9×10^{-7} m
    6. 5.1×10^{-30}

Ch 16: Electric Circuits—Advanced

1.
    1. 4.9×10^{-5} H
    2. -9.8×10^{-5} V
2. Zero
    1. Yes
    2. No
    3. Because they turn current flow on and off.

    1. 0.5 V
    2. 0.05 A
    3. 0.05 A
    4. 5.5 V
    5. 8.25V
    6. $11\times$

    1. *On*
    2. *On*
    3. $On, on, off, on, off, off, on, on$
3. (b) 10.9μ F (c) 195 Ω (d) 169 Ω (e) 1.39 A (f) $-42°$ (g) 115Hz

Ch 17: Light

1. .
2. .
3. 2200 blue wavelengths
4. 65000 $x-$rays
5. . 6×10^{14} Hz6.3.3 m
6. .
7. .
8. (b) vacuum air (c) 1.96×10^8 m/s
9. 6.99×10^{-7} m; $5.26 \times 10^{-}7$ m
10. .
11. .
12. Absorbs red and green.
13. 25°
14. .
15. 33.3°
    1. 49.7°
    2. No such angle
    3. 48.8°
16. (b) 11.4 m (c) 11.5 m
17. 85 cm
18. (c) +4 units (e) -1
    1. 6 units
    2. bigger; $M = 3$

19. (c) 1.5 units (d) $2/3$
20. (c) 3 units (e) $-2/3$
21. (c) 5.3 units
22. .
23. (b) 22.5 mm
24. .
25. 32 cm
    1. 10.2°
    2. 27 cm
    3. 20 cm
26. (a) 0.72 m
27. .
28. $54 \text{ cm}, 44 \text{ cm}, 21 \text{ cm}, 8.8 \text{ cm}$
29. .
30. 13.5°
31. 549 nm

Ch 18: Fluids

1. 0.84
2. 1.4×10^5 kg
    1. 90% of the berg is underwater
    2. 57%
3. (b) 5.06×10^{-4} N (c) 7.05 m/s^2
4. 4.14 m/s
5. 6. 40 coins
6. (b) upward (c) 4.5 m/s^2 (d) Cooler air outside, so more initial buoyant force (e) Thin air at high altitudes weighs almost nothing, so little weight displaced.
7. (a) At a depth of 10 cm, the buoyant force is 2.9 N (d) The bottom of the cup is 3 cm in radius
    1. $83,000$ Pa
    2. 104 N
    3. 110 N

    1. 248 kPa
    2. 591 kPa
    3. 1081 kPa
8. .
9. .0081
    1. 12500 J/m^3
    2. 184 kPa
    3. 1.16 kW
    4. 2.56 kW
    5. 11.8 A
    6. $12.60

    1. 611 kPa
    2. 6 atm

10. (b) $500,000$ N
    1. $27\ \mathrm{m/s^2}, (2.7\ \mathrm{g})$ upward
    2. 1600 N
    3. 2200 N

    1. 10 N
    2. 10.5 N
    3. 11 N
    4. 11 N
11. (a) "The Thunder Road" (b) 2.0 m (note: here and below, you may choose differently) (c) 33.5 $\mathrm{m^3}$ (e) 3.5 million N (f) 111 MPa

Ch 19: Thermodynamics and Heat Engines

1. .
2. .
3. .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. .
12. .
13. .
14. .
15. .
16. .
17. .
18. 517 m/s
19. 1.15×10^{12} K
20. .
21. 40 N
22. $\approx 10^{28}$ molecules
    1. $21,000$ Pa
    2. Decreases to $61,000$ Pa
    3. 5.8 km

    1. No
    2. allowed by highly improbable state. More likely states are more disordered.

    1. 8.34×10^{23}
    2. 6.64×10^{-27} kg
    3. 1600 m/s
    4. 744 kPa
    5. 4.2×10^{20} or 0.0007 moles
    6. 0.00785 $\mathrm{m^3}$

1. 1.9 MW
2. 0.56 MW
3. 1.3 Mw

1. 54%
2. 240 kW
3. 890 kW
4. 590 kW
5. 630 kg

1. 98%
2. 4.0%
3. 12%

23. 14800 J
24. 12,000 J
25. (b) 720 K, 300 K, 600 K (c) isochoric; isobaric (d) C to A; B − C (e) 0.018 J
26. (b) 300 K, 1200 K

1. 1753 J
2. −120 J
3. 80 J
4. 35 J
5. −100 J, 80 J, 80 J

Ch 20: Special and General Relativity

1. longer
2. ? = ∞, the universe would be a dot
3. 76.4 m, 76.4 m
4. .
5. ? = 1.002
6. 9.15×10^7 m/s
7. 2.6×10^8 m/s

1. 0.659 km
2. 22.4
3. 4.92×10^{-5} m/s
4. 14.7 km

8. 2900 m
9. 1.34×10^{-57} m
10. 0.303 s
11. 2.9×10^{-30}kg, yes harder to accelerate

1. f
2. c

12. 4.5×10^{16} J; 1.8×10^{13} softballs

1. 1.568×10^{-13} J
2. 3.04×10^6 J

Ch 21: Radioactivity and Nuclear Physics

1. .
2. .
3. .
4. .
5. .

    1. Substance A decays faster than B
    2. Substance B because there is more material left to decay.

    1. ${}^{219}{}_{88}\text{Ra} \rightarrow {}^{215}{}_{86}\text{Rn} + {}^{4}{}_{2}\text{He}$
    2. ${}^{158}{}_{63}\text{Eu} \rightarrow {}^{158}{}_{64}\text{Gd} + {}^{0}{}_{-1}e^{-}$
    3. ${}^{53}{}_{22}\text{Ti} \rightarrow {}^{53}{}_{23}\text{Va} + {}^{0}{}_{-1}e^{-}$
    4. ${}^{211}{}_{83}\text{Bi} \rightarrow {}^{207}{}_{81}\text{Tl} + {}^{4}{}_{2}\text{He}$

    1. 5×10^{24} atoms
    2. Decay of a lot of atoms in a short period of time
    3. 2.5×10^{24} atoms
    4. $\frac{1}{2}$
    5. 26.6 minutes

6. The one with the short half life, because half life is the rate of decay.

    1. Substance $B = 4.6$ g and substance $A = 0.035$ g
    2. substance B

7. 1.2 g
8. 125 g
9. 0.46 minutes
10. $t = 144,700$ years
11. 0.0155 g
12. 17 years
13. $49,000$ years

Ch 22: Standard Model of Particle Physics

1. strange
2. some type of meson
3. Electron, photon, tau...
4. Neutron, electron neutrino, Z^0
5. Neutron, because it doesn't have electrical charge
6. No, because it doesn't have electrical charge
7. Two anti-up quarks and an anti-down quark
8. Lepton number, and energy/mass conservation
9. Yes, W^+, W^-, because they both have charge
10. The weak force because it can interact with both quarks and leptons
11. Yes; a,b,c,e; no; d,f
12. The standard model makes verifiable predictions, string theory makes few verifiable predictions.

