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CK-12 Middle School Math
Grade 6
Teacher's Edition

CK-12 Middle School Math - Grade 6 - Teacher's Edition

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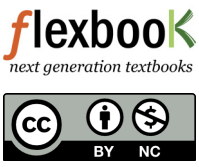


Contents

CHAPTER 1 Grade 6 - Teaching Tips

Chapter Outline

1.1 Number Sense and Variable Expressions

This first chapter *Number Sense and Variable Expressions* takes students from whole number operations into the world of beginning Algebra. This Teaching Tips flexbook is designed to assist you, the teacher, in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Operations with Whole Numbers
- Whole Number Estimation
- Powers and Exponents
- Order of Operations
- Variables and Expressions
- A Problem-Solving Plan
- Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Operations with Whole Numbers –Day 1 and Day 2
- Whole Number Estimation –Day 3 and Day 4
- Powers and Exponents –Day 5 and Day 6
- Order of Operations –Day 7
- Variables and Expressions –Day 8 and Day 9
- A Problem-Solving Plan –Day 10
- Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math –Day 11

Day 12 can be used as a review day.

Day 13 would be a testing day.

Operations with Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Adding Whole Numbers
- Subtracting Whole Numbers
- Multiplying Whole Numbers
- Dividing Whole Numbers

Relevant Review

The focus of this lesson is on whole number operations. To review these skills, students can begin on Day 1 with addition and subtraction. To review these skills, you can practice some simple mental math with the students. Beginning with mental math will help to strengthen this skill and integrates with the last lesson of the chapter on *Problem Solving Strategies*. Begin with sums and differences that are simpler and then work up to more complicated problems. For example:

a. $6+7=13$

b. $27+19=46$

You can also review doubles:

$4+4=8$

$5+5=10$

$6+6=12$

Be sure to include examples that involve carrying and borrowing. This will also provide you with an assessment tool by providing you with an opportunity to see who is able to complete these skills in their head.

$27-8=19$

On Day 2, when working with division and multiplication of whole numbers, it is time to review times tables. This can be done as a timed drill or simply with flashcards either as a whole group or in pairs.

Study Skill Tip

It would be helpful for students to keep a vocabulary notebook or have a separate section in their notebooks dedicated to vocabulary words and their definitions. This could also be a "picture dictionary" where students can draw examples. This will be useful when working with geometry concepts in later chapters.

Teaching Strategies

Each lesson begins with a real world story problem. These problems can be used to help the students to see how the content of the lesson applies to their lives. These engaging themes and problems use the same characters and students will engage in problem solving with the characters in each story problem. These problems are best used as an opener and closer to the lesson. Students can begin by grappling with the problem. Then they learn the content of the lesson, and finally at the completion of the lesson, the students see the solution to the problem with the characters. You can create a bulletin board where this story problem is posted at the beginning of the week. Then the solution is posted at the end of the week. Each day students can have a few minutes to work on trying to solve the story problem. This could be part of the warm –up for the lesson.

A teaching tip for addition and subtraction is to require the students to line up the digits according to place value. This will help to reinforce student understanding of place value as well as help them to keep the digits straight when carrying and borrowing.

Multiplying and dividing becomes simpler when students know their times tables. Continuing to review these on a regular basis will help students in their work. Again, work with students on the spacing of their digits. Organizing numbers in a clear way will help students to keep track of their work.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

Here are a few videos for students to watch on the four operations.

http://www.mathplayground.com/howto_regroupI.html - This video focuses on regrouping when subtracting.

http://www.mathplayground.com/howto_learnmultfacts.html - This is a video on learning times tables. It would be helpful for students who are still struggling with these facts.

http://www.youtube.com/watch?v=eKld7lQHKRg - This is a youtube video which looks at how to figure out long division in a new way. It uses a fifth grade math problem. A good review video for the beginning of sixth grade.

Whole Number Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Estimating sums and differences of whole numbers using rounding
- Estimating products and quotients of whole numbers using rounding
- Estimating to find approximate answers to real-world problems
- Using estimation to determine whether given answers to real-world problems are reasonable.

Relevant Review

When beginning this lesson, it is helpful to review operations with whole numbers. You can do this by creating four review problems of your own or by selecting four review problems from the *Time to Practice* in the last lesson. Because the students are going to be estimating whole number operations, it is helpful for them to review these skills.

Reviewing times tables is always helpful to students. This can be accomplished in a mental math review for five minutes at the beginning of each lesson.

Study Skill Tip

Be sure to have the students include the following vocabulary words into their notebooks.

Estimation
to find an approximate answer to a problem

Sum
the answer in an addition problem

Difference
the answer in a subtraction problem

Round
to change a number to a base ten, hundred or thousand etc.

Product
the answer to a multiplication problem

Quotient
the answer to a division problem

Factor
the numbers being multiplied in a problem

Compatible number

a number that is easily divisible by the divisor in an estimation problem.

Teaching Strategies

Read the real world story problem on *Penguin Estimation* as a whole class. This is a great way to begin the day. It will help to connect the students to real world applications of the material.

When teaching this lesson, be sure to take the time to review whole number computation using the four operations. This way the students have reviewed actually adding, subtracting, multiplying and dividing values *before* they are asked to estimate.

Asking students to copy down the rounding rule in their notebooks is an excellent way to help them to incorporate the rule to memory. You can also write the rounding rule on the board or write it on a card and put it up on a bulletin board. Rounding is the first method of estimation taught in this lesson.

Take the time to review place value when working on rounding to the nearest ten or to the nearest hundred or thousand. You can ask students why it makes sense to round to this value. You want students to come to the realization that it makes sense to round to the largest place. Rounding the tens place in a number in the thousands doesn't help you when you add, subtract, multiply or divide. You are estimating thousands, so you have to round to thousands.

Compatible numbers are tricky for students because they require students to think about what makes sense. If the number doesn't make sense for easy division, then we have to find one that does. Compatible numbers are very challenging for students who don't know their times tables. This is why it is necessary for students to really know those tables. Students with learning disabilities may need a times tables chart glued in their notebooks to help them.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/FV_MultiplicationFacts.html - This is a math video on math facts and has strategies for learning math multiplication facts. It is a great video for students who are still struggling with their times tables.

http://www.schooltube.com/video/ddfa72b3da74fc9ffa13/Math-Project-Using-Estimation-to-Solve-Problems - This is a schooltube video which shows students how they can use estimation to solve problems.

http://www.youtube.com/watch?v=qr-xHNNrZeA - This is a youtube video from yourteacher.com which shows how students can estimate sums and differences.

Powers and Exponents

Goal

The goals of this lesson can be found in the following objectives.

- Distinguish between a whole number, a power, a base and an exponent
- Write the product of a repeating factor as a power
- Find the value of a power
- Compare values of powers with different bases and exponents

- Solve real-world questions using whole number powers

Relevant Review

Review identifying whole numbers. You can use the term "counting numbers" to help students to remember the difference between a whole number and other types of numbers.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Whole number
a number that represents a whole quantity

Base
the whole number part of a power

Power
the value of the exponent

Exponent
the little number that tells how many times we need to multiply the base by itself

Squared
the power of the exponent 2

Cubed
the power of the exponent 3

Notation Note

Be sure to teach the students *how* you want them to write their exponents. Many students will write them the same size and this is challenging for teachers and students to distinguish which value is the base and which is the exponent. You can use "half as big" as a model for writing an exponent.

Teaching Strategies

The real world story problem *The Tiger* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When teaching the lesson, be sure to use the same color coding for the base and exponent as in the book. This will help the students to keep the two digits straight. After working with the concept for a little while in class, the color coding won't be as essential, but in the beginning stages, it is very helpful.

When referring to bases and exponents in terms of powers, have the students practice reading them aloud. For example, you can write the following on the board.

6^7 and ask the students to read the value as *six to the seventh power*. Asking them to read it out loud will help to reinforce the power and role of the exponent.

Pay close attention to the RED ALERT where students will try to multiply the base times the exponent. You will need to consistently remind students that the exponent tells you how many times to multiply the base by itself.

Evaluating values is an important concept that will continue throughout the text. Be sure that students use the term correctly.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.slideshare.net/taleese/powers-and-exponents - This is a series of slides where students can click through them to review all of the information on exponents presented in the lesson. It provides students with another way of accessing content.

Order of Operations

Goal

The goals of this lesson can be found in the following objectives.

- Evaluating numerical expressions involving the four arithmetic operations
- Evaluating numerical expressions involving powers and grouping symbols
- Using the order of operations to determine if an answer is true
- Inserting grouping symbols to make a given answer true
- Writing numerical expressions to represent real-world problems and solve them using the order of operations.

Relevant Review

Begin this lesson by reviewing the four operations and by reviewing the times tables with the students. You can do this in a variety of ways. You can choose to use review sheets or flashcards. Students can work on their times tables in a whole group or with a partner.

Review the previous day's lesson on exponents. Be sure that the students remember that the exponents tell you how many times to multiply the base by itself. Ask the students to evaluate exponents using a few examples.

$6^3 = 216$

$2^8 = 256$

Ask student to identify the base and the exponent.

Study Skill Tip

Be sure to have students write the following vocabulary words and the order of operations down in their notebooks.

Expression
an number sentence with operations and no equals sign.

Equation
a number sentence that can be solved. It has an equals sign in it.

Order of Operations
the order that you perform operations when there are more than one in an expression or equation.

P - parentheses

E - exponents

MD - multiplication/division in order from left to right

AS - addition and subtraction in order from left to right

Grouping Symbols

Parentheses or brackets. Operations in parentheses are completed first according to the order of operations.

Teaching Strategies

The real world story problem *The Aviary Dilemma* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When teaching the order of operations to the students, be sure to take your time teaching them without the exponents and the grouping symbols. If the students have had a good amount of time to practice the order of operations without the exponents and the grouping symbols, then their work will be more accurate when you incorporate the grouping symbols and the exponents into the lesson.

Now the students have practiced with the four operations and are ready to incorporate exponents and grouping symbols. Some students will think of grouping symbols as only parentheses. If you can break this habit here in the sixth grade, it will help the students greatly in the long run when brackets are also used as grouping symbols. Remind students of exponent rules as many times as possible.

When students work on inserting parentheses to make a statement true, they may become very frustrated. This is a great time to use individual white boards or small chalkboards so that the students can erase and try again. Encourage students to be patient while working on these problems.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/howto_pemdas.html - Here is a video on the order of operations. It begins without using exponents and grouping symbols just like the lesson.

http://www.brightstorm.com/math/algebra/pre-algebra/order-of-operations/ - This is a Brightstorm video on the order of operations using exponents and grouping symbols.

http://teachertube.com/viewVideo.php?video_id=79967 - This is a rap done by a teacher to help his students learn the order of operations. Video quality is not the best, but the rap is excellent.

Variables and Expressions

Goal

The goals of this lesson can be found in the following objectives.

- Evaluating single variable expressions with given values for the variable
- Evaluating multi-variable expressions with given values for the variable
- Using given expressions to analyze and solve real-world problems

Relevant Review

Review the previous lesson on the order of operations. Students always need time to go back and see past material again. You can also take five minutes at the beginning of class to work through some mental math. This is especially useful for those students who are still learning their times tables.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Evaluate
to figure out an expression that does not have an equals sign.

Variable
a letter, usually lowercase, that is used to represent an unknown quantity.

Expression
a number sentence that uses operations but does not have an equals sign

Variable Expression
a number sentence that has variables or unknown quantities in it with one or more operations and no equals sign.

Revenue
means money

Teaching Strategies

The real world story problem *The Ticket Revenue Dilemma* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson begins to use content that will remain without the students throughout the rest of their journey in mathematics. For many students, this is the first time that the concept of a variable, an unknown, will be introduced. Now students have been working with unknowns for a long time. Anytime they would solve a problem it would involve an unknown. The big difference is that this is the point in learning when the *variable* is used. Students will need to shift their thinking to make this connection.

Be sure to stress the importance of this shift to the *variable*. These students are middle schoolers now and are getting ready to learn mathematics that is more challenging. Assure them that they are up to the task.

It is recommended that you spend two days on this lesson. The first day will focus on single variable expressions. The second day will focus on multi-variable expressions. If you have a class with many special needs students, you may even wish to spend a third day on this lesson. It is building block for Algebra and therefore it is important for the students to have a good grasp of the content.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathvids.com/lesson/mathhelp/1232-variables-expressions-and-equations - This video is an advanced video. It not only works with expressions and variables, but also with beginning equations. This would be an excellent video for students who are looking for a challenge and are up to the task.

http://www.mathplayground.com/mv_defining_variables.html - This Brightstorm video is an excellent beginning video on defining variables in expressions.

A Problem-Solving Plan

Goal

The goals of this lesson can be found in the following objectives.

- Four Part Problem Solving Plan
  - Read and understand a given problem situation.
  - Make a plan to solve the problem.
  - Solve the problem and check the results
  - Compare alternative approaches to solving the problem.
- Solve real-world problems using this plan.

Study Skill Tip

Be sure to have students copy down the Four Part Problem Solving Plan in their notebooks.

Teaching Strategies

This lesson is different from the ones before it because it uses the real world problem in the Introduction to work through the Problem Solving Plan. *The Orangutan Adoption* will provide students with a way to practice the four parts of the Problem Solving Plan in a way that is relevant and connected to their lives.

Begin this lesson by reading the entire story problem aloud as a class.

In the *Teaching Time* section of the text, each part of the Problem Solving Plan is introduced. It is recommended that you work through the story problem in small groups. Assign the students each a small group to work in. Then present each step of the Problem Solving Plan. After presenting each step, allow students time to work with the story problem and this part of the plan.

Continue through each section of the Problem Solving Plan in this way. First, present the content from the text, then allow students time to work. This allows students the chance to use the Problem Solving Plan, not just hear about it.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Goal

The goals of this lesson can be found in the following objectives.

- How to read and understand a given problem situation
- How to develop and use the strategy: Guess, Check and Revise
- How to develop and use the strategy: Use Mental Math

- Plan and compare alternative approaches to solving problems
- Solve real-world problems using selected strategies as part of a plan.

Relevant Review

Review the Four Part Problem Solving Plan with the students prior to beginning this lesson. In this lesson, you will be combining the Four Part Problem Solving Plan with different strategies.

Teaching Strategies

This lesson takes the Four Part Problem Solving and applies it to the real world problem presented in the Introduction. *The Elephants Weigh In* provides the students with an opportunity to use different problem solving strategies in connection with the Four Part Problem Solving Plan.

Begin this lesson by having the students read the problem aloud as a whole group. Then divide the students into small groups to work through the problem.

In each group, first, use the content from the text to help students use each strategy in problem solving. The first strategy, Guess, Check and Revise will provide certain information. Ask students to work through the information in the text and apply it to the problem. Then allow time for sharing.

The second strategy will use the mental math. If you have been using mental math as a warm –up daily, then this provides the students with a chance to apply their mental math. Again, allow time for students to work in small groups and then encourage students to share their findings.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.phschool.com/atschool/academy123/english/academy123_content/wl-book-demo/ph-016s.html - This is a Pearson video on using Guess, Check and Revise when problem solving.

1.2 Statistics and Measurement

This second chapter *Statistics and Measurement* introduces students to all of the concepts associated with measurement. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Measuring Length
- Perimeter and Area
- Scale Drawings and Maps
- Frequency Tables and Line Plots
- Bar Graphs
- Coordinates and Line Graphs
- Circle Graphs and Choosing Displays
- Mean, Median and Mode

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Measuring Length –2 Days
- Perimeter and Area - 2 Days. The first day focuses on the figuring out the area and perimeter of the figures. The second day focuses on problem solving and using formulas to find missing dimensions when given the area or perimeter.
- Scale Drawings and Maps –1 Day
- Frequency Tables and Line Plots –1 Day
- Bar Graphs –1 Day
- Coordinates and Line Graphs –1 Day
- Circle Graphs and Choosing Displays –2 Days.
- Mean, Median and Mode –1 Day

Day 12 could be a review day.

Day 13 could be a test day.

Measuring Length

Goal

The goals of this lesson can be found in the following objectives.

- Measure length in customary units.

- Measure length using metric units.
- Choose appropriate tools given measurement situations
- Choose appropriate units for given measurement situations.

Required Materials

It is recommended that you have many rulers in customary units and metric units. Also that you have tape measures, yard sticks and meter sticks. It is helpful for students who learn in a kinesthetic way to be able to handle and use these tools.

Study Skill Tip

Be sure that the students copy the following vocabulary words in their notebooks.

Measurement
: using different units to figure out the weight, height, length or size of different things.

Length
: how long an item is

Customary units of length
: units of measurement such as inches, feet, yards and miles

Metric units of length
: units of measurement such as millimeter, centimeter, meter and kilometer.

Inches
: the smallest customary unit of measurement, measured best by a ruler

Feet
: a customary unit of measurement, there are 12 inches in 1 foot

Yards
: a customary unit of measurement, there are 3 feet in 1 yard

Miles
: a customary unit for measuring distances, there are 5280 feet in 1 mile

Millimeter
: the smallest metric unit of length

Centimeter
: a small metric unit of length, best measured by a ruler

Meters
: a unit compared with a foot or yard. 1 meter = a little more than 3 feet

Kilometer
: a metric unit for measuring distances

It would also be helpful to require students to draw a copy of a customary ruler and a metric ruler in their notebooks. This could be used to refer to when necessary.

Teaching Strategies

The real world story problem *The Tomato Plants* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Most students are familiar with customary rulers. They may be confused about how to measure each part of an inch, in other words, whole inch measurements and half inches are usually quite easy to distinguish, but $\frac{1}{4}$ *inch*, $\frac{1}{8}$ etc will be more challenging. When working on this section, make it as hands on as possible. By handing out rulers, you will encourage students to investigate the different measures themselves. Make a list of things that students will measure with their customary rulers on the board.

For example, a notebook, a pencil, an eraser, a math book, etc.

When moving on to metric measurement, you can hand out rulers with metric measurements on them. Again, the students can work with this content in a hands–on way. Provide students with the same list as they were given when working with customary units, then ask them to measure them in metric units. When finished, you can allow sharing for students to compare their findings.

One of the most challenging parts of this lesson is when the students are required to distinguish the correct units and the correct tools to use. This requires reasoning and thinking. Some students will be looking for the rules so that they can find the correct answers. Avoid doing this for them. Requiring students to reason and think through problems is a big part of middle school problem solving. This lesson provides you with an excellent vehicle for cultivating these skills.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/2_Estimation_of_Length/index.html - This video is a links learning video on measuring length.

http://www.ehow.com/video_4755255_measuring-length.html - This is an ehow video using a math teacher that focuses on measuring using customary units of measure. He refers to it as the English system and this should be clarified for students.

Perimeter and Area

Goal

The goals of this lesson can be found in the following objectives.

- Finding the perimeter of squares and rectangles using formulas
- Finding the area of squares and rectangles using formulas
- Solving for unknown dimensions using formulas when given the perimeter or the area
- Solving real-world problems involving perimeter and area, including irregular figures made of rectangles and squares.

Relevant Review

When beginning this lesson, it would be helpful to review customary units of measurement. Since this lesson does focus on area and perimeter, students will be using these units to find different measures.

Also, area uses multiplication. Taking five minutes at the beginning of each class to practice times tables as mental math will be helpful to all students.

Study Skill Tip

Be sure to ask students to copy the following vocabulary words into their notebooks.

Perimeter
the distance around the edge of a figure.

Square
a figure with four congruent sides

Formula
a way or method of solving a problem

Rectangle
a figure that has opposite sides that are congruent

Area
the space inside the edges of a figure

Dimensions
the measurements that define a figure

Also require that students write down the formulas for perimeter and area as they are presented in the lesson.

Teaching Strategies

The real world story problem *The Garden Plot* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. This story problem has the wonderful capacity of lending itself to drawings. Students can work to draw the garden plot in their notebooks or on large sheets of paper. Then these drawings can serve as an activity either in class or out of class. There is flexibility in the way that these real world problems are integrated into the class.

Throughout this lesson there are questions that are posed in the text. You can use these questions as thinking prompts for the students. By posing the questions, you require students to think about each situation. Then you can use the text to answer the questions which will correct or expand upon what the students have already shared.

Use the prompts in the text to help the students to copy the formulas into their notebooks. The students will see a picture of a small notebook on the student page. Alert them to this and support the students in noticing when these visual cues pop –up. This will help to create independent learners who are responsible for copying important information into their notebooks.

Solving for unknown dimensions using formulas is the problem solving part of this lesson. It requires thinking and not just using formulas. Students must “think backwards” to accomplish this task. I would recommend working on this section on Day 2 of the lesson. This gives the students time to practice using the formulas before they must use reasoning and apply them in “thinking backwards.”

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/howto_sameareadiffperimeter.html - This is a problem solving video which connects perimeter and area together. It would be an excellent video for students to use to see the relationship between perimeter and area.

http://www.youtube.com/watch?v=D5jTP-q9TgI - This is a youtube video on perimeter and area that uses a rock song to teach. It is very engaging for students.

Scale Drawings and Maps

Goal

The goals of this lesson can be found in the following objectives.

- Finding actual distances or dimensions given scale dimensions.
- Finding scale dimensions given actual dimensions.
- Solving real-world problems using scale drawings and maps.

Relevant Review

Because this lesson will work with scale in inches, it is necessary to begin by reviewing measuring in inches and parts of inches. This could be accomplished through a homework assignment the evening before or as a warm –up at the beginning of class.

Study Skill Tip

Be sure that the students copy the following vocabulary words down in their notebooks.

Scale
a ratio that compares a small size to a larger actual size. One measurement represents another measurement in a scale.

Ratio
the comparison of two things

Proportion
a pair of equal ratios, we cross multiply to solve a proportion

Teaching Strategies

The real world story problem *Alex's Garden Design* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. This story problem has the wonderful capacity of lending itself to drawings. Students can work to draw the garden plot in their notebooks or on large sheets of paper. Then these drawings can serve as an activity either in class or out of class. There is flexibility in the way that these real world problems are integrated into the class.

There are examples of maps and scales in this lesson, however, it will be more relevant to the students if you use maps from your area or state in addition to the ones referred to in the text. You can begin by presenting the content from the lesson in the text, and then move to new maps. This can be done in group work.

Teach the students how to find and read the scale. Then they can work on figuring out distances.

Once the students have figured this out, move to using the fraction bar to show them how to compare the scale with the actual measurement. This will help them when they move into solving for measurements using proportions. If the process is allowed to develop organically, then most students will naturally make the leap to solving the proportion for the accurate measure. Group work will also help to include those students who have difficulty with the concepts.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.dailymotion.com/video/xeajfp_how-to-find-the-measurement-of-scal_tech - This is a problem solving video using measurement and scale drawings. It has a real world problem that is solved in a blackboard presentation.

http://www.mathplayground.com/howto_ratios.html - This is a video on ratios. Ratios are a building block to this lesson.

http://pbskids.org/cyberchase/forreal/124_for_real.html - This is a fun pbs video on scale and measurement. It does require real player for viewing.

Frequency Tables and Line Plots

Goal

The goals of this lesson can be found in the following objectives.

- Make a frequency table to organize and display given data.
- Make a line plot given a frequency table.
- Make a frequency table and line plot given unorganized data.
- Collect, organize, display and analyze real-world data using frequency tables and line plots

Relevant Review

Because this lesson introduces a brand new concept, there is not any relevant review material.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Frequency
how often something occurs

Data
information about something or someone-usually in number form

Analyze
to look at data and draw conclusions based on patterns or numbers

Frequency table
a table or chart that shows how often something occurs

Line plot
Data that shows frequency by graphing data over a number line

Organized data
Data that is listed in numerical order

Teaching Strategies

The real world story problem *Working in the Garden* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson begins with an introduction to the term *data*. You can begin by brainstorming all of the different places where people may see data.

There are three steps for students to keep track of when working with frequency tables and line plots. The first step is to collect the data. The second is to organize it into a table. The third is to draw a line plot to display the data. You can write these three steps on the blackboard or on a bulletin board as a visual reminder for students. Many will want to skip the second step of organizing the data. This will produce a display that is not accurate. Encourage students to follow the steps.

Key teaching regarding to the line plot is to be sure that the intervals are even. Here is where the students can once again use their rulers. Decide on the measure of the intervals and then ask the students to create the line plot before putting in the data.

Finally ask students to check each other's work to be sure that the intervals and data marks match the data in the frequency table. The line plot and the frequency table should match.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.ehow.com/video_4738954_make-frequency-table.html - This is an ehow family video on how to make a frequency table.

http://www.youtube.com/watch?v=G6eTMRXHhmE - This is a youtube video from yourteacher.com which focuses on pictographs and line plots.

Bar Graphs

Goal

The goals of this lesson can be found in the following objectives.

- Make a bar graph to display given data.
- Make a double bar graph to display and compare given data.

- Make multiple bar graphs to display and compare given data.
- Collect, organize, display and analyze real-world data using frequency tables and bar graphs.

Relevant Review

Begin this lesson with a review of the data discussion from the previous lesson. Help the students to make connections between the real world and all of the places where we rely on data.

Because this lesson also requires students to use frequency tables, it would be helpful to review frequency tables with the students. Also, review scale with the students. Students will need to use their reasoning skills to select a scale when creating a bar graph or a double bar graph.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Bar graph
a way to organize data using bars and two axes. One axis represents the number of each item and the other axis represents the item that was counted.

Double Bar Graph
A graph that has two bars for each item counted. It still uses a scale, but is designed to compare the data collected during two different times or events. A double bar graph is a tool for comparisons.

Teaching Strategies

The real world story problem *The Harvest* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When creating a double bar graph, use the color coding that is presented in the text. This will help the students to make connections between the colors and the data. You can even have them create a copy of the same bar graph as the one in the text. This is a way to practice before they are required to make their own.

Most students really enjoy creating bar graphs. Point out that in the single bar graph and in the double bar graph that the size of the bars needs to be the same. This is a place where a ruler is a very handy tool.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=6GK5176auVw - This is a math made easy video on how to create a bar graph.

http://www.teachersdomain.org/resource/vtl07.math.data.rep.inventbarg/ - This is a pbsCyberchase video on bar graphs. You will require real player for viewing.

Coordinates and Line Graphs

Goal

The goals of this lesson can be found in the following objectives.

- Identify elements of a coordinate grid (origin, vertical and horizontal axes, ordered pairs.)
- Graph given points on a coordinate grid (1^{st} Quadrant)
- Make a line graph to display given data over time.
- Collect, organize, display and analyze real-world data using line graphs.

Relevant Review

There isn't any review necessary for beginning this lesson.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Coordinate grid
a visual way of locating points or objects in space.

Coordinates
the x and y values that tell us where an object is located.