Ch 23: Feynman Diagrams

1. Allowed: an electron and anti-electron(positron) annihilate to a photon then become an electron and anti-electron(positron) again.
2. Not allowed: electrons don't go backward though time, and charge is not conserved
3. Not allowed: lepton number is not conserved
    1. Allowed: two electrons exchange a photon
    2. Not allowed: neutrinos do not have charge and therefore cannot exchange a photon.

    1. Allowed: an electron and an up quark exchange a photon
    2. Not allowed: lepton number not conserved
4. Not allowed: quark number not conserved
5. Allowed: electron neutrino annihilates with a positron becomes a W^+ then splits to muon and muon neutrino.
6. Allowed: up quark annihilates with anti-up quark becomes Z^0, then becomes a strange quark and anti-strange quark
7. Not allowed: charge not conserved
8. Allowed: this is a very rare interaction
9. Not allowed: electrons don't interact with gluons
10. Not allowed: neutrinos don't interact with photons
11. Allowed: the electron and the positron are exchanging virtual electron/positron pairs
12. Allowed: this is beta decay, a down quark splits into an up quark an electron and an electron neutrino via a W^- particle.
13. Allowed: a muon splits into an muon neutrino, an electron and an electron neutrino via a W^- particle.

Ch 24: Quantum Mechanics

1.
    1. $6.752 \times 10^{-26} J, 2.253 \times 10^{-34}$ kgm/s
    2. $5.96 \times 10^{-20} J, 1.99 \times 10^{-28}$ kgm/s
    3. $4.90 \times 10^{-28} J, 1.63 \times 10^{-36}$ kgm/s

    1. 1.94 eV, 1.04×10^{-27} kgm/s
    2. 12.7 eV, 6.76×10^{-27} kgm/s
    3. 5.00 eV, 2.67×10^{-21} kgm/s

    1. .0827 nm
    2. 4.59×10^{-4} nm
    3. .942 nm
2. 1.03×10^{-20} m
    1. 36 nm
    2. no
    3. 380 nm, 73 nm, 36 nm, 92 nm, 39 nm
3. .80 V
4. .564 nm
    1. .124 nm
    2. .00120 nm
5. 24,600 m/s

6. 1.84×10^8 m/s
    1. .491 m/s
    2. 3.1410^7 J
    3. 64 Mw
    4. 1.55 pm
7. 3.27 eV
8. .
9. (b) 15 (c) 182 nm, 188 nm, 206 nm, 230 nm
10. -10.3 eV, -3.82 eV, -2.29 eV, -1.83 eV
    1. 4.19×10^7 m/s
    2. 1.70×10^{-11} m
    3. $1.95°$
    4. .068 m

    1. 1.89 V
    2. 1.60 A
    3. 1.25 Ω

    1. 4.40×10^{-24} kgm/s
    2. 1.17×10^{-24} kgm/s
    3. 3.23×10^{-24} kgm/s
    4. 3.76×10^7 m/s

    1. 1.1365×10^{-22} kgm/s
    2. 5.860 pm
    3. ^{242} Cu $\rightarrow^4$ He $+^2 38$Pu
    4. 238.0497 amu
    5. 17.7 cm
    6. $-y$
    7. $+y$, 34.2 N/C

Ch 25: Global Warming

1. (a) 5.1×10^{14} W (b) 1.8×10^{15} kWh (c) 1.6×10^{22} J (d) 7.0×10^{15} J (e) About 2.3 million bombs

CHAPTER 30 Equations and Fundamental Constants Version 2

Simple Harmonic Motion and Wave Motion

$$T = 1/f$$
$$T_{sp} = 2\pi\sqrt{\frac{m}{k}}$$

$$v = \lambda f$$
$$T_P = 2\pi\sqrt{\frac{L}{g}}$$

$$f_n = \frac{nv}{2L} \qquad \text{nodes at both ends}$$
$$f_n = \frac{nv}{4L} \qquad \text{(n is odd)node at one end}$$
$$f_{beat} = |f_1 - f_2|$$

$v_{sound} = 343$ m/s (in air at 20 C)

A note: 440 Hz

C note: 524 Hz

D note: 588 Hz

E note: 660 Hz

G note: 784 Hz

Fluids and Thermodynamics

$$3/2\ kT = < \frac{1}{2}\,\text{mv}^2 >_{avg}$$
$$P = F/A$$
$$P = P_0 + \rho gh$$
$$\triangle P + \triangle(\rho gh) + \triangle\left(\frac{1}{2}\rho v^2\right) = 0$$
$$\phi = A \cdot v$$
$$^\circ C = ^\circ K + 273.15$$

$$PV = NkT = nRT$$
$$F_{buoy} = -(\rho_{water} V_{displaced})g$$
$$Q_{in} = W + \triangle U + Q_{out}$$
$$W = P\triangle \text{ V}$$
$$k = \frac{1}{2}\rho v^2; u = \rho gh$$
$$\eta = W/Q_{in}; \eta_{Carnot} = 1 - (T_{low}/T_{high})$$

$$k = 1.381 \times 10^{-23} \text{ J/K}$$
$$\rho_{air} = 1.29 \text{ kg/m}^3$$
$$R = 8.315 \text{ J/mol-K}$$
$$\rho_{water} = 1000 \text{ kg/m}^3$$
$$P_{ATMOSPH} = 101,000 \text{ N/M}^2$$
$$N_{avo} = 6.022 \times 10^{23} \text{ mol}^{-1}$$

Properties of Fundamental Particles

$$m_{proton} = 1.6726 \times 10^{-27} \text{ kg}$$
$$q_{electron} = -q_{proton} = -1.602 \times 10^{-19} \text{ C}$$
$$r_{hydrogen\ atom} \approx 0.529 \times 10^{-10} \text{ m}$$

$$m_{electron} = 9.109 \times 10^{-31} \text{ kg}$$
$$1 \text{ amu} = 1.6605 \times 10^{-27} \text{ kg} = 931.5 \text{ Mev/c}^2$$
$$\triangle \text{ E} = \triangle \text{ mc}^2$$

$$m_{neutron} = 1.6749 \times 10^{-27} \text{ kg}$$

Radioactivity, Nuclear Physics, and Quantum Mechanics

$$(\triangle x)(\triangle p) \approx h/4\pi$$
$$\lambda = h/p$$
$$N = \text{N0} \left(\frac{1}{2}\right) t/t_H$$
$$1 \text{ ev} \rightarrow 1240 \text{ nm}$$

(energy of a photon)

$$(\triangle E)(\triangle t) \approx h/4\pi$$
$$E_{photon} = hf = pc$$
$$Kmax = qV = hf + \phi$$

$$h = 6.626 \times 10^{-34} \text{ J.s}$$
$$^{A}Z = \text{element Z with A nucleons}$$
$$^{14}C : t_H = 5,730 \text{ years(half life} = t_h)$$
$$^{239}Pu : t_H = 24,119 \text{ years}$$
$$E_o = -13.605 \text{ ev(Hydrogen ground state)}$$

Light

$$\lambda_{blue} \approx 450 \text{ nm}$$
$$\lambda_{green} \approx 500 \text{ nm}$$
$$\lambda_{red} \approx 600 \text{ nm}$$

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$
$$c = 2.998 \times 10^8 \text{ m/s}$$
$$m\lambda = \text{dsin}(\theta)$$

$$n_{air} \approx n_{vacuum} = 1.00$$
$$n_{water} = 1.33$$
$$n = c/v_{material}$$

primary: Red, Green, Blue

secondary: Magenta, Cyan, Yellow

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{di} \qquad M = h_i/h_o = d_i/d_o$$

Electricity and Magnetism

$$F_E = k\ q_1 q_2/r^2$$
$$E = F_E/q$$
$$E = -\triangle V/\triangle x$$

$$F_B = qv \times B = \text{ qvB}\sin(\theta)$$
$$B_{wire} = \mu_o I/2\pi r$$
$$F_{wire} = \ell(I \times B) = \ell \text{ IB}\sin(\theta)$$

(direction: RHR)

(direction: RHR)

(direction: RHR)

$$k = 8.992 \times 10^9 \text{ N}\cdot\text{ m}^2/C^2$$
$$\mu_o = 4\pi \times 10^{-7} \text{ T}\cdot\text{ m/A}$$
$$\phi = \text{BA}\cos(\theta)$$

$$U_{el} = q\triangle V$$

Point charges: $E(r) = kq/r^2$ and $V(r) = kq/r$

($k = 1/4\pi\varepsilon_o$ where $\varepsilon_o = 8.854 \times 10^{-12}\ C^2/\text{ N}\cdot\text{ m}^2$)

$$V = -\triangle\phi/\triangle t = \text{ Blv}$$

Electric Circuits

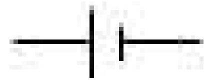
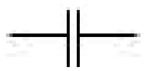
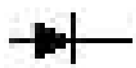
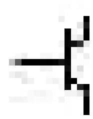
$$\triangle V = IR$$
$$I = \triangle q/\triangle t = \triangle V/R$$
$$\tau = RC$$

$$P = \triangle E/\triangle t = I\triangle V = I^2R = V^2/R$$
$$R = \rho l/A$$
$$V = -L(\triangle I/\triangle t)$$

$$Q = C\triangle V$$
$$C_{parallel\ plate} = k\varepsilon A/d$$
$$C_{parallel} = C_1 + C_2 + \ldots$$

$$R_{series} = R_1 + R_2 + \ldots$$
$$1/R_{parallel} = (1/R_1) + (1/R_2) + \ldots$$
$$1/C_{series} = (1/C_1) + (1/C_2) + \ldots$$

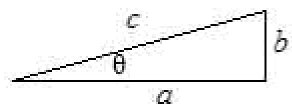
TABLE 30.1:

Name	Symbols		Unit	Typical examples
Voltage Source	$\triangle V$		volt (V)	9 V (cell phone charger); 12 V (car); 120 VAC (U.S. wall outlet)
Resistor	R		Ohm (Ω)	$144\Omega(100\,\text{w}, 120v\,\text{bulb}); 1k\Omega$ (wet
Capacitor	C		Farad (F)	RAM in a computer, 700 MF (Earth)
Inductor	L		Henry (H)	7 H (guitar pickup)
Diode	by type		none	light-emitting diode (LED); solar panel
Transistor	by type		none	Computer processors

Equation Sheet

Mathematics

$$\sin(\theta) = b/c \rightarrow b = c \cdot \sin(\theta)$$
$$\cos(\theta) = a/c \rightarrow a = c \cdot \cos(\theta)$$
$$\tan(\theta) = b/a \rightarrow b = a \cdot \tan(\theta)$$
$$c^2 = a^2 + b^2$$



$$180^\circ = \pi \text{ radians}$$
$$C_{circle} = 2\pi R$$
$$A_{circle} = \pi R^2$$
$$V_{sphere} = (4/3)\pi R^3$$
$$V_{cylinder} = \pi R^2 h$$

If X is any unit, then . . .

$$1 \text{ mX} = 0.001 \text{ X} = 10^{-3} \text{ X}$$
$$1\mu\text{X} = 0.000001 \text{ X} = 10^{-6} \text{ X}$$
$$1 \text{ nX} = 0.000000001 \text{ X} = 10^{-9} \text{ X}$$

$$1 \text{ kX} = 1000 \text{ X} = 10^{3} \text{ X}$$
$$1 \text{ MX} = 1000000 \text{ X} = 10^{6} \text{ X}$$
$$1 \text{ GX} = 1000000000 \text{ X} = 10^{9} \text{ X}$$

If $ax^2 + bx + c = 0$, then . . .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

% difference = |(measured – accepted) / accepted | $\times 100\%$

vector dot product: $a \cdot b = ab\ \cos\theta$ (product is a scalar)—θ is angle between vectors

vector cross product: $a \times b = ab\ \sin\theta$ (direction is given by RHR)