Origin
where the x and y axis meet, has a value of (0, 0)

X **axis** - the horizontal line of a coordinate grid

Y **axis** - the vertical line of a coordinate grid

Ordered pair
$(x; y)$ the values where a point is located on a grid

Line Graph
a visual way to show how data changes over time

Teaching Strategies

The real world story problem *Garden Data and Graphs* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are questions about the content of the lesson posed to the reader throughout the text. You can use these questions as discussion points prior to teaching the content. In this way, the students will engage with the material on a different level. They will be working with the material in such a way as to be thinking and problem solving.

Notice also that there are key words used to describe each type of graph. In this lesson, the line graph is described as showing a change in data over time. This is an important thing for students to remember. Be sure to have them make a note of it in their notebooks.

When creating line graphs, refer students back to the horizontal and vertical axis that is part of the coordinate grid. This will help them to make a connection between the two. The coordinate grid and the line graph is similar. Point out that while the two are similar that they are different too. The coordinate grid uses points to show where something is in space. The line graph uses points to show how data has changed over time.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for

the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=E2FqyJL2w8U - This is a math made easy video on youtube which shows students how to identify coordinates by looking at points that have been graphed on the coordinate grid.

http://www.mathplayground.com/locate_aliens.html - This is an interactive activity where students use coordinates to locate aliens that have been graphed on the coordinate plane. Very fun and interactive!

http://www.youtube.com/watch?v=K7d_i3NU2dM - This is a yourteacher.com video on youtube which shows students how to create a line graph.

Circle Graphs and Choosing Displays

Goal

The goals of this lesson can be found in the following objectives.

- Interpret given circle graphs
- Use circle graphs to make predictions
- Use data from a circle graph to make a bar graph
- Select among frequency tables, line plots, bar graphs and line graphs for best displays of given data.

Relevant Review

It is useful to review the topic of data and to have students continue to talk about different forms of data or to identify places where data is displayed in everyday life. This could be an ongoing assignment through this lesson or it could be an enrichment project.

Review decimals and percentages because these are used in this lesson as they are related to circle graphs. Key things to focus on would be that a decimal is in the base ten system. Review place value with one decimal example. Then talk about how percents are seen as out of 100. Then you can show a conversion.

For example:

$.45 = \frac{45}{100} = 45\%$

Review frequency tables, line plots, bar graphs, and line graphs. Discuss how to make them and the types of data that are best displayed on them.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Circle graph
a visual display of data that uses percentages and circles.

Decimals
a part of a whole represented by a decimal point.

Percentages
a part of a whole written out of 100 using a % sign

Predictions
to examine data and decide future events based on trends.

Teaching Strategies

The real world story problem *Vegetable Totals* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson begins by assigning students the task of interpreting circle graphs that have been designed for them. This will help student to think in terms of a whole. They first look at the whole and then think about the parts. It makes more sense than having them start with the parts and then work towards the whole.

Be sure that you have taken some time reviewing decimals and percentages. This will help to keep you from going forwards and backwards. When students have reviewed the relevant content, then it helps them to make progress when new material is presented.

I recommend spending the first day on the first three objectives of this lesson as they pertain to circle graphs. In this way, students will have a full class of working with circle graphs as they learn to interpret them and use them to make bar graphs. On the second day, you can work on selecting among all of the different data displays.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/piechart.html - This is an interactive math page about creating a circle graph. It walks students through using data to design a circle graph.

http://static1.tenmarks.com/static/albums/Collecting-Displaying-and-Analyzing-Data/Circle-Graphs-Line-Graphs-and-Scatter-Plots-video-lesson.html - This is a video that goes through creating several different types of graphs. Could be used for an advanced student looking for more challenge.

Mean, Median and Mode

Goal

The goals of this lesson can be found in the following objectives.

- Find the mean of a set of data.
- Find the median of a set of data.
- Find the mode of a set of data.
- Identify the range of a set of data.
- Select the best average to represent given sets of data.

Relevant Review

It would be useful to review the four operations especially multiplication and division prior to completing this lesson because these measures require students to use the four operations.

Study Skill Tip

Be sure to have students copy down these vocabulary words in their notebooks.

Mean
: the average of a set of numbers. The mean gives us a good overall assessment of a set of data.

Maximum
the greatest score in a data set

Minimum
the smallest score in a data set

Median
the middle score in a data set

Mode
the number or value that occurs most often in a data set

Range
the difference between the smallest value in a data set and the greatest number in a data set

Measures of Central Tendency
a way of selecting which value in a data set best expresses the set of data.

Teaching Strategies

The real world story problem *The Carrot Review* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The key to this lesson is in the student's ability to remember the vocabulary introduced. When the students are working with the vocabulary words and have memorized each one and what it stands for, then their work will be accurate. For example, if a student does not remember what the word "mean" refers to or how to find the mean, then their work will not be accurate. Once they have learned each term, then it is simply a matter of practice.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=81zcjULlh58 - This is a video on youtube by yourteacher.com which teaches students all about the different measures of central tendency.

1.3 Addition and Subtraction of Decimals

This third chapter will help students develop an understanding of decimals as they learn to add and subtract them. It will also help to reinforce the work on place value that was started in the initial chapter of this book. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Decimal Place Value
- Measuring Metric Length
- Ordering Decimals
- Rounding Decimals
- Decimal Estimation
- Adding and Subtracting Decimals
- Stem-and-Leaf Plots
- Use Estimation

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Decimal Place Value –1 Day
- Measuring Metric Length –1 Day
- Ordering Decimals –1 Day
- Rounding Decimals –1 Day
- Decimal Estimation –1 Day
- Adding and Subtracting Decimals –2 Day
- Stem-and-Leaf Plots –1 Day
- Use Estimation –1 Day

Day 10 Review

Day 11 Test

Decimal Place Value

Goal

The goals of this lesson can be found in the following objectives.

- Express numbers given in words or hundredths grids using decimal place value.
- Express numbers in expanded form given decimal form.

- Read and write decimals to ten-thousandths place.
- Write combinations of coins and bills as decimal money amounts.

Relevant Review

Review that when we are talking about whole numbers that we are talking about counting numbers. Ask the students to name some whole numbers.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Whole number
a number that represents a whole quantity

Decimal
a part of a whole

Decimal point
the point in a decimal that divides parts and wholes

Expanded form
writing out a decimal the long way to represent the value of each place value in a number

Teaching Strategies

The real world story problem *The Ice Cream Stand* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Many students will have used a hundreds grid at some point in time and so using one will be familiar to them. One of the key things to point out is that the values to the right of the decimal point represent the parts of a whole and we use *ths* to show this when we write the decimal in words.

Take the time to ask the students to name the decimals and also to write the names of the decimals out in words. In this way, you can review and practice place value at the same time. You will find that many students will mix up the words even though you just taught it a few minutes prior. You can use this to help to reinforce the accurate wording and reading of each decimal.

Once students have some understanding of the value of each digit to the right of the decimal point, then you can add in the whole numbers which are found to the left of the decimal point. Once the digits on both sides of the decimal point have been explained, then it is time to practice, practice, practice.

One suggestion is to have students work in small groups and try to stump each other. The students can take turns saying different numbers and the others write the value of the decimal in words. Then they can self-correct their work. This is also a way to have students who have a strong understanding of the concepts help their peers.

Working with money is fun especially if you have pretend money for students to work with. Using some pretend money will turn this section into a fun hands–on activity.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.ehow.com/video_4754298_understanding-decimal-place-value.html - This is a video on decimals and place value.

http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/3_Place_Value/ - This is an excellent Links learning video on place value.

Measuring Metric Length

Goal

The goals of this lesson can be found in the following objectives.

- Adding Whole Numbers
- Subtracting Whole Numbers
- Multiplying Whole Numbers
- Dividing Whole Numbers

Relevant Review

A relevant review for this lesson would be to review the basics of customary measurement. You can talk about how the customary system of measurement is the one that is used most often in the United States, but that if you travel to other countries or work in the Sciences that you will need to understand the Metric system of measurement.

Study Skill Tip

Be sure to have the students copy down the following vocabulary words in their notebooks.

Metric System
a system of measurement more commonly used outside of the United States

Length
the measurement of a object or distance from one end to the other

Millimeter
the smallest metric unit of measuring length, found on a ruler

Centimeter
a small unit of measuring length, found on a ruler

Meter
approximately 3 feet, measured using a meter stick

Kilometer
a measurement used to measure distances, the largest unit of measuring length

Also have students copy down the following chart into their notebooks.

Here is a chart to help us with the conversions.

$$1\ km \quad 1000\ m$$
$$1\ m \quad 100\ cm$$
$$1\ cm \quad 10\ mm$$

There are also rules regarding which tools to use in different situations. Here are some rules for students to memorize.

If the object is very tiny, use a ruler for millimeters.

If the object is less than 100 cm use a ruler for centimeters.

If the object is greater than 100 cm use a meter stick.

If the object is greater than 100 meters, use a metric tape measure.

If the object is a distance, use a kilometer odometer.

Teaching Strategies

The real world story problem *The Kid's Area* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

One of the biggest things that will help students in working with conversions and with metric units of measurement is to remember that when we convert from smaller units to larger units we divide and when we convert from larger to smaller units, we multiply. If you can helps students to remember these two things then a lot of the difficulty with metrics can be avoided.

That being said, they must understand the differences in the sizes of the units. Millimeters is smaller than meters. You can ask students to try to figure out how much smaller. Be sure that you have a ruler with millimeters and centimeters on it. Then have a meter stick. This is a way to provide the students with a hands–on example of the difference.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/converting-within-the-metric-system?playlist=Developmental%20Math - This is a khan academy video which teaches students how to convert units within the metric system.

http://www.mathplayground.com/howto_Metric.html - This is a math playground video on understanding the metric system.

Ordering Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Comparing Metric lengths
- Comparing decimals

- Ordering decimals
- Describing real-world portion or measurement situations by comparing and ordering decimals.

Relevant Review

Prior to beginning this lesson, it would be helpful to have students review place value as it pertains to decimals. Students will need to be able to identify each place of a decimal including the whole number part of a number which is to the left of the decimal point. Here is an example to begin the lesson.

6.4691

Possible leading questions:

1. Which digit is in the tenths place?
2. What is the place value of the nine?
3. What is the place value of the six?

By beginning with an example like this one, color coded, you will help everyone to review including students who face learning challenges.

Review the symbols that are used for greater than, less than, greater than or equal to and less than or equal to. In fact, you may want to write them in a corner of the board for easier reference.

Study Skill Tip

Be sure to have the students copy down the following vocabulary words in their notebooks.

Equivalent
means equal

Comparing
using greater than, less than or equal to so that we can compare numbers

Decimals
a part of a whole represented by a decimal point

Order
writing numbers from least to greatest or greatest to least

Teaching Strategies

The real world story problem *Sizing Up Ice Cream Cones* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

A big tip for students when working with this lesson is for them to add zeros so that decimals that are being compared have the same number of digits. When the decimals have the same number of digits, the decimals can be looked at as a whole. The comparisons are simpler.

For example:

.45 _____ .067

One decimal has two digits after the decimal point and the other has three. If students learn to add a zero to the first decimal to make it three digits, then it is easier to compare the two quantities.

.450 _____ .067

.45 is >.067

Adding zeros is also helpful when ordering decimals from least to greatest and from greatest to least. By making sure that each decimal has the same number of digits, students will be able to figure out the accurate order in simpler way.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=JKxde_7YLlY - This is a math video from yourteacher.com on ordering decimals.

Rounding Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Round decimals using a number line.
- Round decimals given place value.
- Round very small decimal fractions to the leading digit
- Round very large numbers to decimal representations of thousands, millions, etc.

Relevant Review

Begin this lesson by reviewing number lines with whole numbers with the students. You can use the reference of a ruler to help students to think about number lines. Students have had practice using rulers and measuring in earlier lessons. This brings past information into the current lesson and helps them to use already acquired information. Students can then move to identifying numbers on a number line.

Begin this lesson with a five minute review on place value and digit value as it connects to decimals. Be sure that students can identify the value of each digit to the right of the decimal point by using tenths, hundredths, thousandths, etc.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Round
: to use place value to change a number whether it is less than or greater than the digit in the number

Decimal
: a part of a whole written to the right of a decimal point. The place value of decimals is marked by THS.

Leading Digit
: the first digit of a tiny decimal that is not a zero

Small decimals
: decimals that have several zeros to the right of the decimal point before reaching a number.

Here are a few rules for students to copy down in their notebooks regarding rounding decimals.

1. Look at the digit to the right of the place value you are rounding.
2. If the digit to the right is a five or greater, you round up.
3. If the digit to the right is less than 5, you round down.

Teaching Strategies

The real world story problem *The New Ice Cream Sign* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The big teaching tip in this lesson is about place value. If students do not understand place value, then rounding decimals is very challenging. They must learn the rules and the places of each digit to the right and left of the decimal point. This will be their biggest help in rounding decimals.

The idea of a leading digit is one that is probably new to most students. Remind students that it is the first digit in a decimal that is not a zero. You can have them practice identifying leading digits by writing a few examples on the board. Here are a couple.

.098

.123

.000034

Ask students to identify the leading digit and then discuss why it is or isn't a leading digit.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/rounding-decimals?playlist=Developmental%20Math - This is Khan Academy video on rounding decimals.

http://www.youtube.com/watch?v=aIbMWR5Ft9M&feature=relmfu - This is a video by yourteacher.com on rounding decimals.

Decimal Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Estimate sums and differences of decimals using rounding
- Estimate sums and differences of decimal numbers using front –end estimation
- Compare results of different estimation methods
- Approximate solutions to real-world problems using decimal estimation

Relevant Review

Review the word *estimation* and talk with the students about what it means to estimate. You can talk about how an estimation must "make sense" or be reasonable. What does that mean? This can begin a very lively discussion at the beginning of the lesson. Identify real world examples of estimation, you can even remind students of the zoo problems from an earlier chapter. These problems required whole number estimation, in this lesson, the students are going to work with decimal estimation.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Estimate
to find an answer that is reasonable and close to an exact answer.

Sum
the result of an addition problem

Difference
the result of a subtraction problem

Front end estimation
estimating by adding the front ends of each number in the problem, then rounding and adding the decimal parts of each number. Works well with smaller numbers

Rounding
converting a number to its nearest whole number. Works well with larger numbers

Teaching Strategies

The real world story problem *Recycling* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

One of the key teaching tips for this lesson is to help students to see how helpful estimation can be when finding a sum. It is quick and easy. The answer is often close to the real one and students can find a solution and move on. When students really understand the value of estimating, then they will use estimation more readily.

When you work with estimating sums, then you can also review the rules of rounding. Rounding is the first estimation method taught. We want the students to be able to use the rules of rounding easily to estimation.

The second method taught is front –end estimation. This one is trickier for students to grasp. Ask students to copy the following rules down in their notebooks.

Here are the steps for front –end estimation.

1. Keep the digits of the two highest place values in the number.
2. Insert zeros for the other place values.

Here are the steps to front –end estimation with decimals.

1. Add the front digits of the numbers being added or subtracted.
2. Round off the decimals of the numbers being added or subtracted.
3. Combine or subtract the results.

Use the rules in the lesson to help you as you guide students through the content. There is even a part where a character in the lesson says "That sounds confusing" you can use this as an opportunity to slow down and work through the example at a pace that is slow enough to include everyone.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/estimation-with-decimals?playlist=Developmental%20Math - This is a Khan Academy video on estimation with decimals.

http://www.youtube.com/watch?v=_aJPI-Z-DFA - This is a Math Made Easy video on estimation with decimals and whole numbers.

Adding and Subtracting Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Adding and Subtracting Decimals by rewriting with additional zero place holders.
- Using mental math to add/subtract decimals
- Identifying the commutative and associative properties of addition in decimal operations, using numerical and variable expressions
- Solving real world problems involving decimal addition and subtraction

Relevant Review

Begin this lesson with a review of place value. If you have been consistent in this review each day, then the review should be short at this point. There may be a few children who are still struggling with place value, pair those children up with others who have the concept firmly in hand.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Properties
a rule for specific mathematical situations.

Associative Property
a property that states that changing the grouping in an addition problem does not change the sum.

Commutative Property
a property that states that changing the order of the numbers in an addition problem does not change the sum.

Teaching Strategies

The real world story problem *The Broken Cash Register* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day

where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Point out to students that when you need an EXACT answer that you can not use estimation. This may seem to make perfect sense, but you would be surprised how many students won't realize it. You can use Julie, the character from the story problem as an example. Exact answers will require you to find a sum or a difference. Sums involve adding and differences require subtraction. Be sure to have student copy these two definitions down in their notebooks. They can include the symbols (+ and -) in their notebooks as well.

Rule for students is to line up the decimal points and each digit when adding and subtracting decimals. This is for vertical addition and subtraction. Use color coding on the board to show students how to do this. For example, you can write a decimal point in red for simple identification for students.

Some students will notice that some examples have different numbers of digits. Just as they did when comparing and rounding, students can add zeros to the end of the numbers so that the numbers that they are adding and subtracting have the same number of digits. Practice this a lot until the students remember to do it without your reminding them. The practice of adding zeros to create equal numbers of digits helps avoid many mistakes.

The introduction of properties can sometimes confuse students. Use this lesson as a simple introduction. Students will continue to work with them throughout higher mathematics.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/adding-decimals?playlist=Developmental%20Math - This is a Khan Academy video on adding decimals.

http://www.khanacademy.org/video/subtracting-decimals-word-problem?playlist=Developmental%20Math - This is a Khan Academy video on subtracting decimals. It is a great example that involves a word problem.

http://www.khanacademy.org/video/subtracting-decimals?playlist=Developmental%20Math - This is a Khan Academy video that teaches students how to subtract decimals. It does cover borrowing in subtraction.

Stem-and-Leaf Plots

Goal

The goals of this lesson can be found in the following objectives.

- Organize a set of data in a stem-and-leaf plot.
- Use a stem-and-leaf plot to find the range of a set of data.
- Use a stem-and-leaf plot to find the mean, median and mode of a set of data.

Relevant Review

Review place value prior to beginning this lesson. Be sure that students review the place value of digits found to the left of the decimal point. In other words, students will be required to organize values according to tens, hundreds, thousands, etc in this lesson. Be sure that they can identify these digits in place value.

Review mean, median, mode and range. Students will be required to identify and use these values in connection with stem-and-leaf plots.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Stem-and-leaf plot
a way of organizing numbers in a data set from least to greatest using place value to organize.

Data
information that has been collected to represent real life information

Ascending
from smallest to largest

Descending
from largest to smallest

Interval
a specific period or arrangement of data

Range
the difference from the largest value to the smallest value

Teaching Strategies

The real world story problem *Ice Cream Counts* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Once students have a good grasp of place value, many enjoy creating stem-and-leaf plots. First, it makes sense to them and it is a visual display of the data. You can use the real life story problem as the model throughout this whole lesson.

One helpful hint to point out is that repeated values are listed in the chart. You can talk with the students about having every value represented. An example of this that would directly relate to students is test scores. If you scored a 98% twice, you would want both of them to count. Just because a value is repeated, they are both important.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/stem-and-leaf-plots?playlist=ck12.org%20Algebra%201%20Examples - This is a Khan Academy video on stem-and-leaf plots. While some of the data moves into Algebra I content, it is still a useful video. It would be beneficial for an advanced student or one who is seeking challenge.

http://www.mathplayground.com/howto_stemleaf.html - This is a good basic video that teaches students how to read a stem-and-leaf plot.

Use Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations
- Develop and use the strategy: Use Estimation
- Plan and compare alternative approaches to solving problems
- Solve real-world problems using selected strategies as part of a plan

Relevant Review

Begin this lesson with a review of the four part problem solving plan.

Review estimation and the two ways of finding an estimate by using rounding and by using front –end estimation.

Study Skill Tip

The only study skill tip for this lesson is to be sure that the students remember how to round and how to use front –end estimation when finding sums and differences.

Teaching Strategies

The real world story problem *Summer Job Benefits* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson is all about practice. It is about the application of the concepts taught in earlier lessons. I recommend having students work through this lesson on their own. They can work in small groups. Then you will be able to walk around and observe student reasoning and the application of the skills learned. Students can ask questions as they work.

Allow time for students to present their findings at the end of the lesson. This is especially important for explaining *how* each group solved the problems.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

1.4 Multiplication and Division of Decimals

This fourth chapter *Multiplication and Division of Decimals* helps students continue on their journey into the world of decimals. Having practiced adding and subtracting decimals, we now move on to working with the operations of multiplication and division. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Multiplying Decimals and Whole Numbers
- The Distributive Property
- Multiplying Decimals
- Dividing Decimals by Whole Numbers
- Multiplying and Dividing Decimals by Powers of Ten
- Dividing by Decimals
- Metric Units of Mass
- Converting Metric Units

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Multiplying Decimals and Whole Numbers –1 Day
- The Distributive Property –2 Days (one on the property, one on finding areas)
- Multiplying Decimals –2 Days (one on multiplying decimals, one on area)
- Dividing Decimals by Whole Numbers –1 Day
- Multiplying and Dividing Decimals by Powers of Ten –1 Day
- Dividing by Decimals –1 Day
- Metric Units of Mass –1 Day
- Converting Metric Units –1 Day

Day 11 Review

Day 12 Test

Multiplying Decimals and Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Multiply decimals by whole numbers
- Use and compare methods of estimation to check for reasonableness in multiplication of decimals by whole numbers

- Identify and apply the commutative and associative properties of multiplication in decimal operations, using numerical and variable expressions.
- Solve real-world problems involving decimal multiplication

Relevant Review

Begin this lesson with a brief review of estimation. Key points are that students can use estimation when they don't need an exact answer. However, the answer must be reasonable. Ask students to define the word "reasonable".

Review the properties of the commutative and associative properties. Commutative property mixes up the order of the values. The associative property mixes up the groupings, often using parentheses.

Study Skill Tip

Be sure that students copy down the following vocabulary words in their notebooks.

Multiplication
a shortcut for addition, means working with groups of numbers

Product
the answer from a multiplication problem

Estimate
an approximate answer-often found through rounding

Properties
rules that are true for all numbers

The Commutative Property of Multiplication
it doesn't matter which order you multiply numbers, the product will be the same.

The Associative Property of Multiplication
it doesn't matter how you group numbers in a multiplication problem, the product will be the same.

Teaching Strategies

The real world story problem *The Science Museum Field Trip* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Once students understand that when multiplying a decimal by a whole number, that we are taking that decimal and repeating it, then you can move on to estimation. Your goal in teaching this lesson is to help students to see that if they are able to estimate the decimal to a whole number, that then their multiplication is simple because they are simply multiplying whole numbers.

Example:

3.89×4

3.89 is close to 4, so we can round to 4. This is our decimal estimation.

Now it is simply a matter of multiplying.

$4 \times 4 = 16$

Our estimate is 16. It is reasonable estimate.

Even though you reviewed the properties prior to starting the lesson, be sure that the students understand them when working through this last section of the lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/multiplying-decimals?playlist=Developmental%20Math - This is a Khan Academy video on multiplying decimals when working with a word problem.

The Distributive Property

Goal

The goals of this lesson can be found in the following objectives.

- Write numerical expressions for the product of a number and a sum
- Identify and apply the Distributive Property to evaluate numerical expressions
- Evaluate products using mental math.
- Apply the Distributive Property to evaluate formulas using decimal quantitites.

Relevant Review

Because this lesson requires students to use mental math to evaluate products, it is a good time to review the times tables. This can be done through a timed test, through partner work with flash cards or through a five minute "spelling bee" type of game. Either way, the times tables will be reviewed.

The last section on area has an example that involves multiplying decimals by whole numbers. Review this skill from the last lesson so that students can apply what they have already learned to this example.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Numerical expression
a number sentence that has at least two different operations in it.

Product
the answer in a multiplication problem

Sum
the answer in an addition problem

Property
a rule that works for all numbers

Evaluate
to find the quantity of values in an expression

The Distributive Property

the property that has you take the product of the sum of two numbers. You take the number outside the parentheses and multiply it by each term in the parentheses.

This lesson also uses the formula for area of a rectangle. Have students copy these formulas down in their notebooks.

$A = lw$

Teaching Strategies

The real world story problem *The Omni Theater Dilemma* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In this lesson, students will begin learning how to write numerical expressions from words when there are parentheses involved. To help students with this, remind them of some key words that will help to identify different operations and to identify when parentheses are required.

Sum means addition

Product means multiplication

Difference means subtraction

Quotient means division

When students see the words "and the" with two operations, they can be sure that there is a set of parentheses required to help students to identify where the parentheses are placed.

The Distributive property makes sense when students understand the word "distribute". When students can see that "to distribute" means to hand–out or pass out, then "handing out" numbers makes sense. Remind students that a property is a rule, a way of doing something in mathematics.

When working with area, remind students that area is the inside space of a figure. Then show how $units \times units$ is always square units. Students can practice these skills when working with the examples in the text.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/the-distributive-property?playlist=Developmental%20Math - This is a Khan Academy video on the distributive property.

http://www.youtube.com/watch?v=u3dexXyyb3c - This math video is a youtube video that is an introduction to the distributive property. It is a good basic video for students.

Multiplying Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Multiply decimals by decimals using area models (hundredths grid).

- Place the decimal point in the product and confirm by estimation.
- Multiply decimals up to a given thousandths place.
- Solve real-world problems involving area of rectangles with decimal dimensions.

Relevant Review

Begin this lesson with a review of multiplying decimals by whole numbers. This will provide students with a foundation for working with multiplying decimals with other decimals.

Review area and how to find the area of a rectangle. Review the formula for area of a rectangle as $A = lw$.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Hundreds grid
a grid of one hundred boxes used to show hundredths when working with decimals.

Product
the answer in a multiplication problem.

Vertically
written up and down in columns

Horizontally
written across

Area
the surface or space inside a perimeter

Teaching Strategies

The real world story problem *The Triceratops Skeleton* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

One of the best ways to help students work with models is to have them construct their own models. This website http://www.teachervision.fen.com/graphs-and-charts/arithmetic/44543.html has a download of a hundreds grid that you can download, print and then copy for the students. I would recommend copying five per student. Then when you work through problems that require students to multiply decimals written in the hundredths, you can help them create models to show the product of the decimals that they are multiplying. This is a hands–on way to work with the information in the first section of the lesson.

When multiplying and then checking work through estimation, students will often avoid estimation because it involves an additional step. They will multiply and then want to be done. It is important for you to help them to complete this step anyway because it is an excellent way to teach students to check their work. Then they can determine if their answer is reasonable or not.

In the last lesson, students worked on the finding the area of rectangles. Now they are going to apply this information to finding the area of rectangles when there are decimal lengths and widths involved. Remind students that area is always measured in square units. That can be square feet, square inches, square meters, etc.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for

the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/area-and-perimeter?playlist=Geometry - This is a Khan Academy video on area and perimeter. It covers area and perimeter of squares, rectangles and also triangles.

http://www.khanacademy.org/video/multiplying-decimals?playlist=Developmental%20Math - This is a Khan Academy video on multiplying decimals.