Kinematics Under Constant Acceleration

$$\triangle x = x_{final} - x_{initial}$$
$$\triangle \text{ (anything)} = \text{final value} - \text{initial value}$$
$$v_{avg} = \triangle x / \triangle t$$
$$a_{avg} = \triangle v / \triangle t$$

$$x(t) = x_0 + v_0 t + \frac{1}{2} a_x t^2$$
$$v(t) = v_0 + at$$
$$v^2 = v_0^2 + 2a(\triangle x)$$

$$(x = x_0 \text{ and } v = v_0 \text{ at } t = 0)$$

$$g = 9.81 \text{ m/s}^2 \approx 10 \text{ m/s}^2$$
$$1 \text{ km} = 1000 \text{ m}$$
$$1 \text{ meter} = 3.28 \text{ ft}$$
$$1 \text{ mile} = 1.61 \text{ km}$$

Newtonian Physics and Centripetal Motion

$$a = F_{net}/m F_g = mg$$
$$F_{net} = \Sigma F_{all\ forces} = ma$$

$$f_k = \mu_k F_N F_{sp} = -k(\triangle x)$$
$$f_s \leq \ \mu_s F_N F_G = G m_1 m_2 / r^2$$
$$F_C = mv^2/r$$

$$G = 6.672 \times 10^{-11} \text{ N} \cdot \text{ m}^2/kg^2$$
$$1 \text{ kg} = 1000 \text{ g} = 2.2 \text{ lbs}$$
$$1 \text{ N} = 1 \text{ kg} \cdot \text{ m/s}^2$$

Momentum and Energy Conservation

$$\Sigma p_{initial} = \Sigma p_{final}$$
$$E_{initial} = E_{final}$$
$$E = K + U + W$$

$$p = mv$$
$$K = 1/2 \text{ mv}^2$$

$$F_{avg} = \triangle p / \triangle t$$
$$U_g = mgh$$
$$U_{sp} = \frac{1}{2}k(\triangle x)^2$$
$$U_g = -Gm_1m_2/r$$

$$W = F \cdot \triangle x$$
$$P = \triangle W / \triangle t$$
$$P = F \cdot v$$

$$1 \text{ J} = 1 \text{ N} \cdot \text{ m}$$
$$1 \text{ W} = 1 \text{ J/s}$$
$$1 \text{ food Calorie} = 4180 \text{ J}$$
$$1 \text{ ev} = 1.602 \times 10^{-19} \text{ J}$$
$$1 \text{ kwh} = 3.600 \times 10^{6} \text{ J}$$

Rotational Motion

$$d = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$
$$\omega = 2\pi/T$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega(t) = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha(\triangle\theta)$$
$$a_C = -r\omega^2$$

$$\tau = I\alpha$$
$$L = r \times p = I\omega$$
$$\tau = r \times F = \triangle L / \triangle t$$

$$K = 1/2I\omega^2$$

$$I_{\text{ring about cm}} = \text{MR}^2$$
$$I_{\text{disk about cm}} = \frac{1}{2}\text{MR}^2$$
$$I_{\text{rod about end}} = (1/3)\text{ML}^2$$
$$I_{\text{solid sphere about cm}} = (2/5)\text{MR}^2$$

Astronomy

$$P_* = 4 \times 10^{26} \text{ W}$$
$$M_* = 1.99 \times 10^{30} \text{ kg}$$
$$R_* = 6.96 \times 10^{8} \text{ m}$$

$$1 \text{ light-year}(ly) = 9.45 \times 10^{15} \text{ m}$$
$$M_{Earth} = 5.97 \times 10^{24} \text{ kg}$$
$$R_{Earth} = 6.38 \times 10^{6} \text{ m}$$

$$\text{Earth} - \text{Sun distance} = 1.496 \times 10^{11} \text{ m}$$
$$M_{Moon} = 7.35 \times 10^{22} \text{ kg}$$
$$R_{Moon} = 1.74 \times 10^{6} \text{ m}$$
$$\text{Earth} - \text{Moon distance} = 3.84 \times 10^{8} \text{ m}$$

CHAPTER 31 Random Walks 1

Chapter Outline

31.1 Introduction

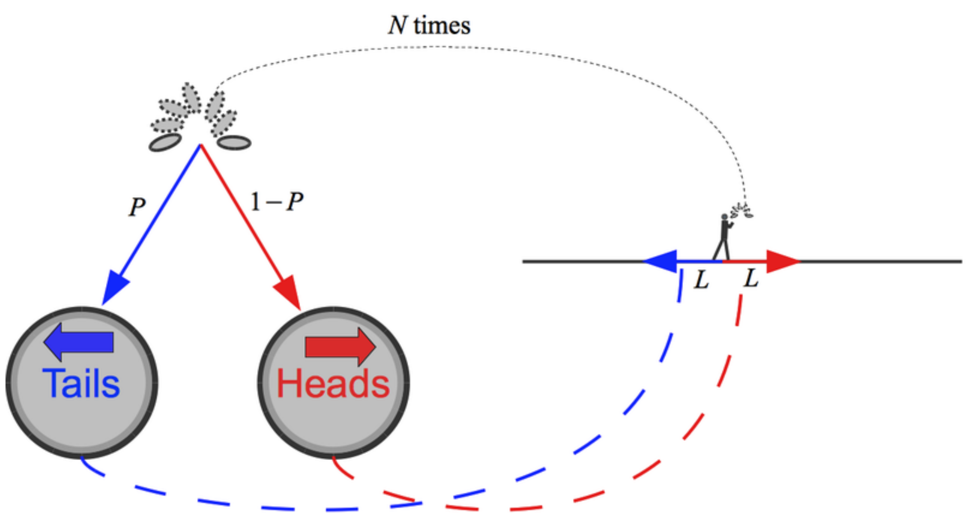
This chapter provides an introduction to the random walk, a model widely used in physics and other fields. It explains the theory behind the basics of one-dimensional random walks, and the next chapter then shows how to model them in Python — an open source programing language widely used within the scientific community.

Much of modern physics relies on computer simulations for results, yet this field is largely left out of high school physics classrooms. It is the goal of this chapter to bridge this gap by providing instructions for running simple models side by side with the theory they mirror. In this sense, the chapter is both an overview of random walks and a short introduction to computational programing.