Dividing by Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Divide decimals by whole numbers.
- Find decimal quotients of whole numbers using additional zero placeholders.
- Divide decimals by whole numbers and round to a given place.
- Solve real-world problems involving the division of decimals by whole numbers.

Relevant Review

Because this lesson teaches students to divide decimals, an excellent review would be to review the basics of dividing whole numbers. Key words indicating division are quotient, split up and divided.

Review the rules of rounding because students will use rounding once again when they divide decimals and whole numbers.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Divide
to split up into groups evenly.

Divisor
a number that is doing the dividing. It is found outside of the division box.

Dividend
the number that is being divided. It is found inside the division box.

Quotient
the answer in a division problem

Teaching Strategies

The real world story problem *The Discount Dilemma* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When teaching this lesson, the vocabulary is very important. Students will need to be able to understand and identify the dividend, the divisor and the quotient. Quotient will be the easiest one to remember because most students have

heard this term in earlier lessons. Students are often confused with the dividend and the divisor. You can tell them that the "divisor" does the work. It "divides" into the dividend. Then the resulting answer is our quotient.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/dividing-decimal?playlist=Arithmetic - This is a Khan Academy video on dividing decimals.

http://www.mathplayground.com/howto_dividedecimals.html - This is a math playground video that helps students learn the necessary steps for dividing decimals.

Multiplying and Dividing by Decimal Powers of Ten

Goal

The goals of this lesson can be found in the following objectives.

- Use mental math to multiply decimals by whole number powers of ten.
- Use mental math to multiply decimals by decimal powers of ten.
- Use mental math to divide decimals by whole number powers of ten.
- Use mental math to divide decimals by decimal powers of ten.
- Write in scientific notation.

Relevant Review

This lesson requires a lot of mental math. This is an excellent opportunity to review the times tables through multiplication and simple division. This can be done by having students work with partners, by using flash cards or by using a "spelling bee" type of game.

Review exponents since this lesson requires students to think about and use positive and negative exponents.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Power of ten
: 10, 100, 1000, 10,000 - you can think of them as multiples of ten.

Scientific notation
: a way to write decimals and numbers by writing a number sentence that shows a power of ten using an exponent.

Whole numbers have positive exponents.

Decimals have negative exponents.

Teaching Strategies

The real world story problem *The Earth's Diameter* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where

the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The real world story problem provides students with a real application of scientific notation, exponents and negative exponents. It shows why scientific notation can be so useful especially in terms of science. When students have an understanding of why certain mathematical concepts are important, it helps them find math valuable and want to use it.

This lesson is great because it is about short cuts and students love short cuts. It will also help you as a teacher to evaluate each student's understanding of place value. Moving decimal points can be challenging if you don't understand the base ten system. Use this to assess student understanding and provide remedial help to students who haven't grasped this concept.

Key things to help students remember:

Multiplying by base ten we move the decimal point to the right.

Dividing by base ten we move the decimal point to the left.

Students should be able to distinguish the patterns in the text. You also want them to conclude that multiplying by base ten creates a larger number and dividing creates a smaller number.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/dividing-a-decimal-by-a-power-of-10?playlist=Developmental%20Math - This is a Khan Academy video about how to divide a decimal by a power of ten.

http://www.khanacademy.org/video/multiplying-a-decimal-by-a-power-of-10?playlist=Developmental%20Math - This is a Khan Academy video about how to multiply a decimal by a power of ten.

http://www.mathplayground.com/howto_dividedecimalspower10.html - This is a math playground video on how to divide a decimal by a power of ten.

Dividing by Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Divide decimals by decimals by rewriting divisors as whole numbers.
- Find quotients of decimals by using additional zero placeholders.
- Solve real-world problems involving division by decimals.

Relevant Review

Begin this lesson with a review of how to divide a decimal by a whole number. In this lesson students will be learning how to divide decimals by decimals. This review is important so that students can build on what they have already learned.

Review dividing by 10's because in this lesson it is used to make the divisor into a whole number. If you have reviewed this at the beginning of class, it will help the students to use this information when they arrive at new problems.

Study Skill Tip

Be sure to ask students to copy the following vocabulary words down in their notebooks.

Divisor
the number doing the dividing, it is found outside of the division box.

Dividend
the number being divided. It is found inside the division box.

Quotient
the answer in a division problem

Teaching Strategies

The real world story problem *The Sand Experiment* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When you teach the section on using zero placeholders, take your time to help students to understand that now we are working in parts. When they learned to divide in earlier lessons, they used remainders for values that were "left over". Now they are working with decimals, which are parts of a whole, so they need to add zeros to figure out the exact part of the whole. Using a remainder does not work anymore, students need to understand that they are in higher levels of math and so they need to find exact answers.

Decimals are a tool to help us to find exact answers and values.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/dividing-decimals?playlist=Developmental%20Math - This is a Khan Academy video on dividing decimals by decimals.

http://www.mathvids.com/lesson/mathhelp/1113-dividing-decimals - This is a mathvids video on dividing decimals.

Metric Units of Mass and Capacity

Goal

The goals of this lesson can be found in the following objectives.

- Identify equivalence of metric units of mass.
- Identify equivalence of metric units of capacity.
- Choose appropriate metric units of mass or capacity for given measurement situations.
- Solve real-world problems involving metric measures of mass or capacity.

Relevant Review

Review metric units of length as they were taught in earlier lessons. You can review millimeters, centimeters, meters and kilometers.

You can have students refer back to the key equivalencies that they wrote in their notebooks during the earlier lesson.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Customary System
the system of measurement common in the United States, uses feet, inches, pounds, cups, gallons, etc.

Mass
the weight of an object

Capacity
the amount of liquid an object or item can hold

MASS –students need to learn the following metric units of mass. Here they are written from largest to smallest. You will also see them written in colored boxes in the text.

Kilogram

Gram

Milligram

EQUIVALENCIES

$$1\ Kilogram = 1000\ grams$$
$$1\ gram = 1000\ milligrams$$

CAPACITY –this is also written in colored boxes in the text. Capacity is connected to liquid measure.

EQUIVALENCIES

$1\ Liter = 1000\ Milliliters$

Teaching Strategies

The real world story problem *The Metric Park* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

First, students need to understand the difference between mass and capacity. Then using the recommended study skill tips, students will need to learn the units from largest to smallest as well as the equivalencies.

Another key for students to remember is the following:

When converting from large units to small units, you multiply.

When converting from small units to large units, you divide.

There are important things for students to learn throughout the text. You will see small pictures of notebooks and colored text boxes that highlight certain key topics and information. You can use these to help you in your teaching, and also to help students to identify when there is something that they need to write down or commit to memory.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/u-s–customary-and-metric-units?playlist=Developmental%20Math - This is a Khan Academy video on customary and metric units of measurement.

Converting Metric Units

Goal

The goals of this lesson can be found in the following objectives.

- Convert metric units of length, mass and capacity using powers of ten.
- Compare and order given metric measurements of length, mass or capacity.
- Solve real-world problems involving conversion of metric measures of length, mass and capacity.

Relevant Review

Review multiplying and dividing by decimal powers of ten.

Review the metric system of measurement. Include measures for length, mass and capacity.

Study Skill Tip

A study skill tip is to be sure that the students understand each unit of measure, what it measures and the equivalencies connected to it. When students understand this, they are able to apply the concepts in problem solving situations.

Teaching Strategies

The real world story problem *The Metric Park* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

As students work through this lesson, remind them that when we convert from a smaller unit to a larger unit that we divide and when working from larger to smaller that we multiply. Consistently reminding students of these facts will help them in their problem solving.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/converting-within-the-metric-system?playlist=Developmental%20Math - This is a Khan Academy video on converting units within the metric system.

http://www.youtube.com/watch?v=d7Q-tS3s9kk - This is a youtube video from ehow.com on how to convert units within the metric system.

1.5 Number Patterns and Fractions

This fifth chapter *Number Patterns and Fractions* continues to help students learn about parts of whole quantities. Having just finished decimals, it is a natural bridge to the world of fractions. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Prime Factorization
- Greatest Common Factors
- Equivalent Fractions
- Least Common Multiple
- Ordering Fractions
- Mixed Numbers and Improper Fractions
- Changing Decimals to Fractions
- Changing Fractions to Decimals

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Prime Factorization –2 Days
- Greatest Common Factors –1 Day
- Equivalent Fractions –2 Days
- Least Common Multiple –1 Day
- Ordering Fractions –1 Day
- Mixed Numbers and Improper Fractions –1 Day
- Changing Decimals to Fractions –1 Day
- Changing Fractions to Decimals –2 Days

Day 12 Review

Day 13 Test

Prime Factorization

Goal

The goals of this lesson can be found in the following objectives.

- Find factor pairs of given numbers.
- Use divisibility rules to find factors of given numbers.

- Classify given numbers as prime or composite.
- Write the prime factorization of given numbers using a factor tree.

Relevant Review

This lesson focuses on factors and divisibility. To help students with this lesson, review the times tables and simple division that can be completed through mental math. Five minutes at the beginning of each lesson will help students to strengthen their skills in these two important areas.

Review exponents, especially which value is the base and which is the exponent.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Factor
the number multiplied together to equal a product.

Divisibility Rules
a list of rules which help you to determine if a number is evenly divisible by another number.

Prime
a number that has two factors, one and itself.

Composite
a number that has more than two factors.

Prime Factorization
writing a number as a product of its primes.

Factor Tree
a diagram for organizing factors and prime factors.

Teaching Strategies

The real world story problem *The Sixth Grade Social* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are many key things in this lesson that students will need to keep track of and learn. These key components of the lesson are written in colorful text boxes throughout the lesson. The content of these boxes can be brought to the students in a couple of different ways. Students can write them down in their notebooks. Also, you and the students can create a bulletin board in the classroom. This bulletin board would contain divisibility rules, definitions and maybe a sample factor tree. This will help students to recall information as they work on problems in the lesson.

Factoring is breaking down values. This is a possible way to present this material to students. We break down many things in life or take things apart. This is a way of taking apart or breaking down a number into it's smaller parts. Brainstorm some things that students might take apart in everyday life. Possibilities include: fixing a bike, breaking down food in their bodies to smaller parts, taking apart a construction of some kind etc.

Present factoring as breaking down into values that are multiplied together.

Prime and composite numbers deserve some attention. A hands–on activity would be great here. http://www.uen.org/Lessonplan/preview?LPid=18882 - This website has a great activity outlined to help students to work with

factors. It is based on the idea of a detective and fingerprints to factorprints. It also has extensions into other lessons in this chapter.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/prime-factorization?playlist=Developmental%20Math - This is a Khan Academy video on prime factorization.

http://www.khanacademy.org/video/prime-numbers?playlist=Arithmetic - This is a Khan Academy video on identifying prime numbers.

Greatest Common Factors

Goal

The goals of this lesson can be found in the following objectives.

- Find the greatest common factor of two or more numbers using lists.
- Find the greatest common factor of two or more numbers using factor trees.
- Solve real-world problems involving greatest common factors.

Relevant Review

Review factors and prime factorization. Be sure that the students can identify factors as numbers that are multiplied together to create a product. It would be great to warm –up with a few examples and have students identify possible factors.

Examples:

The factors of $48 = 6 \times 8, 12 \times 4, 48 \times 1, 24 \times 2$

The factors of $16 = 4 \times 4, 16 \times 1, 8 \times 2$

You should also take one of these examples and build a factor tree all the way to it's prime factors.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Factor
a number multiplied by another number to get a product.

Greatest Common Factor
the greatest factor that two or more numbers has in common.

Product
the answer of a multiplication problem

Teaching Strategies

The real world story problem *The Basketball Game* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In this lesson, the students are introduced to greatest common factors and have to find greatest common factors. As you begin the lesson, you can talk about how in the last lesson you broke down or factored one number into its smaller parts. This lesson is about breaking down or factoring two or more numbers into smaller parts. Then the next task is to find the larger factor that all of the numbers have in common.

Greatest common factors are found with two or more numbers. That is the only distinction between factors and greatest common factors.

If the students seem challenge by jumping right to greatest common factors, you can always take it a step back and do common factors first. Once they have common factors of two or more numbers, you can move back to greatest common factors.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/greatest-common-divisor?playlist=Arithmetic - This is a Khan Academy video on greatest common factors. They are called greatest common divisors in the video title, but the content is the same.

http://www.mathplayground.com/howto_gcflcm.html - This is a math playground video that incorporates greatest common factors with least common multiples. Could be used in this lesson or in the lesson on least common multiples.

Equivalent Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Write fractions equivalent to a given fraction.
- Write given fractions in simplest form.
- Describe real-world fractional portions by writing in simplest form.

Relevant Review

This lesson is the beginning of many lessons on fractions. There isn't any review necessary for this lesson.

Study Skill Tip

Be sure to have the students copy down the following vocabulary words into their notebooks.

Fraction
a part of a whole.

Equivalent
means equal

Numerator
the top number in a fraction

Denominator
the bottom number in a fraction

Simplifying Fractions
dividing a numerator and a denominator by the GCF to create a fraction that is in simplest form. An equivalent fraction is created.

Reducing
another way to say simplifying

Teaching Strategies

The real world story problem *The Cake Dilemma* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This is the first of many lessons on fractions. Taking your time as you introduce this lesson is key to helping students to understand fractions and how to identify fractions. I have heard many students report that all was well in math until they got to fractions. Why is this? We can think about our teaching of the concept, but also number sense. If the student has a weak sense of whole numbers and whole number operations, then their understanding of fractions will be compromised. Continue working on those basic skills as you introduce new concepts.

A fun way to introduce fractions is with food. The book "Eating Fractions" is a fun way for students to think about how to divide things into different fractional parts. Again, they are taking a whole and breaking it up into parts. I encourage you to make this as hands–on as possible. Another possibility is http://www.superteacherworksheets.com/fractions.html - this website has all kinds of downloadable fraction strips and worksheets for students to use to begin exploring fractions.

http://www.gamequarium.com/fractions.html has some great fraction games that students can play on the computer.

There are so many ways to work with fractions, and it can be fun for the students too.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/fractions-in-lowest-terms?playlist=Developmental%20Math - This is a Khan Academy video on writing fractions in their lowest terms.

http://www.khanacademy.org/video/identifying-fraction-parts?playlist=Developmental%20Math - This is a Khan Academy video on identifying fraction parts.

Least Common Multiple

Goal

The goals of this lesson can be found in the following objectives.

- Find common multiples of different numbers.
- Find the least common multiple of given numbers using lists.
- Find the least common multiple of given numbers using prime factorization.
- Find two numbers given the greatest common factor and the least common multiple.

Relevant Review

Review factors and factoring values.

Review prime and composite numbers and prime factorization.

You can use one value to complete all of these tasks. For example, 64 is a large enough number to determine factors, prime and composite factors and to use an example of prime factorization.

Study Skill Tip

Be sure to have students copy down the information in colored text boxes as it is presented throughout the lesson.

Teaching Strategies

The real world story problem *The Decoration Committee* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Students are often confused by multiples and factors. Multiples are related to the operation of multiplication. You can remind students that when you multiply two factors, the result is a larger number. When factoring, you are breaking down a value into smaller parts. Multiples are larger than the numbers that you start with.

Help students to keep all of the different anacronyms straight by listing them out on the board with their definitions. Many students will be confused GCF and LCM.

http://www.math-play.com/Factors-and-Multiples-Jeopardy/Factors-and-Multiples-Jeopardy.html - This website has a factors and multiples Jeopardy game that is free for students to use. You would need computers or it is something students could work on at home.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/least-common-multiple?playlist=Arithmetic - This is a Khan Academy video on finding least common multiples.

http://www.khanacademy.org/video/least-common-multiple–lcm?playlist=Developmental%20Math - This is another Khan Academy video on least common multiples. This one begins to work with adding fractions. It is a bit ahead of this lesson, but could be used for an advanced student.

Ordering Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Compare fractions using lowest common denominator.

- Order fractions using lowest common denominator.
- Describe real-world portion or measurement situations comparing and ordering fractions.

Relevant Review

In this lesson students will be expanding on the basic fractions that they have just learned. Begin this lesson with a review of identifying fractions. Then review creating equivalent fractions and the terms least common multiple and greatest common factor.

Students are also asked to compare fractions in this lesson using greater than, less than or equal to. Review the symbols used to identify each of these comparisons.

Study Skill Tip

Be sure to ask students to copy these vocabulary words down in their notebooks.

Equivalent Fraction
two equal fractions

Denominator
the bottom number of a fraction

Numerator
the top number of a fraction

Like Denominator
when two or more denominators are the same, can also be called common denominator.

Lowest Common Denominator
the least common multiple of two or more denominators. It becomes the lowest common denominator.

Teaching Strategies

The real world story problem *The Ice Cream Sundae Fundraiser* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson combines a few skills that students have learned in earlier lessons. First, students use comparing symbols. This is simple when the denominators are the same. But then, they aren't always the same, are they? This presents the students with a new challenge. Now they have to find a common denominator so they can compare the fractions. Least common multiples now are used as common denominators. Then students are asked to recall how they created equal fractions to rename each fraction in terms of denominators.

Work through each of these steps slowly. Be sure to check in with students so that you know that they are with you as you progress.

You can use these hints to help students.

1. Identify whether or not the fractions have common denominators.
2. If not, then identify the least common multiple. This is the new denominator for both fractions.
3. Create equal fractions with each fraction and the new denominator.
4. Compare using <, >or =.

You may want to write them on the board and/or have students write them down in their notebooks. They will need them in later lessons when adding/subtracting fractions with unlike denominators.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/equivalent-fractions?playlist=Arithmetic - This is a Khan Academy video on finding equal fractions and least common multiples.

http://www.khanacademy.org/video/comparing-fractions?playlist=Developmental%20Math - This is a Khan Academy video on comparing fractions and requires that students simplify the fractions to lowest terms first.

http://www.youtube.com/watch?v=vjjv1CVjwso - This is a musical video on comparing and ordering fractions.

Mixed Numbers and Improper Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Measure lengths to a fraction of an inch.
- Rewrite mixed numbers as improper fractions.
- Rewrite improper fractions as mixed numbers.
- Compare and order mixed numbers and improper fractions.

Relevant Review

This lesson has a section on measuring lengths to a fraction of an inch. Because of this, begin this lesson with a review of rulers and measuring lengths using customary units. The last lesson on measurement used metrics, so it would be wise to remind students of the customary units of measurement.

Review the basics of identifying fractions. Be sure that students can easily identify the numerator and denominator of a fraction.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Mixed Number
a number made up of a whole number and a fraction

Improper Fractions
a fraction where the numerator is greater than the denominator

Equivalent
means equal

Teaching Strategies

The real world story problem *The Pizza Party* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When working with the first section on measurement, be sure to use real rulers. Students can look at the picture in the text, but there is nothing like actually measuring something and figuring out the actual exact measurement using a ruler.

Two key terms are introduced here mixed number and improper fraction. Begin by explaining to students that when the numerator is smaller than the denominator that we have a proper fraction. Then introduce improper fractions as a fraction where the numerator is larger than the denominator.

Once students have learned "how" to convert improper fractions to mixed numbers and mixed numbers into improper fractions, it is simply a matter of practice. http://www.math-aids.com/Fractions/Fractions_Improper.html - This website will construct different worksheets for you on mixed numbers and improper fractions. You can also differentiate the lesson by selecting the level of difficulty that you would like the worksheet to be.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/mixed-numbers-and-improper-fractions?playlist=Arithmetic - This is a Khan Academy video on mixed numbers and improper fractions.

http://www.khanacademy.org/video/converting-mixed-numbers-to-improper-fractions?playlist=Developmental%20Math - This is a Khan Academy video on converting mixed numbers to improper fractions.

Changing Decimals to Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Write decimals as fractions.
- Write decimals as mixed numbers.
- Write decimals as equivalent fractions and mixed numbers.
- Describe real-world portion or measurement situations by writing decimals as fractions.

Relevant Review

Review identifying fractions both proper and improper. Review mixed numbers and how to identify mixed numbers.

Because this lesson is going to combine fractions and decimals for the first time, review place value and decimals with the students.

Study Skill Tip

Be sure to have students copy the following vocabulary words in their notebooks.

Decimal
: a part of a whole written using place value and a decimal point to separate wholes and parts.

Fraction
a part of a whole written with a fraction bar dividing the numerator and the denominator.

Mixed Number
a number that has a whole number and a fraction.

Equivalent
means equal

Teaching Strategies

The real world story problem *The Map Disaster* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This is an exciting lesson because students are going to see the connection between fractions and decimals. In this first of two lessons on conversion, students are going to begin with decimals and learn how to convert decimals into fraction. This is where place value will be important. Tenths is a denominator of 10. Students will begin to see this connection and this will help to simplify the process of conversions.

Students will also apply simplifying or reducing to the fraction form of decimals. Many students forget this step. Always remind them to check and see if there is any way to make the numerator and denominator smaller by dividing by the same factor. Again the word "factor" is used to help students to see that some values can be broken down into smaller parts.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/decimals-and-fractions?playlist=Developmental%20Math - This is a Khan Academy video on converting decimals into fractions. It also includes simplifying.

Changing Fractions to Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Write fractions as decimals.
- Write mixed numbers as decimals.
- Write fractions and mixed numbers as repeating decimals.
- Compare and Order decimals and fractions.

Relevant Review

Review converting decimals into fractions and mixed numbers.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Terminating Decimal

decimal that can be found dividing a numerator and denominator and by adding a decimal point and zero placeholders.

Repeating Decimal

a decimal where the digits in the quotient repeat themselves, can be indicated by putting a small line over the second repeating digit.

Teaching Strategies

The real world story problem *The Survey* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are key points written in colored text boxes throughout this lesson. Be sure to identify this information for students and use the colored text boxes as ways to point out things that are helpful hints.

Point out that there are two ways to convert a fraction to a decimal. Both ways are fine and will produce an accurate answer. The first is to use place value. The key to this first method is that the denominator of the fraction must be written in base ten. The second way is to divide the numerator by the denominator. While this may seem a bit more complicated, it produces an accurate answer including additional decimal places. It is the most useful method when you don' t have a denominator written in a base ten value.

Review the vocabulary words "terminating" and "repeating" then show how there can be terminating and repeating decimals. This can be a challenging idea for students, be sure to go slowly so that all understand.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/converting-fractions-to-decimals?playlist=Arithmetic - This is a Khan Academy video on how to convert a fraction to a decimal.

http://www.mathplayground.com/howto_fractions_decimals.html - This is a math playground video on how to change a fraction to a decimal.

1.6 Addition and Subtraction of Fractions

This sixth chapter *Addition and Subtraction of Fractions* will teach students how to take the basic information about fractions from the last chapter and use that information for adding and subtracting fractions. This chapter will build on the knowledge that students' acquired in the last chapter. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Fraction Estimation
- Adding and Subtracting Fractions with Like Denominators
- Adding and Subtracting Fractions with Different Denominators
- Adding and Subtracting Mixed Numbers
- Subtracting Mixed Numbers by Renaming
- Elapsed Time
- Box-and-Whisker Plots
- Problem-Solving Strategies-Draw a Diagram

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Fraction Estimation –1 Day
- Adding and Subtracting Fractions with Like Denominators –1 Day
- Adding and Subtracting Fractions with Different Denominators –2 Days
- Adding and Subtracting Mixed Numbers –2 Days
- Subtracting Mixed Numbers by Renaming –1 Day
- Elapsed Time –1 Day
- Box-and-Whisker Plots –1 Day
- Problem-Solving Strategies-Draw a Diagram –1 Day

Day 11 –Review

Day 12 –Test

Fraction Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Round fractions to the nearest half.
- Round mixed numbers to the nearest whole number.

- Estimate sums and differences of fractions and mixed numbers.
- Estimate sums and differences involving real-world fractional amounts.

Relevant Review

This lesson is going to teach students how to use estimation in their work with fractions. It would be helpful to review the definition of the word "estimation" and to remind students that an estimate must make sense.

Review the two terms "sum" and "difference". Sum being a word used when addition is the key operation, and "difference" being a word used when subtraction is the key operation.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Fraction
a part of a whole written with a fraction bar, a numerator and a denominator.

Estimate
to find an approximate answer that is reasonable and makes sense given the problem.

Mixed number
a number made up of a whole number and a fraction.

Sum
the answer in an addition problem.

Difference
the answer is a subtraction problem.

Teaching Strategies

The real world story problem *Building a House* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

As you prepare to teach this lesson, notice that there are colorful text boxes throughout the lesson. These text boxes provide students with a helpful hint or strategy for the lesson. When teaching point these out to the students. When they are working on practice problems later, these text boxes can prove to be valuable help when students are stuck or need a reminder.

When rounding in the past, students have used the rules for rounding that they learned in the first chapter. This lesson requires rounding, but those earlier rules do not apply. You will need to point this out to the students. Because we are working with fractions, we have to "reason" and think about which whole number a fraction is closest to. We are rounding to 0, $\frac{1}{2}$ or 1 whole.

When we have a mixed number, we are rounding to the closest value. The rounding rules don't apply here either because the mixed number has a fraction in it. Students need to be shown this connection.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=B-3zp2JxQa8 - This is a Konst math video on estimating fraction sums and differences.

Adding and Subtracting Fractions with Like Denominators

Goal

The goals of this lesson can be found in the following objectives.

- Add fractions with like denominators.
- Subtract fractions with like denominators.
- Evaluate given numerical expressions involving sums and differences of fractions with common denominators.
- Solve real-world problems involving sums and differences of fractions with common denominators.

Relevant Review

This lesson will teach students how to add and subtract fractions with like denominators. Review the previous lesson on estimating fractions to continue to expand student understanding of these concepts.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Like Denominator
when the denominators of fractions being added or subtracted are the same.

Simplifying
dividing the numerator and the denominator of a fraction by its greatest common factor. The result is a fraction is simplest form.

Difference
the answer to a subtraction problem

Numerical Expression
an expression with multiple numbers and multiple operations

Operation
the four operations in math are addition, subtraction, multiplication and division

Evaluate
to find the value of a numerical expression.

Teaching Strategies

The real world story problem *Measuring for Brackets* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Most students will grasp this concept quite simply because all they have to do is work with the numerators when the denominators are common. However, be sure to remind them that they will need to simplify their answers. All answers must be written in simplest form. You can use one of the examples to remind students how to simplify.

The section on evaluating numerical expressions with fractions becomes a bit trickier because students will need to work with more than one operation at a time. This will require some mental math. Encourage students to work through the problem from left to right. They add or subtract the first two fractions, then take that answer and add or subtract it with the third fraction and so on. Keeping things orderly will help students to avoid mistakes.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/adding-fractions-with-like-denominators?playlist=Developmental%20Math - This is a Khan Academy video on adding fractions with like denominators.

http://www.khanacademy.org/video/adding-and-subtracting-fractions?playlist=Pre-algebra - This is a Khan Academy video on how to add and subtract fractions.

Adding and Subtracting Fractions with Different Denominators

Goal

The goals of this lesson can be found in the following objectives.

- Add fractions with different denominators.
- Subtract fractions with different denominators.
- Evaluate given numerical expressions involving sums and differences of fractions with different denominators.
- Solve real-world problems involving sums and differences of fractions with different denominators.

Relevant Review

This lesson builds on the previous lesson by teaching students how to add and subtract fractions with different denominators.

Begin this lesson with a review of how to add and subtract fractions with like denominators. Review simplifying fractions also.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Renaming fractions
: renaming fractions means rewriting them with a common denominator.

Least Common Multiple
: the lowest multiple that two or more numbers have in common.

Lowest Common Denominator
: the least common multiple that becomes the lowest common denominator when adding or subtracting fractions with different denominators.

Equivalent Fractions
equal fractions. Created by multiplying the numerator and the denominator of a fraction by the same number.

Teaching Strategies

The real world story problem *The Layers of a Wall* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Beginning the lesson with a review of the previous day's work is an excellent way to remind students of what they have already learned. This lesson builds upon the previous lesson. The students have learned to add and subtract fractions with like denominators and now they are going to learn to add and subtract fractions with different denominators. Some students may have heard this referred to as "adding and subtracting with unlike denominators" you can tell them that we are talking about the same thing.

There are steps involved in finding common denominators. The first one is to look at the denominators and find the least common multiple of the two denominators. This is an excellent time to remind students of what the least common multiple means. Then we can rename the fractions. Finally, students are able to add or subtract the renamed fractions.

Evaluating numerical expressions with unlike fractions is the most challenging part of this lesson. It is recommended that two days are spent on this lesson. Keep numerical expressions for the second day. In this way, students will have had some practice working with two fractions before a third fraction is added.

Always review and repeat important things to the students. Use colorful text boxes to help the students to remember key rules and steps.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/adding-fractions-with-unlike-denominators?playlist=Developmental%20Math - This is a Khan Academy video on how to add fractions with different denominators.

http://www.khanacademy.org/video/adding-and-subtracting-fractions?playlist=Pre-algebra - This is a Khan Academy video on how to add and subtract fractions.

Adding and Subtracting Mixed Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Add mixed numbers.
- Subtract mixed numbers without renaming.
- Evaluate given numerical expressions involving sums and differences of mixed numbers.
- Solve real-world problems involving sums and differences of mixed numbers.

Relevant Review

Review the previous lesson on how to add and subtract unlike denominators. Then review identifying mixed numbers. Students will work on adding and subtracting mixed numbers in this lesson.

Study Skill Tip

Be sure to have students write the following vocabulary words in their notebooks.

Mixed Number
a number that has a whole number and a fraction.

Numerical Expression
a number expression that has more than one operation in it.

Operation
addition, subtraction, multiplication and division

Teaching Strategies

The real world story problem *The Window* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

One of the first things that might be challenging for students is the two mixed numbers in the story problem. Because the story problem uses large whole numbers of 18 and 46, students may become stuck right away. To help with this, you can remind students that just because the whole number is large doesn't affect the actual procedure of working with the mixed number. We work with $1\frac{1}{2}$ in the same way that we work with $18\frac{2}{3}$. Encourage students not to be discouraged by the size of the numbers.

Remind students of the steps that are written in bold in the text. They need to add the fraction parts of the mixed numbers before the wholes. Use the example in the text to show students that sometimes the fractions will add up to another whole. When this happens, they can't ignore it. This new whole must be added to the other whole numbers.

Working with different denominators adds another step to the process. Review this with the students and remind them that all fractions being added or subtracted must have the same denominator.

Point out the "lightening flash" to students and remind them to always pay special attention to the information in the colored text boxes. For you, as the teacher, these text boxes will also help you to guide students towards information that they need to remember.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/adding-mixed-numbers?playlist=Developmental%20Math - This is a Khan Academy video on adding mixed numbers.

http://www.khanacademy.org/video/adding-mixed-numbers-with-unlike-denominators?playlist=Developmental%20Math - This is a Khan Academy video on adding mixed numbers with unlike denominators. It shows examples with more than two mixed numbers.

Subtracting Mixed Numbers by Renaming

Goal

The goals of this lesson can be found in the following objectives.

- Subtract mixed numbers with renaming.
- Evaluate numerical expressions involving differences of mixed numbers requiring renaming.
- Solve real-world problems involving differences of mixed numbers requiring renaming.

Relevant Review

When beginning this lesson, it is valuable to review subtracting mixed numbers without renaming from the previous lesson.

In addition, review subtracting whole numbers with borrowing. Explain to students that sometimes when we work with mixed numbers, we will need to borrow to subtract the fractions. This will become clearer during the lesson.

Finally, review comparing fractions. If students can't determine whether a fraction is larger or smaller than another fraction, they will have difficulty determining when borrowing is required. This is a necessary skill for this lesson.

Study Skill Tip

Be sure to have students copy the following vocabulary words in their notebooks.

Rename
to write an equivalent form of a whole number or a fraction.

Equivalent
equal

Teaching Strategies

The real world story problem *Measuring Differences* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This is a tricky lesson to teach. Many students are confused when it comes to borrowing to rename fractions so that they can be subtracted. The two key review skills necessary for this lesson to be successful are borrowing and comparing fractions. Be sure that the students understand that when they borrow "a 1" when working with whole numbers, that they are actually borrowing a ten. Tens are what we work with in whole number borrowing. When borrowing with fractions, we borrow a whole from the whole part of the mixed number. Then we write it as a fraction that we can work with.

When comparing fractions, students need to be able to determine whether or not borrowing is necessary. To do this, they must be able to determine whether or not one fraction is smaller than another. Take time to check student understanding of this skill.

Use colors to help students. If working on a white board, write the original problem in black and then use red and blue. Encourage students to do the same until they have the steps down. Once again, point out the important information in the text boxes.

You can have students work in pairs to help each other. Encouraging peer teaching will help you so that you aren't "running" around to every student.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/subtracting-mixed-numbers?playlist=Developmental%20Math - This is a Khan Academy video on subtracting mixed numbers.

http://www.khanacademy.org/video/subtracting-mixed-numbers-word-problem?playlist=Developmental%20Math - Here is a word problem that involves subtracting mixed numbers. However, it has the students work with improper fractions instead of wholes and parts. This is a valuable alternative way to teach the concept as well as the one presented in the text.

Elapsed Time

Goal

The goals of this lesson can be found in the following objectives.

- Add and subtract measures of time.
- Find elapsed time between given start and stop times.
- Solve real-world problems involving elapsed time.

Relevant Review

To review for this lesson, you can review time and units of time such as seconds, minutes and hours. All students should understand these concepts prior to beginning this lesson.

Study Skill Tip

Be sure to have the students copy down the following vocabulary words in their notebooks.

Elapsed Time
the time from the start of an event to the end of the event.

Units of Time
how we measure time using seconds, minutes and hours.

Students should also copy down the units of time and their equivalents from the text boxes that are placed throughout the lesson.

Teaching Strategies

The real world story problem *Beating the Clock* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson begins simply and then becomes more complicated as the lesson progresses. A key in teaching this lesson is to be sure that the students understand the section before moving on to the next section. Students can work in small groups throughout this lesson. It is the best way to encourage peer teaching. Students can work through the examples in each section as a team. Then they can work on the story problem from the introduction by using what they have learned.

Fractional units of time should be familiar to the students. Don't assume that the students know them. Taking the time to teach these to students will help cut down on reteaching.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathvids.com/lesson/mathhelp/1115-elapsed-time - This is a math vids video on calculating and understanding elapsed time.

http://www.youtube.com/watch?v=de4zqJ14qis - This is a teachertube video posted on youtube which explains telling time and calculating elapsed time too.

Box-and-Whisker Plots

Goal

The goals of this lesson can be found in the following objectives.

- Order a set of data to find the median, quartiles and extremes.
- Draw a box-and-whisker plot to represent given data.
- Identify the median, quartiles, and extremes given a box-and-whisker plot.
- Compare and Interpret double box-and-whisker plots of real-world data.

Relevant Review

Review mean, median and mode prior to teaching this lesson.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Median
the middle score of a set of data.

Quartile
dividing a data into four sections.

Upper Quartile
the median of a quartile on the higher end of the range.

Lower quartile
the median of a quartile on the lower range

Extremes
the highest and lowest scores possible in a range of data.

Teaching Strategies

The real world story problem *Homes for Sale* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The real world story problem could also be used as the background for all of the teaching material in this lesson. By using the problem, students can work in small groups to apply the content from the lesson to the story problem. This allows students to use the content as they learn it and apply it directly to the story problem. To have this approach be successful, students should be grouped with diverse learners in each group to encourage peer support and teaching.

There are text boxes with valuable information scattered throughout the lesson. Be sure to point these out to students and ask them to copy the steps for making box–and–whisker plots down in their notebooks.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/box-and-whisker-plot?playlist=ck12.org%20Algebra%201%20Examples - This is a Khan Academy video on box –and –whisker plots and on how to create them.

Problem-Solving Strategy-Draw a Diagram

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Draw a diagram.
- Plan and Complete alternative approaches to solving problems.
- Solve real-world problems using selected strategies as part of a plan.

Relevant Review

Begin this lesson by reviewing the four parts of problem–solving plan. If you have created a poster or diagram in the classroom, point this out to students and review the steps with them.

Study Skill Tip

A study skill tip is included in the review for this lesson. Be sure that the students are clear on the four parts of problem solving plan.

Teaching Strategies

The real world story problem *The Tiled Floor* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Using diagrams as a strategy in problem solving requires students to use reasoning to figure out whether or not it would make sense to draw a diagram. Encourage students to use the story problem to work through the content in this lesson. This is a great opportunity for students to work in groups.

Allow time for students to share their diagrams and how they solved the problem at the end of the lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/word-problem-solving-strategies?playlist=ck12.org%20Algebra%201%20Examples - This is a Khan Academy video on problem solving and on different strategies used in problem solving.

http://www.onlinemathlearning.com/math-word-problems.html - This website has a lot of teaching content on using diagrams to solve problems. You can select the level of the example and the method that you would like the students to use to solve the problem.

1.7 Multiplication and Division of Fractions

This seventh chapter *Multiplication and Division of Fractions* builds upon the last chapter on addition and subtraction of fractions. Moving on to these next two operations is a natural step in the learning process. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Multiplying Fractions and Whole Numbers
- Multiplying Fractions
- Multiplying Mixed Numbers
- Dividing Fractions
- Dividing Mixed Numbers
- Customary Units of Weight and Capacity
- Converting Customary Units
- Problem-Solving Strategy: Choose an Operation

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Multiplying Fractions and Whole Numbers –1 Day
- Multiplying Fractions –1 Day
- Multiplying Mixed Numbers –1 Day
- Dividing Fractions –2 Days
- Dividing Mixed Numbers –1 Day
- Customary Units of Weight and Capacity –2 Days
- Converting Customary Units –1 Day
- Problem–Solving Strategy: Choose an Operation –1 Day

Day 11 –Review

Day 12 –Test

Multiplying Fractions and Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Multiply fractions by whole numbers.
- Multiply whole numbers by fractions.

- Estimate products of whole numbers and fractions.
- Solve real-world problems involving products of whole numbers and fractions.

Relevant Review

Begin this lesson by reviewing multiplication as repeated addition. The text begins with a whole number example, and this is an excellent opportunity to review the basics of multiplication.

Use five minutes at the beginning of class for a review of the times tables. Students can do this using flash cards, partners or a combination of both.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Multiplication
a shortcut for repeated addition

"of"
means multiply in a word problem

Product
the answer in a multiplication problem

Estimate
to find a reasonable answer that is not exact but is close to the actual answer.

Teaching Strategies

The real world story problem *The Rainforest* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are a couple of different skills that are taught to students in this lesson. The first is how to multiply a fraction by a whole number. To do this, students will need to change the whole number to a fraction by placing it over 1. This is an important step and students should copy it down in their notebooks.

Be sure to convert a whole number to a fraction by placing it over 1.

$4 = \frac{4}{1}$

While this is presented in the text, it is valuable to point it out to the students.

Next, the students are taught how to multiply a whole number by a fraction. This is different, because it involves the reverse of multiplication, division. The students learn that multiplying by $\frac{1}{2}$ is the same as dividing by 2. Point out the information in the text box to the students and have them copy it down in their notebook.

These number sense clues are helpful for students as they begin problem solving and working with different types of problems that involve fractions.

Estimating products often involves simplifying or thinking of a fraction in a new way. The example in the text simplified to $\frac{1}{3}$ so it is much easier to work with. Here are some possible examples for students.

$$\frac{12}{24} = \frac{1}{2}$$
$$\frac{32}{48} = \frac{2}{3}$$

Once students begin to simplify, then the reasoning involved in estimating products becomes simpler.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=FLnGUN44Duk - This is a youtube video on how to multiply a fraction by a whole number.

Multiplying Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Multiply two fractions.
- Multiply three fractions.
- Evaluate numerical and algebraic expressions involving products of fractions.
- Solve real-world problems involving products of fractions.

Relevant Review

Review the steps for multiplying a fraction and a whole number. Include the process of multiplying numerators by numerators and denominators by denominators.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Product
the answer to a multiplication problem.

Numerical Expression
an expression that has numbers and operations.

Algebraic Expression
an expression that has numbers, operations and variables.

Teaching Strategies

The real world story problem *Water and the Rainforest* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The groundwork for this lesson began in the lesson prior to this one. The students have learned how to multiply fractions in the last lesson. They just don't know that they have learned it. When multiplying whole numbers and fractions, students had to convert a whole number into a fraction. Then they were essentially multiplying two fractions. Now they are going to take these steps and apply them to working with two and three fractions.

Be sure to review simplifying fractions. Once students have multiplied the fractions, they will need to be sure that their answer is in simplest form.

Point out to students that multiplying three or more fractions follows the same steps as multiplying two fractions. We multiply all of the numerators and then all of the denominators. You can color code the numerators and the denominators if you think this will help the students. Students will simply need to memorize the rule of multiplying fractions.

New information is presented in the section on numerical and algebraic expressions. Students will see parentheses used to denote multiplication as well as two variables next to each other. Remind students that both of these are signs of multiplication. Students will begin to work with these ways of showing multiplication. Pay specific attention to the vocabulary words here. Have students copy them down in their notebooks with examples.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/multiplying-fractions?playlist=Developmental%20Math - This is a Khan Academy video on multiplying fractions.

http://www.youtube.com/watch?v=_PtNsG_kcgM - This is a youtube video by yourmathteacher.com on how to multiply fractions.

Multiplying Mixed Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Multiply mixed numbers.
- Evaluate numerical and algebraic expressions involving products of mixed numbers.
- Solve real-world problems involving products of mixed numbers.

Relevant Review

Review multiplying fractions from the previous section. Be sure that students remember the steps of multiplying numerators by numerators and denominators by denominators.

Review simplifying the product to be sure that the answer is in simplest form.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Mixed Number
a number that has both wholes and parts.

Improper Fraction
a number where the numerator is greater than the denominator.

Numerical Expression
has numbers and operations but no equals sign.

Algebraic Expression
has numbers, operations and variables but no equals sign.

Teaching Strategies

The real world story problem *Losing the Rainforest* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In the first section, students will need to recognize that when we multiply we are working with groups. You can review multiplication as repeated addition to bring this point to the students. Then show the first example as six groups of 1 and one –fourth. Be sure that the students understand this before moving on. One way to check their understanding is to provide them with a different example and ask them to explain it in words.

$5 \times 2\ \frac{1}{3}$ as five groups of two and one –third

Next, you will teach the students how to convert a mixed number to an improper fraction. Use the information in the text box to do this.

Once students are able to complete converting mixed numbers to improper fractions and remember the rule of multiplying numerators by numerators and denominators by denominators, then the rest of the lesson is simply practicing these skills.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/multiplying-fractions-and-mixed-numbers?playlist=Developmental%20Math - This is a Khan Academy video on multiplying fractions and mixed numbers.

http://www.khanacademy.org/video/mixed-numbers-and-improper-fractions?playlist=Arithmetic - This is a Khan Academy video on converting mixed numbers to improper fractions.

Dividing Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Identify and write reciprocal fractions.
- Divide a fraction by a whole number.
- Divide a whole number by a fraction.
- Divide a fraction by a fraction.
- Solve real-world problems involving quotients of fractions.

Relevant Review

Review the basics of division and that division is dividing up quantities into equal parts. Review that division is the inverse of multiplication.

Review the steps for multiplying fractions.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Inverse Operation
opposite operation. Multiplication is the inverse operation of division. Addition is the inverse operation of subtraction.

Reciprocal
the inverse of a fraction. We flip a fraction's numerator and denominator to write a reciprocal. The product of a fraction and its reciprocal is one.

Teaching Strategies

The real world story problem *The Rainforest Game* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

To complete this lesson, students will need to apply several steps to their work. It is for this reason that it is recommended that this lesson be taught over two days. The first day can focus on reciprocals and basic division of fractions and whole numbers. The next day can review the previous day's work and then move on to dividing fractions by fractions. This will provide students with a lot of practice working through the steps.

Once students have the steps for dividing fractions, you must bring them back to looking at an answer and being sure that it is in simplest form. Simplest form also includes that an improper fraction has been converted to a mixed number. Use the text boxes to remind students of the different steps and vocabulary words that are needed for success.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/dividing-fractions?playlist=Pre-algebra - This is a Khan Academy video on dividing fractions.

http://www.mathplayground.com/howto_divide_fractions.html - This is a mathplayground video on dividing fractions.

Dividing Mixed Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Divide a mixed number by a fraction.
- Divide a mixed number by a mixed number.
- Solve real-world problems involving quotients of mixed numbers.

Relevant Review

Begin this lesson by reviewing converting mixed numbers to improper fractions. Then review the steps for dividing fractions. Practice a few examples with the students.

$$\frac{6}{8} \div \frac{1}{2}$$
$$6 \div \frac{1}{3}$$
$$\frac{8}{9} \div \frac{1}{2}$$

Study Skill Tip

Be sure to have students copy the following vocabulary word in their notebooks.

Quotient
the answer of a division problem.

Be sure that the students have copied the steps for dividing fractions down in their notebooks.

Teaching Strategies

The real world story problem *The Snake Comparison* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Once you have completed the relevant review material for this lesson, you are ready to work with students on putting the steps together to divide mixed numbers. Be sure that the students understand that they are combining steps that they have already learned for success.

First, they must convert mixed numbers to improper fractions.

Next, they change the operation to multiplication.

Then invert the second fraction and multiply for the product.

Finally, simplify the answer.

Students need practice and encouragement to commit these steps to memory.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/dividing-mixed-numbers?playlist=Developmental%20Math - This is a Khan Academy video on dividing mixed numbers.

http://www.khanacademy.org/video/dividing-mixed-numbers-and-fractions?playlist=Developmental%20Math - This is a Khan Academy video on dividing fractions and mixed numbers.

Customary Units of Weight and Capacity

Goal

The goals of this lesson can be found in the following objectives.

- Identify equivalence of customary units of weight.
- Identify equivalence of customary units of capacity.
- Choose appropriate customary units of weight or capacity for given measurement situations.
- Solve real-world problems involving customary measures of weight or capacity.

Relevant Review

Review customary units of measurement concerning length that were taught in a previous lesson. Then explain to the students that in this lesson they will be working with customary units of weight and capacity.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Weight
measurement of the heaviness or mass of someone or something

Ounces
the smallest unit of weight, used to measure very small items.

Pounds
the most common unit for measuring weight, most things are measured in pounds.

Tons
the largest unit for measuring weight-very large items are measured by the ton-for example, a car or truck.

Fluid Ounce
the smallest unit for measuring liquid capacity. A baby bottle is measured in fluid ounces.

Cup
a common small unit for measuring liquid capacity.

Pint
2 cups is equal to one pint of liquid capacity.

Quart
2 pints is equal to one quart of liquid capacity.

Gallon
the largest unit of liquid capacity

Use the text boxes to identify other important information. Require that students copy down the different measures of equivalence into their notebooks.

Teaching Strategies

The real world story problem *Weighing In On Animals* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This is a great lesson for hands–on work. The students can use this lesson to use different measures and experiment with weight and capacity. You could split this up into two days, but one is fine. Two days would allow you to do one day on weight and one on capacity. When working with weight, bring in different scales and ask students to weight different things. You can also have them experiment with finding different things with equivalent weight. When working with capacity, bring in different size containers, measuring cups and towels if you are going to work with water. Weighing gallons can be interesting too. Be creative with the content of this lesson.

Remind students that when we convert from a smaller unit to a larger unit, that we divide. When going from a larger unit to a smaller unit, we multiply.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.onlinemathlearning.com/measurement-of-weight.html - This is a math video for sixth graders which focuses on customary units of weight.

http://www.watchknowlearn.org/Video.aspx?VideoID=18071 - This is a rap song to help students to remember the customary units of capacity.

Converting Customary Units

Goal

The goals of this lesson can be found in the following objectives.

- Convert customary units of measure using multiplication.
- Convert customary units of measure using division.
- Add and subtract customary units of measure.
- Solve real-world problems involving customary units of measure.

Relevant Review

Review all of the customary units of measure and their equivalents. The students should have these written in their notebooks. Be sure to have them identify the pages where these notes are located. They will need to refer back to this information throughout the lesson.

Students will be using the operations of multiplication and division in this lesson. Spend five minutes doing a quick speed review of the times tables.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Equivalent
: equal amount or unit

Length
: measuring how long something is-customary units are inches, feet, yards and miles

Weight
: measuring how heavy something is-customary units are ounces, pounds and tons.

Capacity

measuring how much liquid something can hold-customary units are fluid ounces, cups, pints, quarts and gallons.

Teaching Strategies

The real world story problem *The Amazon* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The strategies in this lesson are in the operations. Students have already learned that when working from a larger unit to a smaller unit, that the necessary operation is multiplication. When going from a smaller unit to a larger unit, it is division. Now students will need to use their reasoning skills to determine whether they are going from a smaller unit to a larger unit or from a larger unit to a smaller unit. Then they can apply these skills.

This is a wonderful lesson for small group work. Assign the students the examples in the text. Have them put away their texts and just work on the examples in small groups. Then you can check the work as an entire class. This allows you time for observation and time to assess student learning.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/converting-units-of-length?playlist=Developmental%20Math - This is a Khan Academy video on converting units of length.

http://www.khanacademy.org/video/converting-pounds-to-ounces?playlist=Developmental%20Math - This is a Khan Academy video on converting pounds to ounces.

http://www.khanacademy.org/video/converting-gallons-to-quarts-pints-and-cups?playlist=Developmental%20Math - This is a Khan Academy video on converting units of liquid measure.

Problem-Solving Strategy: Choose an Operation

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Choose an operation.
- Plan and compare alternative approaches to solving problems.
- Solve real-world problems using selected strategies as part of a plan.

Relevant Review

Review the four part problem–solving plan with the students. If you have a bulletin board or chart with the information on it, then use this chance to review each step with the students. Remind them to use skill such as underlining.

Study Skill Tip

Because this lesson focuses on choosing an operation, it would be valuable to refer students back to the list that they made in earlier lessons on words that mean each operation. For example, “of” means multiply and “Less than” means subtraction. The students should have a list of these words to use as reference.

If the students did not copy these words down earlier, then use the words as they are presented in the lesson. Require students to copy them down in their notebooks.

Teaching Strategies

The real world story problem *The Beetles* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Review fraction operations especially addition and subtraction because the story problem *The Beetles* requires the use of these skills.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/word-problem-solving-strategies?playlist=ck12.org%20Algebra%201%20Examples - This is a Khan Academy video on problem solving strategies.

1.8 Ratios, Proportions and Percents

This eighth chapter *Ratios, Proportions and Percents* provides the structure to help students with their reasoning and problem solving. Ratios, proportions and percents are used in everyday life all of the time. Students need to be capable when working with these concepts. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Ratios
- Rates
- Solving Proportions
- Proportions and Scale Drawings
- Understanding Percent
- Percents, Decimals and Fractions
- Finding a Percent of a Number
- Problem-Solving Strategy: Use a Proportion

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Ratios –1 Day
- Rates –1 Day
- Solving Proportions –1 Day
- Proportions and Scale Drawings –1 Day
- Understanding Percent –2 Days
- Percents, Decimals and Fractions –1 Day
- Finding a Percent of a Number –2 Days
- Problem–Solving Strategy: Use a Proportion –1 Day

Day 11 –Review

Day 12 –Test

Ratios

Goal

The goals of this lesson can be found in the following objectives.

- Identify and write different forms of equivalent ratios.
- Write ratios in simplest form.

- Write and compare ratios in decimal form.
- Solve real-world problems involving ratios.

Relevant Review

Review the word equivalent and be sure that the students understand that it means the same as equal.

Review simplifying fractions. Students will need to use this skill in this lesson.

Study Skill Tip

Be sure to have students copy down the following vocabulary words into their notebooks.

Ratio
a comparison between two quantities can be written three different ways.

Equivalent
equal

Simplify
to make smaller

Greatest Common Factor
the largest number that will divide into two or more numbers evenly.

Be sure that the students make note of the three different ways to write a ratio.

Teaching Strategies

The real world story problem *The Milk Comparison* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Notice in the beginning of this lesson, that students are going to learn three different things. They are going to learn how to write ratios, simplify ratios and compare ratios. Therefore, you want to divide your teaching of this lesson into three different sections.

Students learn three different ways to write a ratio. They also need to understand that these are equivalent and can be used interchangeably. In the section that asks students to convert ratios in fraction form to decimals, remind students that they are going to be using the same skills that they have already learned.

Refer students to the important information in the text boxes throughout the lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/ratios-as-fractions-in-simplest-form?playlist=Developmental%20Math - This is a Khan Academy video on how to write ratios as fractions in simplest form.

Rates

Goal

The goals of this lesson can be found in the following objectives.

- Identify and write equivalent rates.
- Write and compare unit rates.
- Solve real-world problems involving rates.

Relevant Review

Review identifying ratios and the three different ways to write a ratio. In this lesson, students will be learning about rates, but will build on what they learned in the previous lesson on ratios.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Rate
a special ratio that compares two quantities. Often uses a unit such as miles, gallons or dollars to describe the rate.

Unit Rate
a rate that is compared to one. A unit rate compares a quantity to one. Rates can be simplified to be unit rates.

Teaching Strategies

The real world story problem *The Measuring Dilemma* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In this lesson, students will need to distinguish between a rate and a unit rate. When explaining this to them, help students to see that a rate compares two quantities, while a unit rate compares a quantity to one. Ask students to copy down the "Facts about Rates" from the colored text box in the first section.