One-Dimensional Random Walks

This type of random walk is conceptually a very simple model. Imagine starting at a point on a line, and then taking a step either to the left or to the right every couple of seconds. This is a typical example of a one-dimensional random walk. Which direction you pick, how big of a step you take, and how often you take steps are all parameters that can differentiate different random walks, but they all have these common basic features.

The 'random' in the title of this chapter refers to the fact that you will pick the direction of your steps randomly. Since the simplest practical idea of randomness that we have is a two-sided coin, we interpret our random walk in terms of that model. Specifically, each time a coin is flipped, the person flipping it takes a step — to the right if it lands on heads, and to the left if it lands on tails:



Notice that the coin doesn't have to be fair: we simply said that there is P probability of it landing on tails, and therefore $1 - P$ probability of it landing on heads. This probability is one of the **parameters** mentioned above. Here's a list of important ones for this type of random walk:

One-dimensional Random Walk Parameters

- Probability of picking left/right (P).

- Step size (L).
- Number of steps (N).

We can now describe our model in the terms above. The person flips a coin, which lands on heads or tails according to the probability p. After each flip, he takes a step of length L in the direction decided by the coin. After N total flips, he stops and records his position.

In general , the value L can vary from step to step, but in this chapter, we're going to focus on random walks with constant step size, which we can just set to $L = 1$ without losing any generality.

Question

Why can we do this?

Answer

Because as long as step sizes are constant, our results would only be off by a proportionality constant for other random walks.

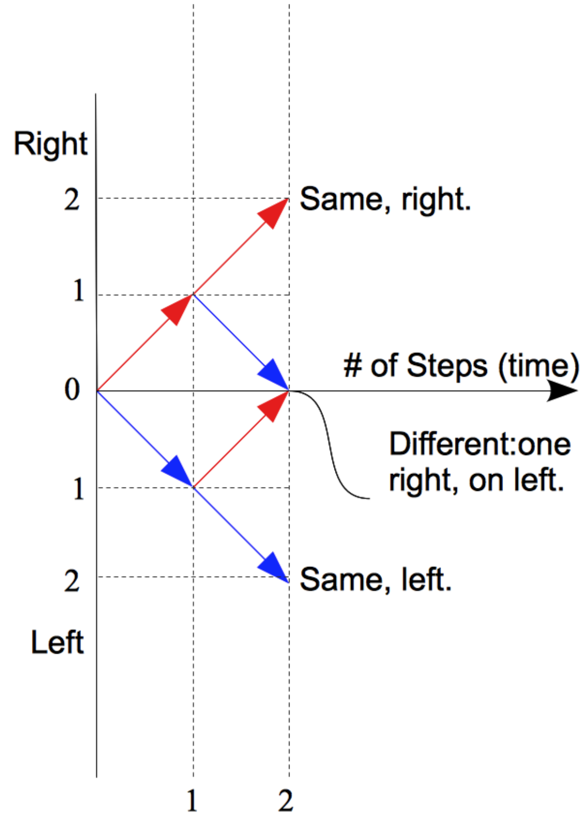
Bookkeeping

Question

Consider a two step random walk (that is, $N = 2$). What are the possible outcomes of such a random walk? How can we keep track of these?

Answer

In a two step random walk, either the steps are in the same direction, or in opposite directions. If they are in the same direction, the walker will be either two to the left, or two to the right of her original position. Alternatively, if they are in the same direction, the walker will remain at her starting position at the end of the walk. We can represent this on a graph where we count the number of steps on the x-axis, and the position on the y-axis (it might seem more natural to use the horizontal axis for position — due to the left/right dichotomy — but since the number of steps is essentially a 'time' variable and time is generally the independent variable, we will follow convention). All the possibilities are represented below:



Question

What is the probability of each of the three possible outcomes?

Answer

The probability of landing two steps to the right is the same as that of rolling two heads with the coin above. This equals $P \times P = P^2$. Analogously, the probability of landing two steps to the left is $(1-P)^2$. The probability of landing in the original position is equal to getting one heads and one tails. You might think this is $P(1-P)$, but it's slightly subtler: since the *order* of the heads and tails doesn't matter, there are two ways to get this outcome: heads first, tails second, or vice versa (right then left, or left then right). Therefore we have to add the probability of one such outcome, $P(1-P)$, to the probability of the other, $(1-P)P$. So the result is $2 \times P(1-P)$.

Note that $P^2 + 2(P)(1-P) + (1-P)^2 = 1$. Why is this important and relevant?

Motivation

You might ask at this point: what's the meaning of this model? Can something so simple actually be useful? And if the model is useful, what exactly are we trying to find?

These are good questions. First, it turns out that the model is useful; first, because many real world phenomena — stock prices, gambling wins/losses, certain quantum phenomena — can actually be modeled as one-dimensional random walks. More importantly, however, many of the one-dimensional results happen to transfer easily to two and three-dimensional random walks, which actually model a much greater range of phenomena.

Question

What situations can be accurately modeled by two and three-dimensional random walks?

Answer

Two dimensional walks can be used to model the spread of insects — mosquitoes, for instance — over an area, while three-dimensional random walks accurately describe the behavior of gas atoms under a wide range of conditions.

Finally, let's try to answer the last question: what exactly are we trying to find here? Remember, the usefulness of any model is measured by its applications: if you model the weather, hopefully you will be able to predict it using your model.

Let's look at the applications we listed above, and think of relevant questions. In terms of stock prices, one might ask: "what is the probability that a given stock will be x dollars above or below its starting point after a given period of time? In the random walk model, this translates to: What is the probability that a walker will be a distance X from the origin after a given number of steps?

In terms of gas molecules, we might ask: if you break a beaker of some gas, how quickly will the gas spread through a room or area? In random walk terms, this becomes: How quickly does the walker tend to move away from the origin. This is a model of diffusion.

Finally, in our insect model, we might ask: how long will it take, on average, for some infectious insects to reach an area some distance away from their starting point? In our model, this becomes: How long will it take, on average, for the walker to reach some distance from the origin?

To recap, some important questions we might try to answer about the random walk model are:

1. What is the probability that a walker will be a distance D from the origin after a given number of steps?
2. How quickly does the walker tend to move away from the origin?
3. How long will it take, on average, for the walker to reach some distance from the origin?

Note: The underlying idea behind all these questions can be summarized in the following manner: **What is the net effect, on average, of the canceling out of steps in opposite directions in a random walk?** Understanding this will not only help us with the mathematics that follows, but is also key to generalizing the model to two and three dimensions.

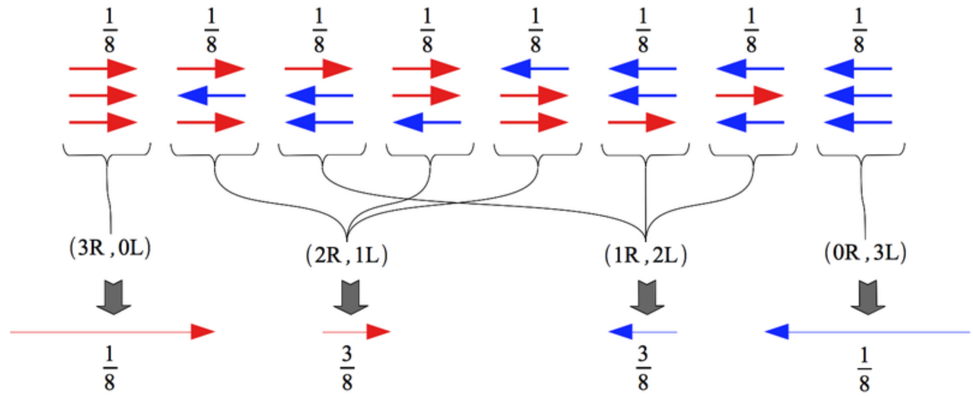
We will only consider the first question theoretically, but the other two can be explored using the computational models of the next chapter.

What are we looking for?

Let's look at the first of the questions above in more mathematical detail — we would like to find is the probability that after N flips, or steps, the walker is D steps to the right (or left) of the origin (starting point)? Remember, at any given step the walker steps to the right if the coin lands on heads (probability P, which is now known) and left if the coin lands on tails (probability $1-P$).

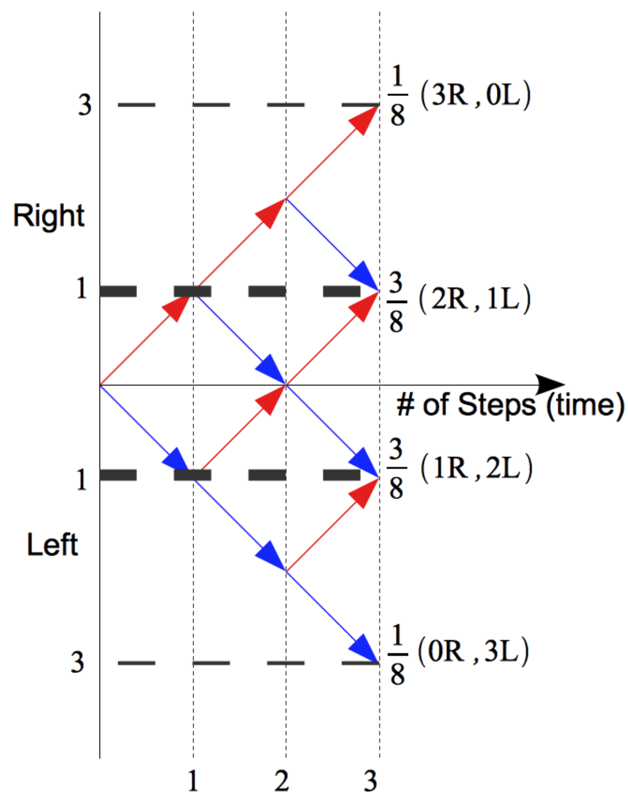
Fair Coin Three-Step Case

In physics derivations, it's often possible to obtain an intuition about the right way to find a general result or formula by considering simple specific cases first. We will use this method here — let's look at the possible outcomes (that is, step sequences) of a three step random walk where $P = 1/2$ (it's a fair coin, the walker is equally likely to step left or right at every step). The various possibilities for this case are illustrated below:



The first case is equivalent to flipping three heads in a row and therefore taking three right step, the second to flipping the sequence heads, tails, heads, and so on. Since in this case the coin is fair, the eight outcomes (step combinations) shown above are equally likely to occur: they each have a probability of $(1/2)^3 = 1/8$.

These outcomes, however, do not all result in different end locations (the four low arrows) for the walker: this is determined by the difference between the number of steps taken to the right and the number taken to the left. So while only one outcome corresponds to an end location of three steps to the right or three to the left, three outcomes correspond to an end location of one step to the right or one to the left, analogously to our calculations in the bookkeeping section above. So the eight equally likely outcomes result in four possible end locations that are clearly not equally likely. This is even clearer when we look at a graph of the possible walks, where we can trace the paths that lead to the same end locations:



Question

Show each of the eight possible step combinations illustrated earlier on the figure above.

As we noted earlier, the probability of stopping at a particular end location (such as one step to the right of the starting point) occurs will equal to the sum of the probabilities of the outcomes that lead to it. This is important, and bears repeating:

Probability of a particular end location

$$P(\text{End Location}) = \sum \text{Probabilities of outcomes that lead to that location}$$

If all such outcomes are equally likely, then

$$P(\text{End Location}) = (\text{Probability of a single outcome}) \times (\text{Number of outcomes})$$

Therefore, the probability of ending one step right is:

$$1/8 + 1/8 + 1/8 = 3 \times 1/8 = 3/8$$

This reasoning allows us to find the probabilities of the other possible end locations as well, noted below the arrows on the graph above. A grouping of possible end locations and their respective likelihoods is an example of a probability mass function. Let's define this important concept.

Probability Mass Function
A list of events with associated probabilities.

In more mathematical language, we can represent of a set of events as

$$(\text{Event}_1, \text{Event}_2, \text{Event}_3, \ldots, \text{Event}_i)$$

.

The associated probabilities, meanwhile, are written as

$$(P(\text{Event}_1), P(\text{Event}_2), P(\text{Event}_3), \ldots, P(\text{Event}_i))$$

.