A big thing for students to notice is that the rates and unit rates have labels associated with them. In the last section on ratios, we were working with numbers only. Now we are working with units, for example, 5 apples for .99. This is comparing apples and money, and not only that, but we are comparing a certain number of apples for a certain amount of money. In this example, we have a rate but not a unit rate.

Students may ask why this is so. You can point out that this example has 5 apples for .99. The quantity if 5, it is not one. If we were figuring out the amount of money for 1 apple, then it would be the unit rate. When we know the unit rate, we can figure out any amount of money for any number of apples. Once we know how much one costs, we can figure out any number of items, in this case, apples.

Notice in the last section that this is the beginning of work with proportions. You don't need to use the word "proportions" because students can become stuck in the vocabulary. Students will use what they have learned to reason through the answer. This is all that is needed for this lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/simplifying-rates-and-ratios?playlist=Developmental%20Math - This is a Khan Academy video on simplifying rates and ratios.

http://www.khanacademy.org/video/finding-unit-rates?playlist=Developmental%20Math - This is a Khan Academy video on how to find unit rates.

Solving Proportions

Goal

The goals of this lesson can be found in the following objectives.

- Use cross products to identify proportions.
- Solve for the unknown part of given proportions using mental math.
- Solve for the unknown part of given proportions using related equations.
- Solve real-world problems involving proportions.

Relevant Review

Review writing ratios and simplifying ratios.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Proportion
two equal ratios.

Ratio
a comparison of two quantities can be written in fraction form, with a colon or with the word “to”.

Cross Products
to multiply the diagonals of each ratio of a proportion.

Teaching Strategies

The real world story problem *Stocking Shelves* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Beginning this lesson with a review of ratios will help students to begin to identify proportions. The key to proportions is the term “equal”. Students need to understand that a proportion is formed by TWO EQUAL RATIOS. Identifying proportions requires that they determine if the two ratios that they are presented are equal or equivalent.

Some students will be able to look at the two ratios and “reason” whether or not they are equal. These students have excellent number sense and/or have learned to simplify quite quickly so that they can determine equivalence without much effort. Allow these students to work in this way so that they can continue to strengthen these skills.

Other students will require a way “how” to figure out if two ratios form a proportion. For these students, cross –multiplying is a definite way to figure it out. We call it cross –products, but cross –multiplying will also work as a definition.

A natural next step is to work on solving proportions. Many students will be able to do this through reasoning and applying what they have learned. Be sure to teach them how to use cross –products so that all students are able to work on solving the proportions.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/writing-proportions?playlist=Algebra - This is a Khan Academy video on writing proportions from a word problem. This video could be used as a challenge for some students.

http://www.khanacademy.org/video/understanding-proportions?playlist=Developmental%20Math - This is a Khan Academy video designed to help students to understand proportions. It could be used for all learners.

Proportions and Scale Drawings

Goal

The goals of this lesson can be found in the following objectives.

- Given scale ratios, find actual or scale dimensions using proportions.
- Given actual and scale dimensions, find scale ratios using proportions.
- Solve real-world problems involving scale drawings and maps using proportions.

Relevant Review

Review ratios and proportions. The relevant review is in solving proportions and that the students really understand that two ratios must be equal to form a proportion.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Proportion
two equal ratios

Scale Drawing
a drawing used when a real life object is too big to draw with its actual dimensions.

Scale
the relationship of a drawing to the measurement of the real object

Scale Factor
the relationship between the measurement of the drawing and the measurement of the real object.

Teaching Strategies

The real world story problem *The Supermarket Display* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day

where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson is a real world application of ratios and proportions. For this lesson to be successful, students need to understand that equal ratios form a proportion and that they must "figure" out the missing value for a proportion to be true. When students understand this, then they will be able to apply proportions in situations requiring reasoning and problems solving such as this one on scale drawings. If students begin having difficulty in this lesson, go back and be sure that proportions and solving proportions are clear to them.

When working with proportions and scale measurement, students can use tools that they are familiar with such as rulers. Once students understand the steps to working through a proportion, they can be allowed to work in a hands–on way. Students can do this by using actual maps or rulers to figure out dimensions. Allow them time to work in small groups or pairs on the problems in the text. Through problem–solving, students develop and strengthen their capacities and skills.

Be sure to have graph paper available for student work.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/scale-and-indirect-measurement?playlist=ck12.org%20Algebra%201%20Examples - This is a Khan Academy video on scale and indirect measurement.

http://www.mathplayground.com/howto_ratiowordproblems.html - This is a math playground video on how to solve ratio word problems.

Understanding Percent

Goal

The goals of this lesson can be found in the following objectives.

- Identify equivalent ratios as fractions, decimals and percents.
- Write percents as decimals.
- Write percents as fractions.
- Read and interpret circle graphs of real-world data.

Relevant Review

Review ratios and equivalent ratios.

Review fractions as part of a whole and decimals as part of a whole.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Percent
: means out of 100, it is a quantity written with a % sign, a part of a whole (100)

Fraction
: a part of a whole, related to decimals and percents.

Decimal

a part of a whole shown by a decimal point, hundredths means two decimal places.

Circle Graph

a visual way of showing percents, a circle graph means 100. The entire circle represents 100 and each percent represents a quantity out of 100.

Teaching Strategies

The real world story problem *The Cereal Reorder* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In this lesson, you are going to bring together three things. First, students have already learned about fractions and decimals. They know how to work with them and how to identify them. Now you are going to add percents to the mix as a part of a whole out of 100. The three are connected and interchangeable. You want the students to realize this as you introduce them to percents.

When students complete the chart on equivalent ratios, take time to review their answers. Clarify any confusion and have the students correct any errors that they may have. Taking the time to do this now will help to prevent reteaching later.

Use the second day of the lesson to review converting fractions, decimals and percents. Then move on to circle graphs. Focus on circle graphs during the second day of the lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/percent-and-decimals?playlist=Arithmetic - This is a Khan Academy video on how to express a percent as a decimal and a decimal as a percent.

http://www.mathplayground.com/howto_perfracdec.html - This is a math playground video on converting fractions, decimals and percents.

Percents, Decimals and Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Write fractions as percents.
- Write decimals as percents.
- Compare and order fractions, decimals and percents.
- Write fractional or decimal real-world data as percents.

Relevant Review

Review the basics of understanding a percent as a part of a whole out of 100. Review the simple examples of fractions as percents and decimals as percents. Students will be building on these simple examples in this lesson.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Percent
means out of 100, it is a quantity written with a % sign, a part of a whole (100)

Fraction
a part of a whole, related to decimals and percents.

Decimal
a part of a whole shown by a decimal point, hundredths means two decimal places.

Equivalent
means equal

Compare
to determine whether a quantity is greater than, less than or equal to another quantity.

Order
to write in order from least to greatest or from greatest to least.

Teaching Strategies

The real world story problem *Sweeping and Mopping* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson is going to build on the previous one. In the last lesson, the denominators of the fractions being converted were 100 or simple denominators to work with. Now students will need to use arithmetic to convert a fraction to a percent. The same thing with a decimals. Alert students to the valuable teaching material in the colored text boxes. These helpful hints can be used to guide them through the lesson.

The lesson touches on fractional percents. This is an introduction for the students so don't worry if they have difficulty with the concept. They will have more opportunity to work with fractional percents in later math lessons.

When comparing fractions, decimals and percents, be sure to review the symbols. Encourage students to convert the values being compared to the same form. This is the simplest way to determine which is larger and then student work will be more accurate.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/percent-and-decimals?playlist=Arithmetic - This is a Khan Academy video on expressions percents as decimals and decimals as percents.

Finding a Percent of a Number

Goal

The goals of this lesson can be found in the following objectives.

- Find a percent of a number.
- Find prices involving discounts.
- Find prices involving sales tax.
- Estimate tips.
- Find amounts involving simple interest.

Relevant Review

Review percents as a quantity out of 100. Review the basics of converting fractions, decimals and percents.

Study Skill Tip

Be sure to have the students copy down the following vocabulary words in their notebooks.

Percent
a part of a whole written out of 100 using a % sign.

Proportion
two equal ratios.

Sales Tax
a percent added to a total

Discount
an amount taken off of an original price

Tip
15 –20% of a total bill paid to a server

Gratuity
another word for tip

Interest
the sum of money a person pays a bank for borrowing money

Principal
the amount of money borrowed

Rate
a percent that the bank charges for borrowing money

Annually
per year

Teaching Strategies

The real world story problem *Discount Day at the Supermarket* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are many things to teach in this lesson and the vocabulary and steps are many, therefore, it is recommended that you take two days to teach this lesson. Begin with the simple percent problems and then move on to the more complicated ones.

First, we are going to find the percent "of" a number. "Of" is a key word that means to multiply. This was taught to students in an earlier lesson. Some will remember this and others will have forgotten it. Therefore, be sure to review that "of" means to multiply. Students can solve this in two different ways. One is by using a proportion and one is by multiplying. There is a colored text box that reviews how to solve a proportion. Be sure to help the students to review these steps when you get to this part of the lesson.

Next, we work with discounts. Finding a discount requires that students use the steps from the first section. First, then find the percent of the number. That is the amount of the discount, then they subtract this from the original amount.

Key points to make during the lesson:

- "Of" means multiply
- Discounts are taken away
- Sales tax is added
- A gratuity or tip is added

The process of finding the amount is the same, but what you do with the amount calculated depends on what it is. Students can write this list in their notebooks.

Interest uses a formula. It is best to work through this step by step and explain each part of the formula to the students. Then using arithmetic, students can solve the problems in the text.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=6AZijeJDmgY - This is a youtube video from yourteacher.com on how to calculate simple interest.

http://www.mathplayground.com/mathatthemall2.html - This is a virtual site from mathplayground.com which helps students to calculate discounts, sales tax and percentages while shopping at the mall.

Problem-Solving Strategy: Use a Proportion

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.

- Develop and use the strategy: Use a Proportion.
- Plan and compare alternative approaches to solving problems.
- Solve real-world problems involving rates, proportions, or percents using selected strategies as part of a plan.

Relevant Review

Review the four part problem–solving plan.

Review identifying rates, proportions and percents so that students are able to apply them when problem solving.

Study Skill Tip

Review solving proportions.

Teaching Strategies

The real world story problem *The Frog Problem* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Use the two dilemmas in this lesson to teach the content of the lesson. You can use the problem with the cheetah and the problem with the frog to help students to apply what they have learned about proportions, rates and percents. Assign one –half of the class one problem and the other half of the class the other problem.

Do not let them look at the lesson in the text. Just simply present each problem. Then allow time for the students to work through them in small groups. They must present all of their findings and illustrate their work. Students can use their notebooks.

You can use this as an assessment class and by the end you will know who still needs more help and who has a firm grasp of the material.

Allow time for students to share their findings at the end of the lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/writing-proportions?playlist=Algebra - This is a Khan Academy video on how to use a proportion to solve a word problem.

1.9 Geometric Figures

This ninth chapter *Geometric Figures* takes students into the world of line and form. Here they will explore different types of lines and angles that then form plane figures. This will expand to sections on symmetry along with similar and congruent figures. This chapter is a building block chapter for later lessons in geometry. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Introduction to Geometry
- Classifying Angles
- Classifying Triangles
- Classifying Quadrilaterals
- Classifying Polygons
- Congruent and Similar Figures
- Line Symmetry
- Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Introduction to Geometry –1 Day
- Classifying Angles –1 Day
- Classifying Triangles –1 Day
- Classifying Quadrilaterals –2 Days
- Classifying Polygons –2 Days
- Congruent and Similar Figures –2 Days
- Line Symmetry –1 Day
- Problem –Solving Strategy: Look for a Pattern; Use a Venn Diagram –1 Day

Day 12 –Review

Day 13 - Test

Introduction to Geometry

Goal

The goals of this lesson can be found in the following objectives.

- Identify points, rays, lines and segments using words and symbols.

- Identify intersecting and parallel lines.
- Identify angles by vertex and ray.
- Draw angles using a protractor.

Relevant Review

This is a new chapter, there isn't a relevant review.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Point
a location in space that does not have size or shape.

Ray
a line that has one endpoint and continues indefinitely in one direction.

Line
a set of connected points without endpoints.

Line Segment
a set of connected points with two endpoints.

Point of Intersection
the point where two intersecting lines meet.

Intersecting Lines
lines that cross or meet at some point

Parallel Lines
Lines that do not cross or meet EVER and are equidistant apart.

Angle
a geometric figure formed by two rays that connect at a single point or vertex.

Vertex
The points on an angle

Protractor
a tool used to measure an angle in terms of degrees.

Teaching Strategies

The real world story problem *The Skateboard Park* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There is a tip given right away in this lesson about keeping a notebook to keep track of definitions and symbols. Be sure that the students draw the symbols for each term into their notebooks. In this way, they have a visual picture to connect with the written definition.

When identifying angles, point out to students that the points name the angle. Then when you work with protractors, use actual protractors and have the students practice measuring each one. For example, you could have them each draw three angles. Then they exchange papers with another friend and measure the angles. Peer exchange allows students the chance to check each other's work and offer suggestions. Be sure that students use rulers so that their work is accurate.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/lines–line-segments–and-rays?playlist=Geometry - This is a Khan Academy video on the basic terms of geometry. It focuses on lines, line segments and rays.

http://www.khanacademy.org/video/using-a-protractor?playlist=Geometry - This is a Khan Academy video on how to use a protractor.

Classifying Angles

Goal

The goals of this lesson can be found in the following objectives.

- Classify angles as acute, right, obtuse or straight.
- Classify angle pairs as supplementary or complementary
- Use real-world angle pair diagrams to find unknown angle measures.

Relevant Review

Before beginning this lesson, review identifying angles and how to name them. Be sure that students understand that when they measure an angle that they are looking for a measurement in degrees. If using protractors was challenging, then review this skill.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Acute angle
an angle less than 90 degrees.

Right angle
an angle equal to 90 degrees.

Obtuse angle
an angle greater than 90 degrees but less than 180 degrees.

Straight angle
a straight line equal to 180 degrees

Supplementary angles
two angles whose sum is 180 degrees.

Complementary angles
two angles whose sum is 90 degrees.

Teaching Strategies

The real world story problem *Moving the Skatepark* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Now that the students understand how to identify an angle, they are going to learn how to classify angles. You can begin this lesson by talking about what the word "classify" actually means. Ask the students for examples for times when they have classified something and/or a time when they have seen someone else classify something. This will give you an excellent introduction to classifying angles.

As with everything concerning angles, they are classified according to degrees. First you begin with single angles. These names need to be learned and memorized by the students. Then you move to pairs of angles. Be sure that these are distinct for the students. In fact, you can point out the two differences –single angle classification and the classification of pairs of angles.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/acute-right-and-obtuse-angles?playlist=Geometry - This is a Khan Academy video on acute, right and obtuse angles.

http://www.khanacademy.org/video/complementary-and-supplementary-angles?playlist=Geometry - This is a Khan Academy video on identifying complementary and supplementary angles.

Classifying Triangles

Goal

The goals of this lesson can be found in the following objectives.

- Classify triangles by angles.
- Classify triangles by sides.
- Draw specified triangles using a ruler and a protractor.
- Find unknown angle measures in given triangles.

Relevant Review

Review measuring angles and identifying them as acute, right and obtuse. Students will need to apply this information to their work with triangles.

Study Skill Tip

Be sure to have students copy the following vocabulary words down in their notebooks.

Triangle
a three sided figure with three angles. The prefix "tri"means three.

Acute Triangle
all three angles are less than 90 degrees.

Right Triangle
One angle is equal to 90 degrees and the other two are acute angles.

Obtuse Triangle
One angle is greater than 90 degrees and the other two are acute angles.

Equiangular Triangle
all three angles are equal

Scalene Triangle
all three side lengths are different

Isosceles Triangle
two sides lengths are the same and one is different

Equilateral Triangle
all three side lengths are the same

Protractor
A tool used to measure angles

Interior angles
the angles inside a figure

Teaching Strategies

The real world story problem *The Bermuda Triangle* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

All students are familiar with triangles by the time they have gotten to the sixth grade. What they aren't familiar with is how we use them in Geometry. This is a building block lesson and the information that students gain from it will be used for a long time. Be sure to cover each concept completely and that the students take accurate notes.

First, students should realize that triangles can be seen in two ways –by the measure of the interior (inside) angles and by the length of the sides. These are the two ways we can classify them. A big part of this lesson is on the part of the students. They have to learn and memorize the terms and what they mean. You can work on this with "Jeopardy" games on vocabulary and with other types of games. But until they have committed these terms to memory, they will struggle on identifying the different types of triangles.

Once the students have practiced classifying triangles, then you can move on to figuring out missing angle measures. This is practice in solving equations. Because this is middle school math, require that the students write out the equation even if they can solve it in their heads. This has them begin to think about variables and unknown quantities in a new way. It will strengthen their algebraic thinking for the future.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for

the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.brightstorm.com/math/geometry/geometry-building-blocks/types-of-triangles/ - This is a Brightstorm video on different types of triangles.

Classifying Quadrilaterals

Goal

The goals of this lesson can be found in the following objectives.

- Classify quadrilaterals by angles
- Classify quadrilaterals by sides
- Draw specified quadrilaterals using ruler and protractor
- Find unknown angle measures in given quadrilaterals

Relevant Review

Students will need to use a protractor to draw specified quadrilaterals. To support this lesson, take the time to review measuring angles using a protractor.

Study Skill Tip

Be sure to write down the following vocabulary words in their notebooks.

Quadrilateral
closed figure with four sides and four vertices.

Trapezoid
Quadrilateral with one pair of opposite sides parallel.

Rectangle
Parallelogram with four right angles.

Parallelogram
Quadrilateral with opposite sides congruent and parallel.

Square
Four congruent sides and four congruent angles.

Rhombus
Parallelogram with four congruent sides.

Parallel
lines that are equidistant apart and will never intersect

Congruent
exactly the same, having the same measure

Teaching Strategies

The real world story problem *The Grind Box* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This is another lesson where knowing the vocabulary will be essential for student success. In addition to writing definitions, it is recommend that students draw an example of each of the terms in their notebooks. This provides students with a visual connection to the term as well as a written definition.

There are two distinct parts of the lesson. The first part involves learning about each specific type of quadrilateral and how to identify each type. This could be one whole lesson. Students are introduced to each type of quadrilateral and use math language to describe each. When you teach the students about these figures, be sure to use accurate math language yourself. Students will be hearing words like "congruent" for many years. It will help if they are introduced to these key terms right from the start.

The second part of the lesson is on measuring and drawing. This could be the second day of the lesson. Students would spend the first day learning all about each type and then work on drawing them accurately on the second day. This then leads to figuring out missing angle measures as well.

Require students to use protractors and rulers so that their work is accurate.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathtrain.tv/play.php?vid=195 - This is a math train video on how to identify different types of quadrilaterals.

Classifying Polygons

Goal

The goals of this lesson can be found in the following objectives.

- Classify polygons.
- Distinguish between regular and irregular polygons.
- Relate sides of polygons to the sum of the interior angles.
- Relate sides of polygons to the number of diagonals form a vertex.

Relevant Review

Review vocabulary words from the previous lessons. This is an excellent way to assist students in their learning of different vocabulary words.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Polygon
A simple closed figure formed by three or more line segments.

Pentagon
five sided polygon

Hexagon
six sided polygon

Heptagon
seven sided polygon

Octagon
eight sided polygon

Nonagon
nine sided polygon

Decagon
ten sided polygon

Regular Polygon
polygon with all sides congruent

Irregular Polygon
a polygon where all of the side lengths are not congruent

Congruent
exactly the same or equal

Diagonal
a line segment in a polygon that connects nonconsecutive vertices

Nonconsecutive
not next to each other

Teaching Strategies

The real world story problem *The Sculpture* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When preparing for this lesson, notice that there are a lot of key vocabulary words for students to learn. You can have the students write these terms on index cards, where they write the definition on one side and draw a picture on the other side. This can be used for quizzing purposes or can create a memory type of game for the students. Learning the vocabulary is key for this lesson.

You can divide this lesson into two days. On the first day, teach all of the polygons, how to identify them and the vocabulary connected with each. Be sure that students understand the difference between a regular polygon and an irregular one. This is the work of the first day.

The next day, go back over each type of polygon and then move into the sections on figuring out missing angle measures. This will get trickier because there is more algebra involved as students work with equations. Point out

the information in the text boxes –especially the "Black Back in Math" box about the angle measures of a triangle and a quadrilateral. Students will need to know this information when problem solving with quadrilaterals.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Congruent and Similar Figures

Goal

The goals of this lesson can be found in the following objectives.

- Identify given triangles as similar, congruent or neither.
- Identify corresponding parts of congruent figures.
- Identify corresponding parts of similar figures.
- Find unknown measures of corresponding parts of similar figures.
- Use similar figures to measure indirectly.

Relevant Review

Review triangles and the different types of triangles prior to beginning this lesson. Review the definition of congruent.

Review ratios as comparing two quantities and proportions as a pair of equal ratios. Then review using cross –products to solve a proportion.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Congruent
having the same size and shape and measurement

Similar
having the same shape, but not the same size. Similar shapes are proportional to each other.

Corresponding
matching-corresponding sides between two triangles are sides that match up

Ratio
a way of comparing two quantities

Proportion
a pair of equal ratios.

Indirect Measurement
using the characteristics of similar triangles to measure challenging things or distances.

Teaching Strategies

The real world story problem *The Pair of Skateboard Ramps* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

This lesson can be divided into two days. The first day will focus on identifying and the second day will focus on using the identification in indirect measurement using ratios and proportions.

The terms *similar* and *congruent* seem simple enough, but they are confused all the time. Students need to understand these terms to be able to work on identifying figures.

Next, the students will need to understand how to identify corresponding parts or sides of figures. You can use the word "matching" to help them to understand these parts. In congruent figures,the corresponding sides have the same measure. In similar figures, the corresponding sides are proportional to each other. This means that while they have different measures, that when you compare the sides, that the sides form a proportion.

When you reach the last section, tell the students that because the corresponding sides of similar triangles form a proportion, that you can solve for missing sides just as they did when working with proportions. Ask them how –the answer should be cross –products. If students struggle with this, then help them to figure out the missing side so that they can see how cross - products will help them to find an accurate answer.

Proportional reasoning is key and often undertaught. Many students don't see the connections when things are similar. Take the time to stress proportions and similar as connected for the students. This will serve them in problem solving throughout higher levels of mathematics.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/similar-triangles?playlist=Geometry - This is a Khan Academy video on similar triangles.

http://www.khanacademy.org/video/understanding-proportions?playlist=Developmental+Math - This is a Khan Academy video on understanding proportions. It would be an excellent review video prior to teaching this lesson.

Line Symmetry

Goal

The goals of this lesson can be found in the following objectives.

- Identify lines of symmetry in figures and objects
- Draw figures with specified symmetry
- Recognize congruence in mirror images

Relevant Review

Review the term congruent and how to figure out whether or not a figure is congruent of similar.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Line Symmetry
when a figure can be divided into equal halves that match.

Bilateral Symmetry
when a figure can only be divided into one equal half. This figure has one line of symmetry.

Lines of Symmetry
the lines that can be drawn to divide a figure into equal halves. Some figures have multiple lines of symmetry. Some figures have one line and some have no lines of symmetry.

Rotational Symmetry
When a figure rotated up to 360° around a fixed point looks exactly the same as it did in the beginning.

Congruent
exactly the same size and shape

Teaching Strategies

The real world story problem *Symmetrical Ramps* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Review the words "vertical" and "horizontal" prior to beginning this lesson. The terms are used in the context of the lesson. Be sure that students clearly understand them.

Asking students to design their own figures with different types of symmetry will cause the students to demonstrate an understanding of the material taught. Require that they use rulers will help their work to be accurate.

Pattern blocks are also a lot of fun when it comes to working with symmetry.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/4_Line_Symmetry/index.html - This is an excellent video on symmetry and on understanding symmetry.

Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Look for a Pattern.

- Develop and use the strategy: Use a Venn Diagram.
- Plan and Compare alternative approaches to solving problems.
- Solve real-world problems using selected strategies as a part of a plan.

Relevant Review

Review the four part problem solving plan introduced in earlier lessons.

Review factoring and prime factorization as it is used in this lesson as a way of looking for patterns.

Study Skill Tip

Ask students to draw a blank Venn diagram in their notebooks. Then they will have a format for working with Venn diagrams when problem solving.

Teaching Strategies

The real world story problem *Skate Park Construction* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are two strategies that students will work with in this lesson. One is on Venn diagrams and the other is on patterns. When looking at Venn diagrams, be sure that students understand that the problem will need to have some things in common and some things not in common. Venn diagrams are used to compare and to organize lists. Teach the students to look for these things when deciding whether or not to use a Venn diagram in problem –solving. Patterns are useful when numbers are involved, especially numbers that repeat in some way.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/patterns-in-sequences-2?playlist=Algebra+I+Worked+Examples - This is a Khan Academy video on patterns.

http://www.youtube.com/watch?v=kti6G0Lqfig&feature=related - This is a youtube video on Venn diagrams and on how to use Venn diagrams when problem –solving math problems. This one begins with an example that uses food at a carnival.

1.10 Geometry and Measurement

This tenth chapter *Geometry and Measurement* continues where chapter nine left off. Students began with identifying geometric figures and learning terms, and now they are moving on to working with measurement through area. Students will also be introduced to solid figures in this lesson. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Area of Parallelograms
- Area of Triangles
- Circumference of Circles
- Area of Circles
- Classifying Solid Figures
- Surface Area and Volume of Prisms
- Surface Area and Volume of Cylinders
- Problem-Solving Strategy: Solve a Simpler Problem

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Area of Parallelograms –1 Day
- Area of Triangles –1 Day
- Circumference of Circles –1 Day
- Area of Circles –2 Days
- Classifying Solid Figures –1 Day
- Surface Area and Volume of Prisms –2 Days
- Surface Area and Volume of Cylinders –2 Days
- Problem –Solving Strategy: Solve a Simpler Problem –1 Day

Day 12 –Review

Day 13 - Test

Area of Parallelograms

Goal

The goals of this lesson can be found in the following objectives.

- Recognize the area of a parallelogram as the area of a related rectangle.
- Find the area of parallelograms given base and height.

- Find unknown dimensions of parallelograms given area and another dimension.
- Estimate actual areas approximated by parallelograms in scale drawings.
- Solve real world problems involving area of parallelograms.

Relevant Review

It would be relevant to review parallelograms and their characteristics. This will help students to apply the qualities of a parallelogram to measurement as we work with area and formula for area of parallelograms.

Review right angles as measuring 90 degrees.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Area
: the space within the perimeter of a figure or place. Area often refers to the surface or covering, the middle of a figure. Area is measured in square units.

Parallelogram
: a quadrilateral with two pairs of opposite congruent sides.

Rectangle
: a parallelogram with two pairs of opposite congruent sides and four 90 degree angles.

In addition, have students begin a section for formulas. This section of their notebook will contain formulas for finding different geometric measures. This list of formulas can serve as an easy reference point for students. Standardized tests will often provide formulas without any indication of what the formula is for –if the students have committed the formulas to memory, then they have an added advantage when it comes to testing.

Teaching Strategies

The real world story problem *The Quilt Block* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The first thing that you want students to notice is that a rectangle is like a parallelogram. You can refer to them as relatives because they have some of the same characteristics, but are not congruent figures. While rectangles have a length and width, parallelograms have a base and height.

Students also need to understand that square units are used to describe area. Point out that the operation for area is multiplication and this is where the units squared or square units comes from. Many students will forget this when they solve problems.

The first part of the lesson focuses on finding the area of parallelograms and rectangles. Then the students are given the area and one dimension and must solve for the other dimension. Remind students that the opposite of multiplication is division. They used multiplication to find the area, so they can use division to find the missing dimension.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/area-and-perimeter?playlist=Geometry - This is a Khan Academy video on area and perimeter.

http://www.brightstorm.com/math/geometry/area/area-of-parallelograms/ - This is a Brightstorm video on how to calculate the area of a parallelogram by using information about rectangles.

Area of Triangles

Goal

The goals of this lesson can be found in the following objectives.

- Recognize the area of a triangle as half the area of a related parallelogram.
- Find the area of triangles given base and height.
- Find unknown dimensions of triangles given area and another dimension.
- Find areas of combined figures involving triangles.

Relevant Review

Review area and how to find the area of a parallelogram. This lesson builds on the previous lesson by showing students how to find the area of a triangle by using the area of a parallelogram.

Study Skill Tip

Students should continue to copy down formulas in the section of their notebooks designated for formulas.

Be sure to have students copy down the following vocabulary words in their notebooks.

Triangle
a polygon with three sides.

Parallelogram
a four sided figure with opposite sides congruent and parallel.

Rectangle
a parallelogram with opposite sides congruent and parallel and with four right angles.

Square
a parallelogram with four congruent, parallel sides and four congruent right angles.

Teaching Strategies

The real world story problem *The Triangle in the Quilt Block* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Take the time to review finding the area of a parallelogram before moving on to finding the area of a triangle. This is important since the lesson builds on the information in the previous lesson. You can use this to review the definitions of parallelogram, square and rectangle. Use this introduction to help students to make sense of the formula for finding the area of a triangle. When you divide a parallelogram in half, it makes it very clear how to figure out the area of a triangle.

One thing to point out is that multiplying by one - half is the same as dividing a quantity by 2. Some students may not know this fact.

After practicing with the formula for area, students will need to work backwards and find missing dimensions. This is done best when the formula is used and students are reminded to work backwards through the formula to find the missing dimension. It will involve division since this is the inverse operation for multiplication.

Combined areas use the information taught in the last two lessons. Remind students to use their formulas and that combined areas are added together for a total area.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.brightstorm.com/math/geometry/area/area-of-triangles/ - This is a Brightstorm video on how to find the area of a triangle.

http://www.mathplayground.com/howto_area_triangle.html - This is a math playground video on how to find the area of a triangle.

Circumference of Circles

Goal

The goals of this lesson can be found in the following objectives.

- Identify ratio of circumference to diameter as pi.
- Find the circumference of circles given diameter or radius.
- Find diameter or radius of circles given circumference.
- Solve real world problems involving circumference of circles.

Relevant Review

Review perimeter as the distance around a polygon. This will help when working with circles. Circles are not polygons because they are not made up of line segments. Therefore, we need a different formula for calculating the perimeter of a circle –which we call circumference.

Study Skill Tip

Remind students to copy all formulas into their notebooks in the section on formulas.

Have students draw a diagram of a circle and label each part of the circle.

Be sure to have students copy down the following vocabulary words in their notebooks.

Circumference
: the measure of the distance around the outside edge of a circle.

Diameter
: the measure of the distance across the center of a circle.

Radius
the measure of the distance half-way across the circle. It is the measure from the center to the outer edge. The radius is also half the length of the diameter.

Pi
the ratio of the diameter to the circumference, 3.14

Archimedes
a Greek mathematician and philosopher who identified 3.14 as pi.

Teaching Strategies

The real world story problem *The Quilting Visitors* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

You will see in the pacing section that this lesson is recommended as a one day lesson. However, if you wanted to do some exploration about pi, you could do that one the first day and then move on to circumference on the second day. There are tons of websites about pi including this one which has games on pi http://www.squidoo.com/free-flash-math-games . You could also celebrate pi day on March 14, and there are many websites about this as well.

Once students understand pi and where it comes from, then you can move on to calculating circumference. The easiest way to do this is to use the formula. There are many examples in the text for the students to practice with.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/circles–radius–diameter-and-circumference?playlist=Geometry - This is a Khan Academy video on circles including radius, diameter and circumference.

Area of Circles

Goal

The goals of this lesson can be found in the following objectives.

- Find areas of circles given radius or diameter.
- Find radius or diameter of circles given area.
- Find areas of combined figures involving parts of circles.
- Display real-world data using circle graphs.

Relevant Review

Review the parts of a circle and how to find the circumference of the circle.

Study Skill Tip

Ask students to copy down all formulas in the section of their notebook designated for formulas.

Be sure to ask students to write the following vocabulary words into their notebooks.

Area
the surface or space of the figure inside the perimeter.

Radius
the measure of the distance halfway across a circle.

Diameter
the measure of the distance across a circle

Squaring
uses the exponent 2 to show that a number is being multiplied by itself. $3^2 = 3 \times 3$

Pi
the ratio of the diameter to the circumference. The numerical value of pi is 3.14.

Teaching Strategies

The real world story problem *The Quilting Survey* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In the last lesson, students learned to find the circumference of the circle. Remind students that the circumference is the measurement of the outside edge and that it has a relationship to the diameter of the circle. Then you can move on to teaching them about figuring out the area of the circle.

Be sure that the students copy down the formula for area into their notebooks and remind them that area is always measured in square units. This is the case if we are working with polygons or with circles.

Notice the steps for working backwards. First students need to divide the given area by pi (3.14). Then they have to find the square root of the quotient. They won't have this terminology yet, so you can talk about figuring out which number times itself or which number squared is equal to the quotient. This is how it is done in the text. You can reinforce this when teaching the lesson.

Taking two days on this lesson will help you to work through all of the content of this lesson. Be sure to review on the second day.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/area-of-a-circle?playlist=Geometry - This is a Khan Academy video on finding the area of a circle.

Classifying Solid Figures

Goal

The goals of this lesson can be found in the following objectives.

- Classify solid figures as prisms, cylinders, pyramids, cones or spheres.
- Identify faces, edges and vertices of solid figures.
- Select real-world examples of given solid figures.

Relevant Review

In this lesson, students will begin learning about solid figures or three –dimensional figures. Review all of the two - dimensional figures that students have been introduced to in previous lessons.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Plane Figure
a flat two-dimensional figure.

Solid Figure
a three-dimensional figure with height, width and depth.

Prism
a solid with two parallel congruent bases.

Cylinder
a solid with two parallel congruent circular bases.

Pyramid
a polygon for a base and triangular faces that meet at one vertex.

Cone
a solid with a circular base and one vertex

Sphere
a three-dimensional circular solid

Face
any flat surface on a solid figure

Edge
when two faces meet in a line segment. The line segment is the edge.

Vertex
when three or more faces meet at a single point.

Polyhedron
a solid figure with flat surfaces that are polygons.

Teaching Strategies

The real world story problem *The Sewing Box* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where

the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The best teaching strategy for this lesson is to have the students build solid figures. This is the best way to understand faces, edges and vertices as well as identification. Using websites or nets will help in this process. Here are some resources.

http://www.senteacher.org/wk/3dshape.php - This is a website with free nets for teachers to print and then use with their students.

http://www.korthalsaltes.com/ - This is a great websites with all kinds of paper models of polyhedra.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.gamequarium.org/dir/Gamequarium/Math/Geometry/Solid_Figures/ - This is a website with interactive games and activities for learning to identify solid figures.

http://www.brightstorm.com/math/geometry/volume/3-d-solid-properties/ - This is a Brightstorm video on properties of three - dimensional solid figures.

Surface Area and Volume of Prisms

Goal

The goals of this lesson can be found in the following objectives.

- Identify surface area of prisms as the sum of the areas of faces using nets.
- Find surface area of rectangular and triangular prisms using formulas.
- Identify volumes of prisms as the sum of volumes of layers of unit cubes.
- Find volumes of rectangular and triangular prisms using formulas.

Relevant Review

Review characteristics of solid figures from the last lesson. Review the difference between a prism, a pyramid, a cylinder and a sphere. Review the vocabulary words face, edge and vertex.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Surface area
: the outer covering of a solid figure-calculated by adding up the sum of the areas of all of the faces and bases of a prism.

Net
: diagram that shows a “flattened” version of a solid. Each face and base is shown with all of its dimensions in a net. A net can also serve as a pattern to build a three-dimensional solid.

Triangular Prism
: a solid which has two congruent parallel triangular bases and faces that are rectangles.

Rectangular Prism
a solid which has rectangles for bases and faces.

Volume
the amount of space inside a solid figure

Teaching Strategies

The real world story problem *The Sewing Box* is used again in this lesson. In the last lesson, it was used to introduce solid figures, now it is used to teach surface area. Students can use what they learned in the last lesson as a foundation for this one. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In the last lesson, you used nets to build solid figures. The students did not know that these patterns were called nets, and now you are going to use them to calculate surface area. This will be an excellent bridge from the last lesson to this one. Now you can use the same nets and show students how to calculate the area of each face, then add them up to find the surface area of the solid.

Notice that students will need to be calculating the area of each face of the solid. This means multiplication. Then all of the areas (the products) are added together to find the final surface area. Remind students that when they see a capital B that it means the area of base –which is different from the lowercase b which they used to find the area of a parallelogram. A parallelogram is a two –dimensional figure and we are working with three –dimensional figures.

It is recommended that you spend two days on this lesson. Students will need to practice finding surface area and volumes of different prisms. You can also make this hands –on by bringing in different boxes. Then students can practice finding the surface area and volume of each of the boxes. This can be done in pairs or small groups.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.brightstorm.com/math/geometry/volume/volume-of-prisms/ - This is a Brightstorm video on figuring out the volume of prisms.

http://www.brightstorm.com/math/geometry/area/surface-area-of-prisms/ - This is a Brightstorm video on how to calculate the surface area of prisms.

Surface Area and Volume of Cylinders

Goal

The goals of this lesson can be found in the following objectives.

- Identify surface area of cylinders as the sum of the areas of faces using nets.
- Find surface area of cylinders using formulas.
- Identify volume of cylinders as the sum of volumes of layers of unit cubes.
- Find volumes of cylinders using formulas.

Relevant Review

Review the characteristics of cylinders and ask students to identify the difference between a cylinder, a prism and a pyramid.

Then review the definitions for surface area and volume. This will help the students to remember which measurements they are calculating.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Volume
the amount of space inside a three-dimensional figure.

Surface Area
the entire outer covering or surface of a three-dimensional figure. It is calculated by the sum of the areas of each of the faces and bases of a solid.

Cylinder
a three-dimensional figure with two congruent parallel circular bases and a curved flat surface connecting the bases.

Radius
the measure of the distance halfway across a circle.

Teaching Strategies

The real world story problem *The Bean Containers* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

It is recommended that this lesson be taught over two days. On the first day, you can focus on surface area and on the second day, you can teach volume. Be sure to show the students how the formula for area of a circle is a part of the formula for surface area. This makes perfect sense, but you will need to point it out to the students.

You can also use nets of cylinders to show students how to calculate surface area. http://www.senteacher.org/wk/3dshape.php - This website has a printable net of a cylinder for teachers. You can use this net to help students with their calculation of surface area.

Volume can first be explored in a hands –on way. You can use the illustration of the beans and the containers from the real –world problem. Then move on to the formula and to figuring out volume as it is shown' in the text.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/cylinder-volume-and-surface-area?playlist=Geometry - This is a Khan Academy video on finding the surface area and volume of a cylinder.

Problem-Solving Strategy: Solve a Simpler Problem

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Solve a Simpler Problem.
- Plan and Compare Alternative Approaches to Solving Problems.
- Solve real-world problems using selected strategies as part of a plan.

Relevant Review

Review the four part problem solving plan that has been introduced in earlier lessons.

Study Skill Tip

There aren't any study skills necessary for this lesson.

Teaching Strategies

The real world story problem *The Bean Containers* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

You can use this story problem as a way to work through the content in the lesson. The problem - solving strategy "find a pattern" at the end of chapter nine links up nicely to this one. You can use the video on patterns and problem solving to help with this lesson.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/patterns-in-sequences-2?playlist=Algebra+I+Worked+Examples - This is a Khan Academy video which focuses on breaking down problems into smaller parts using patterns.

1.11 Integers

This eleventh chapter *Comparing* Integers takes students from Geometry into the world of numbers. These numbers are new and are negative. This expands the world of numbers the students have been exposed to thus far. Integers are building block for Algebra. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Comparing Integers
- Adding Integers
- Subtracting Integers
- Multiplying Integers
- Dividing Integers
- The Coordinate Plane
- Transformations
- Surveys and Data Displays

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Comparing Integers –1 Day
- Adding Integers –1 Day
- Subtracting Integers –2 Days
- Multiplying Integers –1 Day
- Dividing Integers –1 Day
- The Coordinate Plane –1 Day
- Transformations –1 Day
- Surveys and Data Displays –1 Day

Day 9 –Review

Day 10 - Test

Comparing Integers

Goal

The goals of this lesson can be found in the following objectives.

- Write integers representing situations of increase/decrease, profit/loss, above/below, etc.
- Identify opposites of given integers

- Compare and order integers on a number line.
- Compare and order positive and negative fractions and decimals.

Relevant Review

Review whole numbers as counting numbers. Show a number line to the students starting with zero and expanding to the positive counting numbers. The number line will be expanding to include negative numbers during this lesson.

Study Skill Tip

When using the number line to teach positive and negative numbers to the students, ask the students to draw a number line that includes both positive and negative numbers into their notebooks.

Be sure to have students copy the following vocabulary words down in their notebooks.

Integers
: the set of whole numbers and their opposites

Negative Numbers
: numbers that are less than zero

Positive Numbers
: numbers that are greater than zero

Zero
: is a part of the set of integers, but is neither positive or negative

Teaching Strategies

The real world story problem *The Pen Pal Project* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The world of negative numbers opens up a whole new world for the students. Up until this time, they have been working with counting numbers and with parts of numbers including fractions and decimals. Now they are going to begin to think about negative numbers or values that really don't exist. A negative is below zero. This is a new way of thinking for the students.

There are many instances where negative numbers are talked about in real world situations. Use the examples in the text to make this real for the students. They may be able to brainstorm some new examples as well. Always allow time for the students to assimilate new information by making connections to the material that is real for them. Encourage them to share these realizations.

Be sure to review the symbols that we use to compare values (<, >,=, etc). Ask students to think about numbers and their opposites. This shows them that every number has a positive and a negative value associated with it.

A tricky concept is that the closer a negative number is to zero, the larger it is. Take time to repeat this many times and give students many opportunties for mistakes. They will want it to be larger because the digits are larger, this shows a lack of true understanding of the concept of a negative number. Use number lines to help you teach this to the students.

Fractions and decimals can work into this as well. Review ordering positive fractions and positive decimals. Then bring in the negatives as opposites. This will help students as they think about negative fractions and decimals. Use number lines to help make the ordering and comparing clearer.

Repeat and reaffirm that the closer a negative value is to zero the larger the value. This can't be repeated too many times. It is a common place for error.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Adding Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find sums of integers on a number line.
- Identify absolute values of integers.
- Add integers with the same sign.
- Add integers with different signs.

Relevant Review

Review identifying integers on a number line. Remind students that the closer a negative value is to zero, the larger it is.

Study Skill Tip

Be sure to give students a number line to work with at their desks. You can take a strip of cardboard and have each of them create a number line. They can do this with a ruler and then use the number line through all of the work that they are doing.

Be sure to have students copy down the following vocabulary words in their notebooks.

Integer
: the set of whole number and their opposites. Positive and negative whole numbers are integers.

Absolute Value
: the number of units that an integer is from zero. The sign does not make a difference-absolute value is the number of units.

Teaching Strategies

The real world story problem *The Trouble with Time Zones* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When working through this lesson, review the language of sums meaning addition, and use the words “losses” and “gains” when talking about adding negative and positive numbers. This will help students to make a real –world connection to integers.

Absolute value is a topic where students often make mistakes. They forget that an absolute value is neither negative or positive, it is the distance that a value is from zero. Be sure to review and repeat this statement many times so that students really have an understanding of it.

This lesson moves from a concrete example of adding integers to an abstract one. The concrete way of adding is through the use of the number line. The abstract method is through the mental math of adding the integers. Some students will be able to easily make this leap, while others will need more practice with the number line.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/adding-integers-with-different-signs?playlist=Developmental+Math - This is a Khan Academy video that teaches students how to add integers with different signs.

http://www.khanacademy.org/video/absolute-value-of-integers?playlist=Developmental+Math - This is a Khan Academy video on how to find the absolute value of integers.

Subtracting Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find differences of integers on a number line.
- Subtract integers with the same sign.
- Subtract integers with different signs.
- Solve real-world problems involving sums and differences of integers.

Relevant Review

Review adding integers with the same signs and with different signs. Use a number line to review how to use one to find sums. This review is essential since this lesson will build on it with subtraction.

Review absolute value, the symbol for absolute value, and how to find the absolute value of a number by figuring out the distance that the number is from zero.

Study Skill Tip

Be sure to have students use a number line when beginning their work with subtraction. This could be the same number line that they created on strips of poster board in the last lesson.

Be sure to have students copy down the following vocabulary words in their notebooks.

Sum
: the result of an addition problem.

Difference
: the result of a subtraction problem.

Teaching Strategies

The real world story problem *The Football Game* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class. This problem could also be created on the blackboard with a football field. Then students could actually work with losses and gains on a larger format. More problems could also be created using this theme.

There are several great websites with interactive games for students to practice working with integers. Here are a couple of them.

http://classroom.jc-schools.net/basic/math-integ.html - There are several math games on this site for students to choose from and practice with.

http://www.funbrain.com/numbers.html - This is a fun brain game site with many games for students to help them build their math skills.

When there is a key point in the text, be sure to have students write this key point down in their notebooks. Subtraction is a topic where students often struggle. Because of this, students will need more help. It is recommended that you take two days to teach this lesson. On the first day, work with number lines and subtraction. Then on the second day, you can complete the other material in the lesson. Working in a slow way will help students and will help prevent some reteaching.

Pay attention to the helpful hints in the colored text boxes. One key for this lesson is that subtracting an integer is the same as adding it's opposite.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=RDw3Gq-heTw - This is a yourteacher.com video on how to subtract integers. It begins with subtracting integers by using a number line.

Multiplying Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find products of positive integers.
- Find products of positive and negative integers.
- Find products of negative integers.
- Evaluate numerical and algebraic expressions involving integer multiplication.

Relevant Review

Review identifying integers on a number line. Review adding and subtracting integers using a number line. Here are five examples to help you to review.

1. $-9+-3=$ ____ -12
2. $-19+33=$ ____ -14
3. $45-67=$ ____ -22
4. $12--9=$ ____ 21
5. $-12+-18=$ ____ -30

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Product
the result of a multiplication problem.

Integer
the set of whole numbers and their opposites.

Commutative Property of Multiplication
a property that states that it doesn't matter which order you multiply terms. The product will be the same. $ab = ba$

Numerical Expression
an expression that contains multiple numbers and operations.

Algebraic Expression
an expression that contains numbers, variables and operations.

Rules are an important part of multiplying integers. When a text box presents itself, stop the lesson and take the time to have students copy these notes down in their notebooks.

Teaching Strategies

The real world story problem *Welcome to Jafakids* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Multiplying integers becomes easier when the students know their times tables. If students don't know their times tables, then it is important that you take the time to review these tables on a daily basis.

When working with more than two integers, remind students of the order of operations. There are many sets of parentheses in the review problems. Be sure to have students do the work in parentheses first. Working with more than one operation at a time can be challenging for students. Be sure to review adding and subtracting integers so that when the three operations, adding, subtracting and multiplying, are combined that students feel confident in their ability to solve problems.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=-4-yuOPe1eI&feature=relmfu - This is a video from yourteacher.com on how to multiply integers.

http://www.squidoo.com/integers - These are math tutor videos where students can select the topic that they want to review. Operations with integers are one option on this website.

Dividing Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find quotients of positive integers.
- Find quotients of positive and negative integers.
- Find quotients of negative integers.
- Evaluate numerical and algebraic expressions involving integer division.

Relevant Review

Review multiplying integers and the rules associated with multiplying integers. Because division is the opposite of multiplication, the same rules are going to apply to both operations.

As always, take time to review the times tables. They will help the students when working with multiplication and division of integers.

Study Skill Tip

Be sure to have students copy down these vocabulary words in their notebooks.

Quotient
the answer from a division problem

Integer
the set of whole numbers and their opposites

Inverse Operation
the opposite of a given operation

Fraction Bar
the line used to divide the numerator and the denominator of a fraction-also means division

Numerical expression
an expression that combines integers and operations

Algebraic expression
an expression that combines variables, integers and operations.

Teaching Strategies

The real world story problem *The History Test* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

The same rules for multiplying integers apply when we work with dividing integers. Even though these rules were presented in the last lesson, take the time to review the rules for the students. Students will need to be able to apply these rules in their work with integers.

Evaluating numeric and algebraic expressions is challenging for students because it combines more than one operation. Notice that there is a fraction bar used to signify division. You will need to explain this to the students. Students will need to understand that when they see this bar that they are going to divide.

Again, remind students to follow the order of operations. If a problem has multiplication and division in it, they will need to multiply integers and then divide them.

Some students may become discouraged when they see the variables in the last example. You can remind students that all they are doing is substituting values for those variables. It is just an added step, then they can perform the operations with integers as they have always done.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.youtube.com/watch?v=2QJCOnStpy0 - This is a math made easy video on dividing integers. It uses a fraction bar to signify division which is excellent reinforcement for the text.

The Coordinate Plane

Goal

The goals of this lesson can be found in the following objectives.

- Graph ordered pairs of integer coordinates as points in all four quadrants.
- Graph geometric figures given coordinates of vertices.
- Locate places on maps using integer coordinates.
- Describe paths between given points as integer translations.

Relevant Review

Review identifying integers as positive and negative. Review identifying positive and negative integers on a number line.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Quadrant
the four sections of a coordinate grid

Origin
the place where the x and y axis' meet at (0, 0)

Ordered Pair
the x and y value used to locate a point on a coordinate grid (x, y)

x axis
the horizontal axis on the coordinate grid

y axis
the vertical axis on the coordinate grid

Coordinates
the x and y value of an ordered pair

Longitude
vertical measure of degrees on a map

Latitude
horizontal measure of degrees on a map

Teaching Strategies

The real world story problem *The Map* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

In an earlier lesson, students were introduced to graphing in one quadrant. When this happened, they were introduced to the x axis and to the y axis. This is important to review before beginning this lesson. You are going to take this information and build on it. Once you have reviewed one quadrant, draw out or use an overhead transparency to show students a complete coordinate grid. You can explain this as the coordinate plane.

Once it has been presented, ask the students to describe the grid. Students should identify positive and negative values both on the horizontal and vertical axes. Be sure to remind students which axis is horizontal and which is vertical. Some students will need this reminder.

Always encourage students to start at the origin. Students with spatial challenges will be lost otherwise. If they start at the origin, then go horizontal and then vertical, it will cut back on the number of errors that students will have.

This is a fun lesson. Many students really enjoy working on the coordinate plane. There are also many patterns students can create as they practice graphing and identifying values.

http://www.mathplayground.com/locate_aliens.html - This is a website where students can play a math game where they identify aliens on the coordinate plane.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/the-coordinate-plane?playlist=ck12.org+Algebra+1+Examples - This is a Khan Academy video on the coordinate plane and graphing on the coordinate plane.

http://www.khanacademy.org/video/quadrants-of-coordinate-plane?playlist=Developmental+Math+2 - This is a Khan Academy video on identifying quadrants on the coordinate plane.

Transformations

Goal

The goals of this lesson can be found in the following objectives.

- Identify transformations in the coordinate plane as translations (slides), reflections (flips) and rotations (turns).
- Graph paired transformations of figures given coordinates of vertices.
- Describe transformations as $x-$ and $y-$ coordinate changes.
- Identify equivalent transformations with different coordinate changes.

Relevant Review

Review moving on the coordinate plane. You will need to review the location of the x and y axes so that students are able to take what they have learned and apply it to this next lesson on transformations.

Review ordered pairs and that the first value is the x value and the second is the y value.

Remind students to always begin at the origin and move from there to plot the point.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Transformation
when a figure is moved in some way on the coordinate plane, the way that the figure is moved is called a transformation.

Translation
a slide, this when a figure slides on the coordinate plane from one place to the next

Reflection
the flip of a figure. Figures can be reflected over the x axis or over the y axis

Rotation
a turn. A figure can be turned in various directions on the coordinate plane.

Equivalent
another name for equal

Teaching Strategies

The real world story problem *The Clubhouse* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

There are three different types of transformations that are taught in this lesson. First, students work on identifying the three types, then they are going to design and manipulate each type. You can use the nicknames for each type of transformation, for example that a translation is called a slide, but be sure to use the nickname in connection with the real name of the transformation. In this way students will connect the accurate term with the nickname.

Sometimes manipulating the transformations can be difficult for students. To do this in a hands –on way, you can give the students different shaped figures, such as triangles, cut out of construction paper. Students can take these triangles and move them on the coordinate plane to practice sliding them, rotating them and flipping them.

After practicing this in a hands –on way, you can move to the examples in the text where students are only working with the coordinates of the vertices of each figure.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.brightstorm.com/math/geometry/transformations/transformations-and-isometries/ - This is a Brightstorm video on all of the different types of transformations. There are also links on this website to show each of the transformations in detail.

Surveys and Data Displays

Goal

The goals of this lesson can be found in the following objectives.

- Collect and organize real-world survey data.
- Choose an appropriate data display.
- Distinguish which data displays are more effective for a specific purpose.
- Analyze and interpret statistical survey data.

Relevant Review

Review the four part problem - solving plan from earlier lessons on problem –solving.

Study Skill Tip

Be sure to have students copy down the different data displays and what they are best used for. This information should be documented in their notebooks.