In our example, the end locations of the three step random walk described above will can be written as

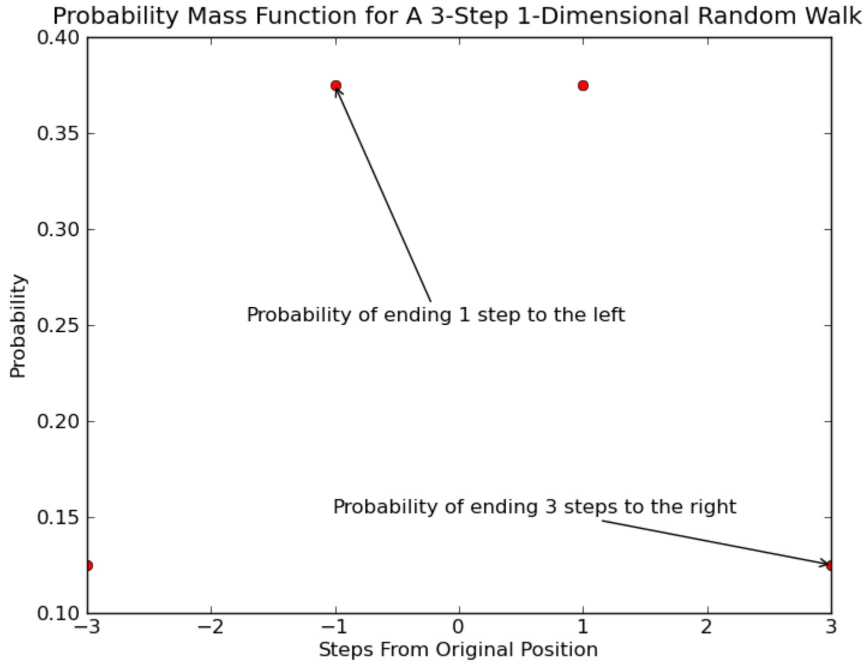
$$(3R, 1R, 1L, 3L)$$

and their probabilities as

$$(1/8, 3/8, 3/8, 1/8)$$

.

We can plot this distribution on a graph where final displacement (alternatively, number of steps — the two quantities will be equal if we set the step lengths to 1, which we can with no loss of generality) from the origin is on the x-axis, while its probability is on the y-axis:



Above, we have answered the question posed at the beginning of this section for the three-step case. That is, we have completely determined what the likelihood of the walker being in any possible location is at the end of this walk.

Question

Find the probability mass function for a single roll of a fair die.

Answer

The outcomes are simply the possible numbers, since it is a fair die, they have equal probability: $1/6$. Therefore the PMF is

$$(1,2,3,4,5,6);(1/6,1/6,1/6,1/6,1/6,1/6)$$

Question

Why are particular outcomes (sequences of steps, not end locations) equally likely in fair-coin random walks with any number of steps?

Answer

Since steps right and left are equally likely, any particular sequence of N steps has a probability of $(1/2)^{-N}$. (Why? Think of the two-step example given in the beginning of the chapter: if $p = 1/2$

$$P^2 = (1-P)^2 = P(1-P) = (1-P)(P) = 1/4$$

.

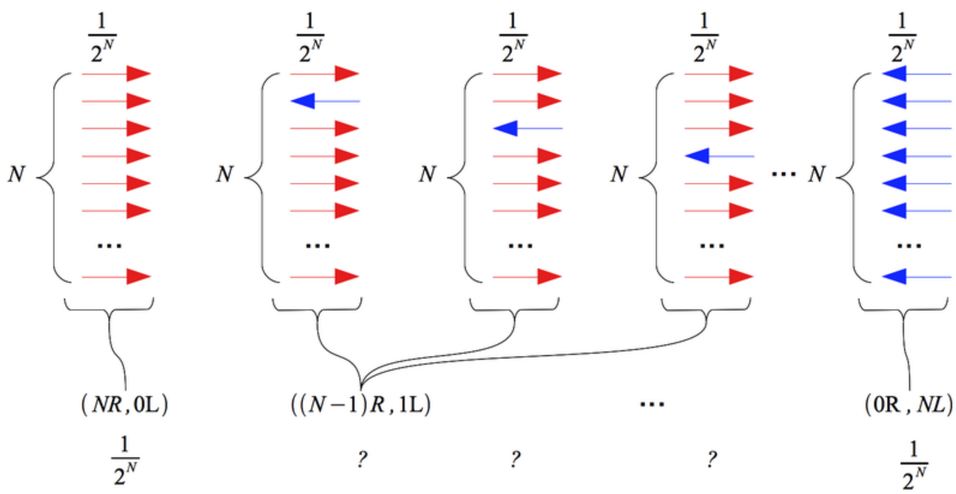
Fair Coin General Case

Now let us try to generalize these results to to a random walk with $P = 1/2$ and N steps. The intuition we obtained from considering the simple case above can be summarized as follows: to find the probability mass function of the end location of a random walker, one should:

1. Consider all the possible outcomes (as we showed above, they will be equally likely)
2. Think which outcomes lead to which end locations — in particular, **how many** outcomes lead to a particular end location.
3. Use the fact that

$$P(\text{End Location}) = (\text{Probability of a single outcome}) \times (\text{Number of outcomes})$$

The diagram below is analogous to the one for three steps, but now with N steps. We can divide the possibilities into 2^Nequally likely outcomes, this time each with probability $1/(2^N)$. So, we found the first part in the left side of the equation above.The question is, 'How many outcomes lead to a given end location, say, D steps to the right (as posed above)?'



There is still only one outcome that leads to each of the two 'extreme' locations, when all steps are taken either right or left. Their probabilities are 2^{-N} — but what about the other locations?

To find their respective probabilities, we need to remember the fact that end locations depend on the **difference** between the number of steps taken to the left and right (and not their order) to pose the problem in a slightly different way.

Let L be the number of steps taken to the left, and R to the right. Since the total number of steps is N,

$$N = L + R \qquad \text{Total of steps adds to } N$$

If the walkers winds up D to the right of the origin, she must have taken D more steps to the right than to the left:

$$D = R - L \qquad \text{Total distance traveled}$$

Solving these two equations (work through them yourself), we find that:

$$R = 1/2(D+N) \qquad \text{Steps to the right}$$
$$L = N - R = 1/2(N-D) \qquad \text{Steps to the left}$$

We have solved for the necessary number of steps left and right in terms of known quantities, N and D. At this point all that remains is finding how many ways there are to take $1/2(D+N)$ steps to the right out of a total of N steps: this will give us the number of outcomes that lead to end location of D steps to the right.

In the three step case, for instance, ending one space right of the origin required taking two steps right and one step left; there are three discrete ways to take two achieve this (the left step can be first, second, or third), and so three outcomes that lead to that location.

For the case of N total steps and $1/2(D+N)$ steps to the right, the correct result will be given by the 'ways of choosing' formula from combinatorics: literally, it is the number of ways to choose $1/2(D+N)$ positions for the right steps out of a total of N positions. This is written as

$$\binom{N}{1/2(N+D)}$$

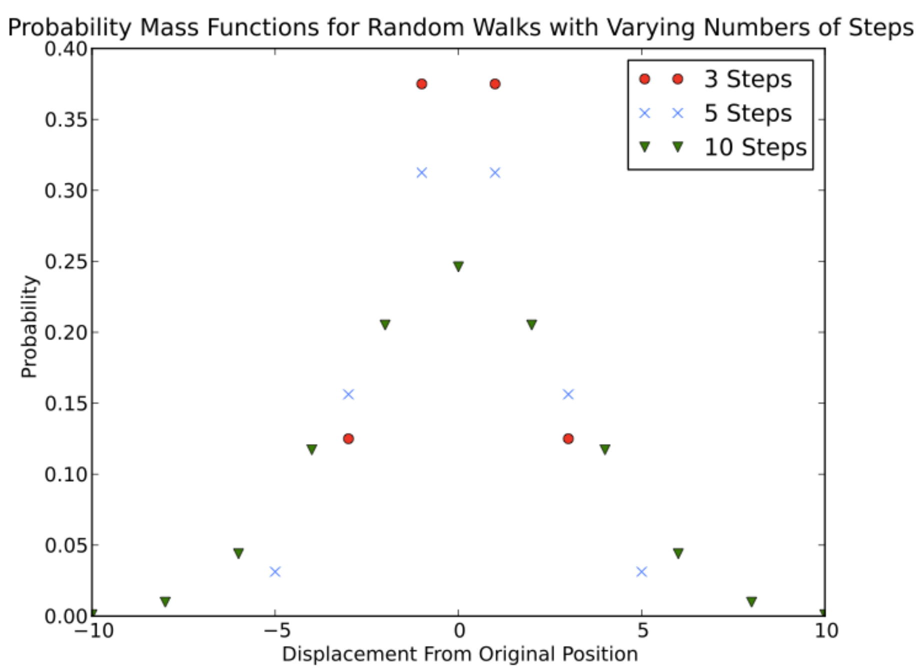
where

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

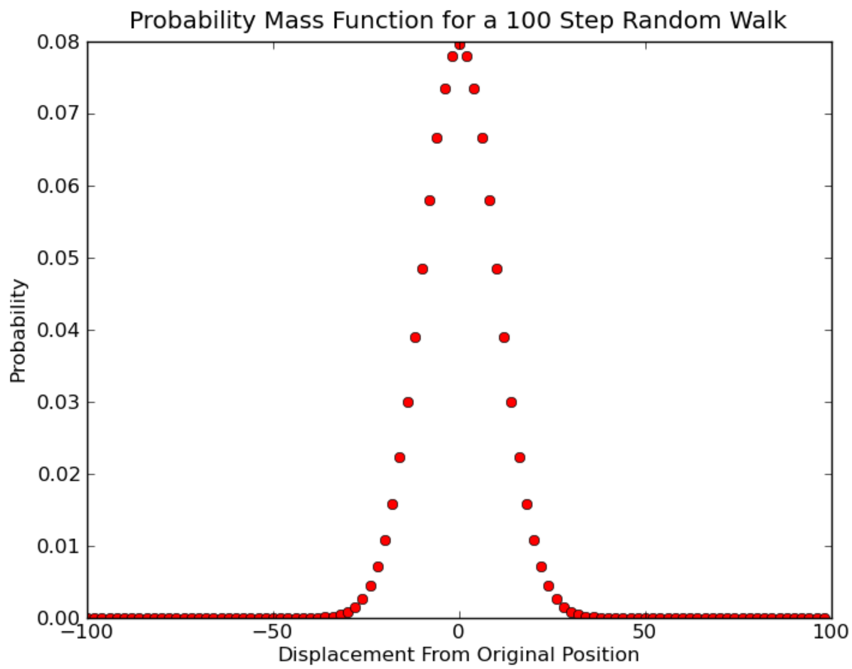
So, according to our earlier result, the probability of finding the walker a distance D steps to the right of the origin is given by the following formula:

$$P(D) = \underbrace{2^{-N}}_{P(\text{one outcome})} \times \underbrace{\binom{N}{1/2(N+D)}}_{\text{Number of outcomes that lead to this end position}}$$

We have now answered our original question (finding the probabilities of various end locations) for all unbiased $(P = 1/2)$ random walks with constant step lengths. Again, we can plot this distribution for several different cases:



When the number of steps becomes large, the distribution begins to look like a bell curve; here is the plot for $N = 100$:



Problems

1. What is the difference, in terms of end probability distributions, between random walks with even and odd numbers of steps?
2. Solve for the probability mass function of end locations for a four-step random walk analogously to the three-step example above (illustrating it also). Then, graph this probability mass function.
3. Our proof for the general case can be called 'right-biased' in two ways. This question settles both:

    a. We found the probability of being D to the right of the origin, but the probability distributions were graphed as symmetrical. First, explain why this must be true in terms of possible outcomes and end locations. Then, show that the formula for $P(D)$ can be used to find probabilities to the left also, that is, prove that $P(D) = P(-D)$ using the formula above and the definition of factorials.
    b. We also found the number of outcomes that lead to an end displacement of D in terms of steps taken to the right. Use the result from the previous part to show that using $1/2(N-D)$ — the number of steps to the left corresponding to a final distance of D steps to the right — in the derivation of the general result would not have changed it.

    a. Derive the probability mass function for a biased random walk (that is, steps in one direction are more likely than in the other, or the coin has a higher probability of landing heads (P than tails $(1-P)$. (Hint 1: Does the number of outcomes for a specific end location change?) (Hint 2: ALL the outcomes will no longer be equally likely, but what about outcomes that lead to specific end locations?) (Hint 2.5: every outcome that leads to a specific location HAS to have the same number of steps left and right).
    b. Graph a few of these distributions.