Ways to Display Data

1. **Bar Graph** –A bar graph displays the frequency of data or how often data occurs.
2. **Double Bar Graph** –Compares the frequency of two sets of data.
3. **Line Graph** –shows how data changes over time.
4. **Double Line Graph** –compares how two sets of data changes over time.
5. **Circle Graph** –shows a percentage out of a whole.

Teaching Strategies

The real world story problem *The Bedtime Survey* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class. This story problem can be used as the model for the lesson. It is an example of a survey that can easily be conducted in the class as well. Use data that is real and relevant to the students so that the surveys and data displays that are created have value for the students.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/surveys-and-samples?playlist=ck12.org+Algebra+1+Examples - This is a Khan Academy video on surveys and samples.

http://www.youtube.com/watch?v=tnAzOp2d67I - This is a teacher tube video on youtube that uses the Boston Red Sox to teach about data displays.

1.12 Equations and Functions; Probability

This final chapter *Equations and Functions; Probability* begins preparing the students for the world of seventh grade math. Here is the introduction to equations and functions as well as a look into the world of probability. This is an excellent opportunity for students to grapple with these new skills. This Teaching Tips flexbook is designed to assist you, the teacher in structuring and designing each lesson. For each section of the chapter, there will be information on pacing, goals, review skills needed and teaching strategies as you guide your students into the world of mathematics.

Lessons

The following lessons are part of this chapter.

- Writing Expressions and Equations
- Solving Equations Using Addition and Subtraction
- Solving Equations Using Multiplication and Division
- Functions
- Graphing Functions
- Introduction to Probability
- Finding Outcomes
- Probability of Independent Events

Pacing

When planning, the pacing of the chapter guides our work. We can think of each day as a 45 minute class period. If you are in a school which features block scheduling, then you can combine two days together to equal one class period.

- Writing Expressions and Equations –1 Day
- Solving Equations Using Addition and Subtraction –2 Days
- Solving Equations Using Multiplication and Division –2 Days
- Functions –1 Day
- Graphing Functions –2 Days
- Introduction to Probability –1 Day
- Finding Outcomes –1 Day
- Probability of Independent Events –1 Day

Day 12 –Review

Day 13 - Test

Writing Expressions and Equations

Goal

The goals of this lesson can be found in the following objectives.

- Write addition and subtraction phrases as single-variable expressions.

- Write multiplication and division phrases as single-variable expressions.
- Write sentences as single-variable equations.
- Model real-world situations with simple equations.

Relevant Review

Review substituting a value for an unknown variable. This can be seen in the following example.

Example $x + 8$ *when* x *is* 10

To solve this problem, the student needs to substitute 10 for the variable and solve the problem. This is how this lesson begins.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Expression
a variable expression has variables or unknown quantities, numbers and operations without an equal sign.

Equation
a variable equation has a variable, numbers and operations with an equal sign.

To be successful in this lesson, students will need to understand key words and the operation connected with each key word. Be sure to have the students copy these key words down in their notebooks.

Addition

Sum, plus, altogether, plus, and

Subtraction

Difference, less than, subtract, take away

Multiplication

Product, times, groups

Division

Split up, quotient, divided

Teaching Strategies

The real world story problem *The Class Trip to the Amusement Park* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When working on deciphering statements from written language to mathematical language, you will need to direct students to the following steps.

1. Identify any numbers
2. Identify the operation involved
3. Identify the variable

Encourage the students to copy these steps down in their notebooks and to follow them. After students have had some practice with expressions, the lesson directs students to writing equations. Students are going to follow the same steps except now they will have an equals sign. There will be an unknown on one side of the equation and a quantity on the other side of the equals. This is the first step in preparing students for the next lesson on solving equations.

Be sure to pay attention to the colored text boxes. There is valuable information for the students in these boxes.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.homeschoolmath.net/teaching/teach-solve-word-problems.php - This is a homeschooling website which teaches students how to understand equations and how to write equations and expressions correctly.

http://www.brightstorm.com/math/algebra/graphs-and-functions/writing-an-equation-to-describe-pictures/ - This is a Brightstorm video on writing equations.

Solving Equations Using Addition and Subtraction

Goal

The goals of this lesson can be found in the following objectives.

- Simplify sums and differences of single-variable expressions
- Solve single-variable addition equations
- Solve single-variable subtraction equations
- Model and solve real-world problems using addition or subtraction equations

Relevant Review

Review the key words from the last lesson that indicate different operations.

Addition

Sum, plus, altogether, plus, and

Subtraction

Difference, less than, subtract, take away

Multiplication

Product, times, groups

Division

Split up, quotient, divided

Remind students of the different ways to show multiplication, using a dot, parentheses or a variable next to a number.

Remind students that division is shown using a division sign or a fraction bar.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Expression
a combination of variables, numbers and operations without an equal sign.

Simplify
to make smaller

Inverse
the opposite. An inverse operation is the opposite operation.

Sum
the answer in an addition problem

Difference
the answer is a subtraction problem

Teaching Strategies

The real world story problem *The Bus* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

It is recommended that you take two days to teach this lesson. Divide the objectives of the lesson between the two days. On the first day, work on combining like terms and solving single variable addition problems. On the second day, work on solving single variable subtraction problems and find a solution to the real world story problem from the introduction.

Simplifying is the first topic and it is combined with like terms. When students simplify they make something smaller by combining. Combining means adding or subtracting. You can point this out to the students. When they have like terms that are either being added or subtracted, the chances are that they can also simplify the terms.

When solving equations, point out the inverse operations or opposite operations for the students. Help students to pay attention to the notes in the Recap box in this lesson. The information in this text box provides them with steps and a review.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/howto_solvevariable.html - This is a math playground video that teaches students how to solve a variable equation. It goes through all four operations, addition, subtraction, multiplication and division.

Solving Equations Using Multiplication and Division

Goal

The goals of this lesson can be found in the following objectives.

- Simplify products and quotients of single-variable expressions.
- Solve single-variable multiplication equations.
- Solve single-variable division equations.
- Model real-world problems using multiplication or division equations.

Relevant Review

Review solving equations using addition and subtraction.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Product
the answer in a multiplication problem

Quotient
the answer in a division problem

Inverse Operation
the opposite operation

Teaching Strategies

The real world story problem *Finding a Bus* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

It is recommended that the content of this lesson be taught over two days. The first day focuses on simplifying products and quotients and on solving single variable equations using multiplication. The second day focuses on solving single variable equations using division.

Some students will be able to use mental math to solve these problems. This is fine, but be sure that they understand the steps so that if there is one that they can't solve in mentally that they still understand how to solve the problem.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/howto_solvevariable.html - This is a math playground video on how to solve variable equations.

http://www.khanacademy.org/video/solving-one-step-equations?playlist=Developmental+Math - This is a Khan Academy video on how to solve one step equations.

http://www.khanacademy.org/video/solving-one-step-equations-2?playlist=Developmental+Math - This is another Khan Academy video on solving equations. This video follows the previous one listed.

Functions

Goal

The goals of this lesson can be found in the following objectives.

- Write an expression for an input-output table.
- Evaluate a given function rule for an input-output table.
- Write a function rule from an input-output table.
- Model and solve real-world problems involving patterns of change.

Relevant Review

Review patterns with the students. http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/5_Patterns/index.html - This video provides students with an excellent review of patterns.

http://abcteach.com/directory/basics/math/patterns/ - This website has many reproducible worksheets on patterns that could be used with the students.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Pattern
a series of pictures, numbers or other symbols that repeats in some way according to rule.

Function
one variable depends on each other and there is only one term for each variable in a function.

Input-Output Table
A table that shows how a value changes and the input value changes according to a rule and becomes the value in the output.

Teaching Strategies

The real world story problem *The Car Wash* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When students work with input/output tables, encourage them to look for the pattern. They need to ask the question "What happened to the input to get the output?" This leads to choosing one or more of the four operations. When writing a rule, it is helpful for students to remember these hints.

Hints for Writing Function Rules

1. Decipher the pattern of the function. What happened to the input to get the output?
2. This is the rule for the function.
3. Write the rule as an expression.

Encourage students to write these hints down in their notebooks. Allow students time to practice writing these rules.

You can have them work in pairs and design their own rules as well.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.mathplayground.com/functionmachine.html - This is a math playground game that allows students the chance to practice with a function machine and a computerized input/output table.

http://www.youtube.com/watch?v=y_aaOS6Q2qo - This is a teachertube video on input/output tables and function rules.

Graphing Functions

Goal

The goals of this lesson can be found in the following objectives.

- Graph linear functions in the coordinate plane.
- Distinguish between linear and non-linear functions.
- Use function graphs to relate perimeter, area and volume to linear dimensions of objects.
- Model and solve real-world problems involving patterns of change with multiple representations of functions.

Relevant Review

Review input/output tables from the previous lesson. Also review how to write a function rule and review the steps for writing a function rule.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Function
one variable is dependent on another. One variable matches exactly one other value.

Linear Function
the graph of a linear function forms a straight line.

Non-Linear Function
the graph of a non-linear function does not form a straight line.

Perimeter
the distance around the outside edge of a figure.

Area
the measure of the surface of a two-dimensional figure

Volume
the measure of the space contained inside a three-dimensional figure.

Teaching Strategies

The real world story problem *Roller Coaster Speed* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

It is recommended that this lesson be divided into two sections. The first section focuses on functions and on graphing functions. This is the work of the first day. The second day moves into the geometry connection with area, perimeter and volume.

When working with functions, you want students to realize that there is a direct connection between the values x and y in a function. When there is a clear pattern, a connection, a relationship, then the function is linear and the graph form a straight line. When a function is non - linear, the graph of the line will not be straight.

When moving on to area, perimeter and volume, begin by reviewing the definitions of these three words. The trickiest part of this section is that students will need to think backwards to find the possible dimensions for the table. They are used to having the length and width given to them so that then they have to find the area. This is not the case here. We are looking for many possible options. This will require them to problem –solve and reason to find an answer. If students are having trouble with this section, then allow them time to work with a partner.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/recognizing-linear-functions?playlist=Algebra+I+Worked+Examples - This is a Khan Academy video on recognizing linear functions from tables.

Introduction to Probability

Goal

The goals of this lesson can be found in the following objectives.

- Recognize the probability of an event as the ratio of favorable outcomes to possible outcomes.
- Describe probabilities of events as fractions, decimals or percents.
- Find the probabilities of complementary events.
- Predict whether specified events are impossible, unlikely, likely or certain.

Relevant Review

Review fractions, decimal and percentages because students will be using them to work with probability and calculate outcomes.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Probability
the chances that something will happen. It can be written as a fraction, decimal or percent.

Ratio
compares two quantities. In probability the ratio compares the number of favorable outcomes to the number of possible outcomes

Complementary Events
For every probability that something will happen, there is a probability that it won't happen. These two ratios are complementary events.

Teaching Strategies

The real world story problem *The Spinner Game* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

You can begin this lesson with a brainstorming session on probability. You can use the opening example of Kevin and the weather to start and then turn it over to the students. If inappropriate content such as gambling comes up, remind the students that this is school and move on to a different example.

There are many great games out there for teaching probability.

http://www.surfnetkids.com/probability_games.htm - This website is focuses on probability websites with games and interactive content for students. There are several sites that are recommended on this page.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/basic-probability?playlist=Probability - This is a Khan Academy video that is an introduction to probability.

Finding Outcomes

Goal

The goals of this lesson can be found in the following objectives.

- Use tree diagrams to list all possible outcomes.
- Find all possible combinations.
- Find all possible permutations.
- Describe real-world situations involving combinations or permutations.

Relevant Review

Review the basics of calculating probability from the previous lesson.

$$P = \frac{\#\ of\ Favorable\ Outcomes}{\#\ of\ Possible\ Outcomes}$$

Use this to review fractions, decimals and percentages too.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Probability
the chances or likelihood that an event will happen.

Outcome
the end result

Tree Diagram
a visual way of showing options and variables in an organized way. The lines of a tree diagram look like branches on a tree.

Combination
an arrangement of options where order does not make a difference.

Permutation

an arrangement of options where order does make a difference.

Teaching Strategies

The real world story problem *The Ferris Wheel* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

Tree diagrams are fun and most students will enjoy working with them. Allow students the chance to work in small groups as they look at the content in this lesson. You can present the material from the text and the dilemma, then allow them time to work on figuring out the answer in their groups. In this way the lesson moves from lecture, to group work , to sharing solutions. Be sure students use a ruler so that their work is accurate and neat.

Continue working in this way on combinations and permutations. This will help the students to talk through the problems in the text as they work together on them.

Finally, ask students to work in small groups on the ferris wheel problem from the introduction.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/permutations-and-combinations-1?playlist=Algebra+I+Worked+Examples - This is a Khan Academy video on permutations and combinations. It is an introduction to working with these ideas.

http://www.youtube.com/watch?v=mkDzmI7YOx0 - This is a basic video on understanding tree diagrams. It begins very simply and helps students understand how to build a tree diagram.

Probability of Independent Events

Goal

The goals of this lesson can be found in the following objectives.

- Find the probability of at least one of two independent events occurring.
- Find the probability of two independent events both occurring.
- Find the probability of three independent events all occurring.
- Use probability to predict the likelihood of real-life events.

Relevant Review

Review the basics of calculating probability from the previous lesson.

$$P = \frac{\#\ of\ Favorable\ Outcomes}{\#\ of Possible\ Outcomes}$$

Use this to review fractions, decimals and percentages too.

Study Skill Tip

Be sure to have students copy down the following vocabulary words in their notebooks.

Probability
the chances or likelihood that an event will occur

Independent Event
when the outcome of one event does not have anything to do with the outcome of another event. One does not alter or impact the other.

Teaching Strategies

The real world story problem *The Amusement Park Survey* can be used as an introduction to the lesson. The solution could be used to close the lesson. It could also be used for a day dedicated to problem solving. This would be a day where the problem would be presented and solved on the same day. There is flexibility in the way that these real world problems are integrated into the class.

When working on this lesson, make it as hands –on as possible. For example, the marble example could be done by the students themselves. Then they can record their own data. This is an excellent way to make the content real for the students.

http://classroom.jc-schools.net/basic/math-prob.html - This website has excellent interactive games for students to practice calculating probability.

http://www.kidsmathgamesonline.com/numbers/probability.html - This is a website with number games for kids.

http://msteacher.org/epubs/math/math6/math.aspx - This is a great social network site for math teachers which also has information on teaching different topics. This page focuses on probability.

Practice Problems and Time to Practice

You will see practice problems throughout each lesson. As concepts change, there are practice problems provided for the students to work with. These problems could be used as a review at the end of the day or as a warm –up for the next day. In other words, you teach addition and subtraction on Day 1, so on Day 2, the practice problems for addition and subtraction are given for the students to solve on Day 2.

Technology Integration

http://www.khanacademy.org/video/independent-events-2?playlist=Algebra+I+Worked+Examples - This is a Khan Academy video on probability and independent events.

CHAPTER 2 Grade 6 - Common Errors

Chapter Outline

2.1 Number Sense and Variable Expressions

This first chapter *Number Sense and Variable Expressions* takes students from whole number operations into the world of beginning Algebra. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Operations with Whole Numbers
- Whole Number Estimation
- Powers and Exponents
- Order of Operations
- Variables and Expressions
- A Problem-Solving Plan
- Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Operations with Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Adding Whole Numbers
- Subtracting Whole Numbers
- Multiplying Whole Numbers
- Dividing Whole Numbers

Common Errors

In the first section of the lesson, students are introduced to whole number operations. These initial problems are written horizontally and students are instructed to write them vertically according to place value. This transition is a common error for students. Students line up the digits incorrectly and add digits with different values together. When students write a problem vertically, be sure that they are lining up the digits according to place value.

A common error when working with subtraction is when there is a problem like this one:

$$800 - 199 = \_\_\_\_$$

A problem such as this one is challenging for students because they forget that they are borrowing from the 8, in other words, they are borrowing a hundred, then a ten etc. Working through an example like this one helps students to borrowing. The answer to this problem is 601.

When multiplying, watch that students use a zero place holder when multiplying by a multi-digit multiplier. Many students are challenged by division, especially long division. It is worthwhile to spend some time going over the steps to division.

Ideas for Assistance

The first common error happens when students take a problem that has been written horizontally and write it vertically. To help with this, students can use grid paper with larger boxes to organize their numbers.

http://www.enchantedlearning.com/math/graphs/graphpaper/ –This website has many different options for graph/grid paper. You are looking for an option where you can put each digit into a different box. Line up the columns and add/subtract successfully. This website has free printing options.

Additional Examples

Here are some additional examples for practice. The answers to the problems are listed below.

Addition Problems –watch for lining up digits correctly with both addition and subtraction.

1. $678 + 112 =$ ____

2. $1,345 + 124 =$ ____

3. $80,976 + 12,345 =$ ____

Subtraction Problems –Watch borrowing.

4. $800 - 167 =$ ____

5. $9000 - 299 =$ ____

Multiplication Problems –Watch for zero place holders

6. $67 \times 14 =$ ____

7. $206 \times 148 =$ ____

8. $1,219 \times 56 =$ ____

Division Problems –Watch for the steps –are they being followed.

9. $4380 \div 12 =$ ____

10. $7112 \div 56 =$ ____

Answer Key

1. 790
2. 1,469
3. 93,321
4. 633
5. 8701
6. 938
7. 30,488
8. 68,264
9. 365
10. 127

Whole Number Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Estimating sums and differences of whole numbers using rounding

- Estimating products and quotients of whole numbers using rounding
- Estimating to find approximate answers to real-world problems
- Using estimation to determine whether given answers to real-world problems are reasonable

Common Errors

The biggest error that students make when estimating is that they don't understand the purpose of estimating, so they work to get an exact answer even though they have been told that they only need an approximate answer, an answer that makes sense and is reasonable. The purpose of estimating is to work with reason, so if students jump right to computation, then they don't really understand the purpose of estimation. If you find that your students are going right to computation, then go back and explain estimating. Work on developing real–world situations where the students might be required to estimate.

Working with compatible numbers is tricky, because students don't understand where to look to find numbers that are compatible. The numbers are easily divisible, so they are often going to look for a number that is even or easily divisible in thirds or quarters. This is challenging because so many students are weak in their times tables. They actually don't know whether or not the number is divisible, so they have difficulty finding the compatible number. Understanding divisibility leads to understanding how to select a compatible number. Compatible numbers are close to the original numbers, but easier to work with.

Additional Examples

When bringing estimation to life, you will be helping the students to come up with real world situations where estimation is valuable. Here is a website to help you with this task.

http://math.free-tuition.com/k2/why-is-it-important-to-estimate/ –This is an NCTM website that has a great fun video on estimation and why it is important to estimate.

When focusing on divisibility, here is a website to help students.

http://www.basic-mathematics.com/what-are-compatible-numbers.html –Here is a website that teaches students how to identify compatible numbers.

Directions: Choose a compatible number for each value.

1. $69 \times 41 =$ ____
2. $33 \times 11 =$ ____
3. $405 \times 210 =$ ____
4. $12 \times 41 =$ ____
5. $87 \div 32 =$ ____
6. $92 \div 11 =$ ____
7. $990 \div 99 =$ ____
8. $204 \div 5 =$ ____

Answer Key

1. 70×40
2. 30×10
3. 400×200
4. 10×10
5. $90 \div 30$
6. $90 \div 10$
7. $1000 \div 100$
8. $200 \div 5$

Powers and Exponents

Goal

The goals of this lesson can be found in the following objectives.

- Distinguish between a whole number, a power, a base and an exponent
- Write the product of a repeating factor as a power
- Find the value of a power
- Compare values of powers with different bases and exponents
- Solve real-world questions using whole number powers

Common Errors

The big common error with exponents is that students forget that the exponent indicates how many times they need to multiply the base by itself, not a times table. For example:

$5^3 = 5 \times 5 \times 5$ NOT 5×3

This is a common mistake that students make and shows that they don't understand the symbol or the job of the exponent. When working on additional examples, you can have them identify the base and exponent, then students can write out the problem in long form before solving each.

Remind students that when they compare quantities written with exponents that they have to evaluate the expression BEFORE then compare them.

For example: 4^2____3^3

Evaluate each expression. $4^2 = 16$ and $3^3 = 27$

$16 < 27$

Additional Examples

Directions: Identify the base, the exponent, write the problem out in long form and then find the solution.

Example: 4^2 The base is 4, the exponent is 2, $4 \times 4 = 16$

1. 3^3

Answer: The base is 3, the exponent is 3, $3 \times 3 \times 3 = 27$

2. 6^2

Answer: The base is 6, the exponent is 2, $6 \times 6 = 36$

3. 4^5

Answer: The base is 4, the exponent is 5, $4 \times 4 \times 4 \times 4 \times 4 = 1024$

4. 2^6

Answer: The base is 2, the exponent is 6, $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

5. 7^3

Answer: The base is 7, the exponent is 3, $7 \times 7 \times 7 = 343$

Order of Operations

Goal

The goals of this lesson can be found in the following objectives.

- Evaluating numerical expressions involving the four arithmetic operations
- Evaluating numerical expressions involving powers and grouping symbols
- Using the order of operations to determine if an answer is true
- Inserting grouping symbols to make a given answer true
- Writing numerical expressions to represent real-world problems and solve them using the order of operations

Common Errors

One of the first common errors is that students forget to complete multiplication and division and addition and subtraction in order from left to right. Writing the order of operations as it is in the text can help with some of this problem.

P

E

MD

AS

You can see that multiplication and division, the M and the D are written next to each other in the order that you perform the operations. Then you have the addition and subtraction, the A and the S written next to each other to stress this point as well.

Problems that have multiplication or division in the center of an expression are particularly challenging for students. Be sure to keep an eye on these –they will need to complete the multiplication or division before the addition and subtraction. See the additional examples to help students practice these skills.

Remind students that the operations are performed in order from left to right. You can't repeat this too many times.

Students need to memorize the order of operations. You can teach this with the common expression "Please Excuse My Dear Aunt Sally" or with the word PEMDAS pronounced pem–dass. These little rhymes help students to remember mathematical rules.

A common error to watch for when inserting grouping symbols to make a statement true is that students have performed the order of operations AND the grouping symbols. They sometimes insert the grouping symbols, don't remember to perform the order of operations and end up with an answer that is not correct. Students must insert the grouping symbols so that the order of operations makes each statement true.

Additional Examples

Directions: Practice solving each expression according to the order of operations.

1. $6+4\times 8-3$

2. $4-3+1\times 9-7$

3. $8+7\times 4+9\times 2$

4. $10+4\div 2+3\times 6$

5. $9+6\div 3+8\times 9$

6. $14-10\div 2+6\times 5$

7. $12+6\times 8+3\div 3-2$

8. $6+8\times 7+5\times 9$

Directions: Practice inserting parentheses to make each a true statement.

9. $4+6-2+9\times 3=28$

10. $6+3\times2-4+5=3$

Answer Key

1. 35
2. 3
3. 54
4. 30
5. 83
6. 39
7. 59
8. 107
9. $4+(6-2)+9\times3-7=28$
10. $6+3\times2-(4+5)=3$

Variables and Expressions

Goal

The goals of this lesson can be found in the following objectives.

- Evaluating single variable expressions with given values for the variable
- Evaluating multi-variable expressions with given values for the variable
- Using given expressions to analyze and solve real-world problems

Common Errors

A common error is that students mix –up the variable x for the times symbol of (x). You can help students by making each X distinct. The times symbol should have an X made with straight lines while the variable letter is in cursive or italics. Remind students that there are several different ways to show multiplication.

$$()()$$
$$x(y)$$
$$x\cdot y$$
$$xy$$

Reminding the students of these different ways will help them to recognize multiplication in an expression and solve the problem correctly.

A different symbol is introduced for division as well. In this lesson, students are shown that a fraction bar can also show division. Some students will wonder why this is division and not a fraction. Explaining this to them can be challenging. What you can say is that when the numerator or top number is larger than the bottom, then we can say that we are going to divide. While this may sound confusing it is in line with our work converting improper fractions and mixed numbers, division is involved in this process. The larger number goes on top when using a division bar to show division.

Additional Examples

Directions: Use a division bar to rewrite the following problems.

1. $15\div3$

2. $24 \div 6$
3. $100 \div 25$
4. $18 \div 6$
5. $144 \div 2$
6. $90 \div 3$
7. $72 \div 8$
8. $99 \div 9$

Answer Key

1. $\frac{15}{3}$
2. $\frac{24}{6}$
3. $\frac{100}{25}$
4. $\frac{18}{6}$
5. $\frac{144}{12}$
6. $\frac{90}{3}$
7. $\frac{72}{8}$
8. $\frac{99}{9}$

A Problem-Solving Plan

Goal

The goals of this lesson can be found in the following objectives.

- Four Part Problem Solving Plan
  - Read and understand a given problem situation.
  - Make a plan to solve the problem.
  - Solve the problem and check the results
  - Compare alternative approaches to solving the problem.
- Solve real-world problems using this plan.

Common Errors

Errors are made in problem solving when students don't follow the problem solving plan.

Encourage students to use the following two questions when they first read a problem.

1. What information have we been given?
2. What do we need to figure out?

Next, they will need to underline the given numbers and valuable information from the problem. The key to problem solving is to follow the steps.

Many students get confused when selecting an operation. Identifying the key words that mean addition, subtraction, multiplication and division will help them to figure out what the main operation is when solving a problem.

Addition –sum, plus, in all, total

Subtraction –difference, less than, more than

Multiplication –product, times, grouping

Division –split up, divided, quotient

Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Goal

The goals of this lesson can be found in the following objectives.

- How to read and understand a given problem situation
- How to develop and use the strategy: Guess, Check and Revise
- How to develop and use the strategy: Use Mental Math
- Plan and compare alternative approaches to solving problems
- Solve real-world problems using selected strategies as part of a plan.

Common Errors

Students often have difficulty with word problems, and it is a common source of tension in math classes. One of the first ways to avoid these challenges is to teach students to read the problem more than once. Then the students should underline any key information or key words. All of this is done before they try to solve the problem. Helping students to work in this way will help them to take their time. Rushing for a solution is a common place where errors are made.

Have students take notes on the different strategies. Look at this information from the text.

1. Guess, check and revise can get you started and you will need to try several different options to get the correct answer.
2. Mental math requires you to think in terms of the divisibility of numbers or multiples.

Help students to keep a problem solving page in their notebooks. On this page, students will write down different strategies. This list of strategies can then be referred back to whenever the students are solving a problem.

2.2 Statistics and Measurement

This second chapter *Statistics and Measurement* will help students become familiar with measurements. Also covered are finding the perimeter and area of rectangles, frequency, creating and understanding graphs, and an introduction to the mean, median, mode, range, and central tendency. Takes students from whole number operations into the world of beginning Algebra. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Measuring Length
- Perimeter and Area
- Scale Drawings and Maps
- Frequency Tables and Line Plots
- Bar Graphs
- Coordinates and Line Graphs
- Circle Graphs and Choosing Displays
- Mean, Median and Mode

Measuring Length

Goal

The goals of this lesson can be found in the following objectives.

- Measure length in customary units.
- Measure length using metric units.
- Choose appropriate tools given measurement situations
- Choose appropriate units for given measurement situations.

Common Errors

When working with rulers, it is important to have real rulers for the students to look at because many students have difficulty finding the different fractional units on a ruler. You can avoid this error by providing students with real rulers and by giving them the opportunity to measure some real items in the class. Provide students with the same items for measure. By doing this, you can compare the measurements of different students to find the correct answer. See below for some suggestions.

When working with millimeters and centimeters, provide students with rulers that have these measurements on them. Be sure to help students to identify which side has inches and which has metrics. Believe it or not, there are many students who will become very confused and select an incorrect unit of measurement. Then ask student to measure the same real objects as they did with inches and record the measurements. Use the group to help students to correct their individual errors. There should be concensus about the length of each item.

You can have fun with this lesson by taking students around the school to measure different lengths. Be sure to provide them with many different tools and lots of practice. If you assign two groups the same thing to measure, then they will be able to discuss which tool was used and the result of the measure.

Additional Examples

Here is a possible list of small items to measure in inches.

1. An unsharpened pencil
2. Paperclips –all the same size so everyone measures the same ones.
3. An unused eraser
4. A brand new piece of chalk
5. A dry erase marker
6. A math book –this will sometimes branch over into feet because the length of the book could be more than 12 inches. As a result, this makes an excellent transition.

Perimeter and Area

Goal

The goals of this lesson can be found in the following objectives.

- Finding the perimeter of squares and rectangles using formulas
- Finding the area of squares and rectangles using formulas
- Solving for unknown dimensions using formulas when given the perimeter or the area
- Solving real-world problems involving perimeter and area, including irregular figures made of rectangles and squares.

Common Errors

A first common error that students make when working with area and perimeter is that they mix up the length and the width. You can teach students that length is *long*. When students have these words to associate with different meanings, it helps them to remember what they are doing.

When working with formulas, this is a new concept for the students. Have the students write down each formula as they are introduced. The biggest mistake students make is to forget which formula to use when. You can overcome this obstacle by having students write down each formula and what it is used to measure. Students will need to commit these formulas to memory.

Students also forget to use square units when area is being measured. Remind students that they need to include units in their answers. The common thing is for students to include this or their answer is not complete. Look at this example.

Example: 36 needs to be 36 sq. in.

Another thing to point out to students, especially if it hasn't been taught yet, is that square units is the same as u^2. So these two are equal or equivalent.

36 sq. feet is the same as 36 ft^2

Have students write this down in their notebooks.

Additional Examples

Directions: Write the formula that is best used for each type of measurement.

1. The perimeter of a square

2. The perimeter of a rectangle

3. The area of a square

4. The area of a rectangle

Directions: Answer each question.

5. When there is a number next to a variable as in $2l$ what operation is required?

6. Why is area measured in square units?

Directions: Find each area. Be sure to use the correct units in your label.

7. A square with a side length of 6 feet.

8. A square with a side length of 7 inches.

9. A square with a side length of 3 meters.

10. A rectangle with a length of 17 feet and a width of 12 feet

Answer Key

1. $P = 4s$
2. $P = 2l + 2w$
3. $A = s^2$ - encourage students to use this formula because it makes a connection to exponents used in the last chapter.
4. $A = lw$
5. multiplication
6. Area is multiplication –$units \times units$
7. 36 square feet
8. 49 square inches
9. 9 square meters
10. 204 sq. ft.

Scale Drawings and Maps

Goal

The goals of this lesson can be found in the following objectives.

- Finding actual distances or dimensions given scale dimensions.
- Finding scale dimensions given actual dimensions.
- Solving real-world problems using scale drawings and maps.

Common Errors

One of the first things that students will need to do is to be able to identify a given scale. Different maps have the scale in different places. On many you will find it in the lower right hand corner, however, this is not a rule. Bring in many different maps and ask the students to find the scale on each map. This is the first step to working with a scale.

Students then need practice writing ratios/proportions and solving them to figure out the scale measurement or the real distance. Remind students that they will need to compare the units of the scale with the distance and do the same with the ratio that they are trying to solve.

$$\frac{units}{real\ distance} = \frac{units}{real\ distance}$$

Students often mix up what is being compared and therefore have incorrect answers. If the scale is $1'' = 300\ feet$. Then here is how the first ratio would look.

$$\frac{1}{300}$$

Remind students that they will need to match up units and distance in the second ratio depending on what information they have been given in the original problem. If they are given the inches, then the numerator is filled in. If they have been given the real distance, then the denominator is filled in. Be sure that the students understand how to set up the problem correctly.

Additional Examples

Directions: Use the scale $4'' = 500$ *meters* to figure out each actual distance.

1. $8''$
2. $12''$
3. $16''$
4. $2''$
5. $6''$

Directions: Use the scale $2'' = 100$ *miles* to figure out the scale measurement given the actual distances.

6. 400 miles
7. 1000 miles
8. 1200 miles
9. 1500 miles
10. 150 miles

Answer Key

1. 1000 meters
2. 1500 meters
3. 2000 meters
4. 250 meters
5. 750 meters
6. $8''$
7. $20''$
8. $24''$
9. $30''$
10. $3''$

Frequency Tables and Line Plots

Goal

The goals of this lesson can be found in the following objectives.

- Make a frequency table to organize and display given data.
- Make a line plot given a frequency table.
- Make a frequency table and line plot given unorganized data.
- Collect, organize, display and analyze real-world data using frequency tables and line plots

Common Errors

Students can become confused about what you are actually measuring with a frequency table. You can point out that we are measuring the number of times that a certain number occurred. Here is an example: If there are 6 sixth grade classes in the building, how many classes have 24 students?

Then present this list.

Class 1 = 22

Class 2 = 24

Class 3 = 24

Class 4 = 18

Class 5 = 21

Class 6 = 24

There are three classes with 24 students. Therefore we can put this data down.

3 classes 24 students

Now we can also measure the number of classes that have 18, 21 and 22. Each of those is 1.

To organize this data, we have to look at the range of students. It goes from 18 –24. This means that the lowest is 18 and the highest is 24. To track the number of classes we have to include all possible values in this range. First create a table.

TABLE 2.1:

Number of Students	Number of Classes

Now fill in the data.

TABLE 2.2:

Number of Students	Number of Classes
18	1
19	0
20	0
21	1
22	1
23	0
24	3

When students design the line plot, they must include all the values. All work must be accurate so that the lines are straight and evenly measured. Requiring that students use a ruler helps with accuracy.

Bar Graphs

Goal

The goals of this lesson can be found in the following objectives.

- Make a bar graph to display given data.
- Make a double bar graph to display and compare given data.

- Make multiple bar graphs to display and compare given data.
- Collect, organize, display and analyze real-world data using frequency tables and bar graphs

Common Errors

There are many websites that can help students as they problem–solve creating graphs.

http://nces.ed.gov/nceskids/createagraph/default.aspx –This is a great website on creating graphs. It provides students with practice creating graphs.

http://www.mathsisfun.com/data/bar-graph.html –This is a website that will help students as they work to create graphs.

One of the biggest problems with bar graphs is that students don't follow the steps for making them. Students will need to begin by drawing the two axes. Then add in the data and be sure that the bars are the same size.

The next common error is that students don't choose the correct scale. Many will try to use the same scale for every bar graph that they create. When teaching, encourage the students to study the data and select the correct scale whether that be tens, hundreds or thousands.

Additional Examples

Directions: Look at each example and choose tens, hundreds or thousands for the correct scale.

1. 200, 230, 300, 500, 700
2. 10, 12, 13, 18, 20, 22, 25
3. 45, 67, 88, 90, 100, 114
4. 500, 546, 625, 700, 899, 902
5. 2000, 2500, 3000, 3600, 4000

Answer Key

1. Hundreds
2. Tens
3. Tens
4. Hundreds
5. Thousands

Coordinates and Line Graphs

Goal

The goals of this lesson can be found in the following objectives.

- Identify elements of a coordinate grid (origin, vertical and horizontal axes, ordered pairs.)
- Graph given points on a coordinate grid (1^{st} Quadrant)
- Make a line graph to display given data over time.
- Collect, organize, display and analyze real-world data using line graphs.

Common Errors

A common error that students make is that they don't begin plotting points at the origin. They start on the horizontal axis and often this leads to miscounting. Remind students that they must always begin at the origin. Then count the horizontal value and then go up to the vertical value.

1. Start at the origin (0, 0)
2. Count across to the x value.
3. Stay there and count up to the y value.

When zero is the x value we don't count any values across. When zero is the y value, then we count across to the x value and then we don't count up because there isn't a y value. This is explained clearly in the text, however, many students many need reminders when plotting these points.

When creating line graphs, the lines between the points are straight lines. This is a common error for students. They will want to draw one line that connects all of the points. This is often not possible when creating line graphs. Therefore, have the students use rulers so that the lines that connect the data points are straight.

Additional Examples

http://www.shodor.org/interactivate/activities/SimpleCoordinates/ –This website has simple coordinate games where students figure out points in the first quadrant. It is an interactive website that will provide additional practice for students in naming and graphing coordinates.

http://www.oswego.org/ocsd-web/games/BillyBug/bugcoord.html –This is a fun website where you help Billy Bug to find his supper by moving him along the coordinate plane. It is graphing in the first quadrant.

http://teachingimage.com/coordinates.php –This website is full of free downloadable worksheets to help students with graphing coordinates. There are pages with varying degrees of difficulty.

Circle Graphs and Choosing Displays

Goal

The goals of this lesson can be found in the following objectives.

- Interpret given circle graphs
- Use circle graphs to make predictions
- Use data from a circle graph to make a bar graph
- Select among frequency tables, line plots, bar graphs and line graphs for best displays of given data.

Common Errors

Understanding the difference between a circle graph and a bar graph is challenging for many students. Errors are made when students forget that the data represented in a circle graph is not represented in the same way in a bar graph.

A circle graph represents percentages, NOT exact values.

A bar graph represents exact values.

When working with data represented in both bar graphs and circle graphs, students may be given a value or a percentage for a problem. The example with Trey and his spending money is used in the text. Students need practice with the arithmetic involved using ratios and proportions. This will be a common place for errors.

Additional Examples

Directions: Solve each problem for the numerical amount.

1. What is 10% of 20?
2. What is 50% of 400?
3. What is 40% of 80?

4. What is 20% of 30?
5. What is 80% of 300?
6. What is 90% of 500?

Answer Key

1. 2
2. 200
3. 32
4. 6
5. 240
6. 450

These websites provide additional practice for students in working with circle graphs and bar graphs.

http://www19.statcan.ca/02/02_015-eng.htm –a website that combines circle graphs and bar graphs. It has questions on each type of graph for students to work with.

http://www.superteacherworksheets.com/graphing.html –This website has many downloadable worksheets on the many different types of graphs available.

Mean, Median and Mode

Goal

The goals of this lesson can be found in the following objectives.

- Find the mean of a set of data.
- Find the median of a set of data.
- Find the mode of a set of data.
- Identify the range of a set of data.
- Select the best average to represent given sets of data.

Common Errors

One of the first common errors when working on median is that students don't want to take the time to write the values in order from least to greatest. As a result, they estimate the median rather than finding the exact median. Remind students that they cannot skip this important step. Remind students that when there is an even set of numbers that they must find the average of the two middle values. They do this by following the steps for finding the mean.

1. Add the values together.
2. Divide by the number of values added, in the case of medians it is 2.

To help students remember mode, you can make this connection with the word "most". The mode of a set of data values is the number that occurs the most. Again it is important for all values to be written in order from least to greatest.

Additional Examples

The following websites have wonderful interactive games or resources.

http://www.gamequarium.com/data.html –This website has fun interactive games on data interpretation. These games would provide an excellent review for the students.

http://www.kidsmathgamesonline.com/numbers/meanmedianmode.html –This website has number games using mean, median and mode.

http://www.math-aids.com/Mean_Mode_Median/ –This is a website that has worksheets on mean, median and mode to be used with the students.

2.3 Addition and Subtraction of Decimals

This third chapter *Addition and Subtraction of Decimals* introduces students to the Decimal System, covering addition and subtraction, rounding and estimation, and ordering decimals as well as introducing stem and leaf Plots. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Decimal Place Value
- Measuring Metric Length
- Ordering Decimals
- Rounding Decimals
- Decimal Estimation
- Adding and Subtracting Decimals
- Stem-and-Leaf Plots
- Use Estimation

Decimal Place Value

Goal

The goals of this lesson can be found in the following objectives.

- Express numbers given in words or hundredths grids using decimal place value.
- Express numbers in expanded form given decimal form.
- Read and write decimals to ten-thousandths place.
- Write combinations of coins and bills as decimal money amounts.

Common Errors

A common error when writing decimals in expanded form is that students will include only the digit and not the place value.

For example: Write 234 in expanded form. The student write $2+3+4$. This is incorrect because each digit represents a different place not a single digit.

One way to help students with this is to have them read the number aloud to themselves. If this is the case, the student would read "Two hundred and thirty –four". This is a step towards accuracy. Now the student would write.

$200+34$

Next, we help them to see that if the 2 represents hundreds, then the three and the four represent other places as well.

Now the students can often move to $200+30+4$. Provide students with opportunities to practice this skill.

Additional Examples

Directions: Write each in expanded form.

1. 334
2. 567
3. 45
4. 1,231
5. 6,789
6. 8,023

Answer Key

1. $300+30+4$
2. $500+60+7$
3. $40+5$
4. $1000+200+30+1$
5. $6000+700+80+9$
6. $8000+20+3$ –Notice there aren't any hundreds to report in this number.

Websites on decimal place value

http://gamequarium.com/decimals.html –This website has many fun decimal games to accompany the students as they work their way through the chapter.

http://www.mrnussbaum.com/placevaluepirates1.htm –This is a great game that has students hunt for pirates while using place value. *Notice, it can be played without decimals as well.*

http://themathgames.com/our-games/decimal-games/place-value/decimal-place-value-math-game –This is a decimal in space game where students shoot asteroids.

Measuring Metric Length

Goal

The goals of this lesson can be found in the following objectives.

- Identify the equivalence of metric units of length
- Measure lengths using metric units to the nearest decimal place.
- Choose appropriate tools for given decimal metric measurement situations
- Choose appropriate decimal units for given metric measurement situations

Common Errors

Common errors happen when converting one metric unit to another. Students need to remember the rules that when we go from a smaller unit to a larger unit, that we multiply and when we go from a larger unit to a smaller unit, we divide. Of course this all depends on whether or not students know the larger units from the smaller units.

Preventing common errors comes with learning the rules about conversions, but also by memorizing metric units in order from least to greatest.

The next tricky part comes when we convert units and end up with a decimal answer. Students can use the following rules to help them.

When dividing by 10, move the decimal point one place to the left.

When dividing by 100, move the decimal point two places to the left.

When dividing by 1000, move the decimal point three places to the left.

Be sure to add extra zeros as needed.

Additional Examples

Directions: Divide the following whole numbers by moving the decimal point the correct number of places. Remember to add zero placeholders as needed.

1. $3 \div 10$
2. $4 \div 100$
3. $1.5 \div 10$
4. $16 \div 100$
5. $18 \div 10$
6. $12 \div 1000$
7. $2 \div 100$
8. $5 \div 1000$
9. $47 \div 10$
10. $47 \div 100$

Answer Key

1. .3
2. .04
3. .15
4. .16
5. 1.8
6. .012
7. .02
8. .005
9. 4.7
10. .47

The following website has practice on metric units of length.

http://www.gamequarium.com/metricsystem.html –This website has many games having to do with the metric system.

Ordering Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Comparing Metric lengths
- Comparing decimals
- Ordering decimals
- Describing real-world portion or measurement situations by comparing and ordering decimals.

Common Errors

This lesson begins by asking students to compare different measurements with different metric units of length. There are two common errors to watch for right away. The first is that the students simply look at the number of the measurement and not at the unit. Therefore, a common error would be the following.

Example: 25 mm ____ >____ 10 cm

Students would mark that 25 mm is greater than 10 centimeters simply because 25 is greater than 10. They haven't done any conversions to be sure that their work is accurate.

The second common error is that the student would compare based on unit and completely disregard the number connected to the unit.

Example: 1 meter ____ >____ 1000 cm

This would be incorrect because 1 meter is equal to 100 centimeters. If you have 1000 centimeters, than 1 meter is definitely less than 1000 centimeters.

Students must be reminded to *convert* both measures to the same units and *then* compare.

When comparing decimals, the common error is to compare without adding zeros. When comparing, students need to be sure that they have the same number of digits in both numbers that they are comparing so that the comparison is accurate.

Example: .5 _____ .46

Students should add a zero to the .5 to help with the comparison. Then both values have the same number of digits.

.50 >.46

This is the correct answer.

It is even more important when you have an example like this one.

Example: .005 _____ .06

Adding a zero to the .06 will show the difference in the values.

.005 <.060

This is the correct answer.

Additional Examples

Directions: Compare the following metric units of length using <, >, or =.

1. 10 mm _____ 3 cm

2. 1000 mm _____ 4 cm

3. 4 m _____ 4000 mm

4. 2 km _____ 400 m

5. 20 mm _____ .2 cm

Directions: Compare the following decimals using <, >or =.

6. .0012 _____ .012

7. .67 _____ .7

8. .23 _____ .1

9. .01 _____ .1

10. .0060 _____ .60

Answer Key

1. <
2. >
3. =
4. >

5. >
6. <
7. <
8. >
9. <
10. <

Rounding Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Round decimals using a number line.
- Round decimals given place value.
- Round very small decimal fractions to the leading digit
- Round very large numbers to decimal representations of thousands, millions, etc

Common Errors

A common error is that students don't look at the accurate place that they are rounding when rounding decimals.

For example: Round .345 to the nearest hundredths place

Given an example like this one, students simply look at the three and then at the whole number to the right which is a 4 and round to 300 because a four is following the three. The student has completely ignored the fact that we aren't rounding the three at all. We are rounding the hundredths place which means that we are rounding the 4. To round the 4 up or down, we look at the five and round up to the 5.

Our final answer is .350

Students will mix this up all the time. You can help them by asking them to underline the place that they are rounding. THEN look at the digit to the right and round up or down based on the rules of rounding.

Remind students that rounding by using leading digits is for decimals with more than 4 decimal places.

Example: .0000678 would be an excellent choice to round by leading digit.

In this example, we would round the first digit not a zero. In this case it is a 6 and look to the right of the 6 and then follow the rules of rounding. Given that the digit following the 6 is a 7, we would round up.

Our final answer is .00007.

Notice that we don't need the last two digits because we have rounded up to get a better sense of the size of the decimal.

Additional Examples

Directions: Round each decimal by using place value.

1. Round .346 to the nearest hundredth.

2. Round .87 to the nearest tenth

3. Round .00587 to the nearest thousandth

4. Round .011 to the nearest hundredth.

5. Round .607 to the nearest hundredth.

Directions: Round each decimal using leading digits.

6. .0000543

7. .00000125

8. .0000088

9. .00001976

10. .0000000354

Answer Key

1. .35
2. .9
3. .006
4. .01
5. .61
6. .00005
7. .000001
8. .000009
9. .00002
10. .00000004

Decimal Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Estimate sums and differences of decimals using rounding
- Estimate sums and differences of decimal numbers using front–end estimation
- Compare results of different estimation methods
- Approximate solutions to real-world problems using decimal estimation

Common Errors

Remind students that a sum is the answer in an addition problem and that a difference is the answer in a subtraction problem. You don't want students to begin by making an error because they have forgotten these two vocabulary words.

Review rounding to the nearest whole number. To do this, we look at the tenths place in the decimal part of the answer. Rounding to whole numbers makes estimating sums and differences much simpler.

Example $34.6 + 80.2$

We round the first to the nearest whole number 34.6 becomes 35.

We round the second to the nearest whole number 80.2 becomes 80

$35 + 80 = 115$

If students can't do this addition in their heads, then they can always add $3 + 8$ mentally which is 11.

110 would also be an acceptable estimate to this problem.

Here is an example for rounding to the nearest whole number when looking for a difference.

Example: $67.9 - 10.2$

We round 67.9 up to 68.

We round 10.2 down to 10

$68 - 10 = 78$

The correct estimate of the difference is 78.

Additional Examples

Directions: Estimate the following sums and differences using rounding.

1. $54.5 - 10.3 =$ _____
2. $678.9 - 99.6 =$ _____
3. $45.6 + 12.1 =$ _____
4. $69.5 + 22.1 =$ _____
5. $70.1 + 50.5 =$ _____
6. $89.9 - 10.2 =$ _____

Answer Key

1. 45
2. 580
3. 58
4. 92
5. 80

The following websites have games/information on estimating decimal sums and differences.

http://www.math-play.com/decimal-math-games.html –This website has many links to all sorts of games involving decimals. There are games on rounding and estimation on the site as well as others.

http://www.khanacademy.org/video/estimation-with-decimals?playlist=Developmental+Math –This is a Khan Academy video on estimation with decimals. It shows how rounding can be helpful when working with decimals.

Adding and Subtracting Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Adding and Subtracting Decimals by rewriting with additional zero place holders.
- Using mental math to add/subtract decimals
- Identifying the commutative and associative properties of addition in decimal operations, using numerical and variable expressions
- Solving real world problems involving decimal addition and subtraction

Common Errors

The first error that students make is that they do not line up the decimal points. This will take a lot of reminding, but it is worth the effort. You can also tell students that they add wholes and parts separately. The decimal point divided the wholes and the parts. This is how they are going to add or subtract the given values.

The second common error is when students don't add zero place holders. You can tell the students that they need to have the same number of digits in each number. If one has two digits before and two digits after, then the other has to also.

Example: $45.67 + 32.4$

In the first number there are two digits before the decimal and two digits after the decimal. In the second, there are two digits before the decimal and one digit after the decimal. This means that we must add a zero place holder after the 4 to even up the digits. The number of digits in each need to match.

$45.67 + 32.40$ is the correct form of the problem.

Next, you can remind students to line up the decimal points when they add these two values vertically.

Additional Examples

Directions: Practice using zero placeholders and then add or subtract the following decimals.

1. $12.3 + 14.56$
2. $18.9 - 12.30$
3. $17.2 + 34.567$
4. $19.66 + 18.1$
5. $78.9 - .123$
6. $45.67 - .23$
7. $88.532 + 12.3$
8. $99.7 - 12.30$
9. $6.78 + 1.98$
10. $14.2 - 5.6$

Answer Key

1. 26.86
2. 6.6
3. 51.767
4. 37.76
5. 78.777
6. 45.44
7. 100.832
8. 87.4
9. 8.76
10. 8.6

Stem-and-Leaf Plots

Goal

The goals of this lesson can be found in the following objectives.

- Organize a set of data in a stem-and-leaf plot.
- Use a stem-and-leaf plot to find the range of a set of data.
- Use a stem-and-leaf plot to find the mean, median and mode of a set of data.

Common Errors

The biggest hurdle students face when working with stem-and-leaf plots is to think about each value in terms of its tens place. Students don't like to divide up digits in one number, so this is the place where they struggle.

For example: 34 is divided into 3 and 4, 3 being the tens place and 4 being the ones.

If there is more than one number in the 30's then there is more than one leaf in the chart.

For example: 33, 33, 34, 35 would all be divided up.

3 | 33 4 5

This is how these three values would look in a stem-and-leaf plot.

Once students understand how to divide up values, then they have fun with this type of data display.

Notice that repeated values are included. This is also important to note. Because 33 is repeated in the data list, it is also repeated in the stems.

Additional Examples

Here are a few websites to help you as you work with students on stem-and-leaf plots.

http://www.khanacademy.org/video/stem-and-leaf-plots?playlist=ck12.org+Algebra+1+Examples –This is a Khan Academy video on stem-and-leaf plots.

http://www.mrnussbaum.com/coolgraphing.htm –This website provides students with an opportunity to create several different types of graphs and data displays. All data displays can be printed for the students.

Use Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations
- Develop and use the strategy: Use Estimation
- Plan and compare alternative approaches to solving problems
- Solve real-world problems using selected strategies as part of a plan

Common Errors

This lesson focuses on problem solving and on using estimation as a strategy when problem solving. To do this, students will need to estimate and not hunt for exact answers. This is the most common error that students make when problem solving. They don't allow estimates to inform their decisions. They go right to finding an exact answer.

Additional Examples

Here are a few websites to help students as they practice using estimation.

http://www.gamequarium.com/estimation.html –This website has great estimation games for students. These games provide students with many opportunities to practice using estimation.

http://pbskids.org/cyberchase/games/ballparkestimation/ –This is a Cyberchase game on estimation. You will need Real Player to be able to access the game.

2.4 Multiplication and Division of Decimals

This fourth chapter *Multiplication and Division of Decimals* furthers a student's understanding of decimals by introducing methods of multiplication and division. Also covered are the Distributive Property and the use and conversion of metric units. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Multiplying Decimals and Whole Numbers
- The Distributive Property
- Multiplying Decimals
- Dividing by Whole Numbers
- Multiplying and Dividing Decimals by Powers of Ten
- Dividing by Decimals
- Metric Units of Mass and Capacity
- Converting Metric Units

Multiplying Decimals and Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Multiply decimals by whole numbers
- Use and compare methods of estimation to check for reasonableness in multiplication of decimals by whole numbers
- Identify and apply the commutative and associative properties of multiplication in decimal operations, using numerical and variable expressions.
- Solve real-world problems involving decimal multiplication

Common Errors

One of the first common errors students make is that they don't understand that multiplication. By this I mean that they don't really understand that we have multiples of the same decimal that are being added together. When you begin this lesson, take a few minutes to remind the students that multiplication is simply repeated addition. Look at this example for an illustration of how this connects.

Example: 4.5×6

This means that we have 4.5 six times.

$4.5 + 4.5 + 4.5 + 4.5 + 4.5 + 4.5$

Understanding this is the first step to working accurately with multiplication and whole numbers.

Another common error is that students forget to put the decimal point into the answer. They will also sometimes try to simply bring it down into the product as they did with addition. You must remind them that multiplication is

a short –cut for addition, so you have to treat a product different from a sum. Another common error is to count decimal places from left to right rather than from right to left. These things to remember are pointed out in the text, you will need to help the students to commit these rules to memory.

Estimating products makes multiplying much easier because students can simply round to a whole number and multiply. Remind students of the rules for rounding so that they round accurately and have a reasonable estimate when finished.

Additional Examples

Directions: Practice multiplying the following decimals and whole numbers. Remember to put the decimal point in the correct place in the product.

1. 2.5×7

2. 6.9×3

3. 15.6×8

4. 7.5×3

5. 12.45×2

6. 14.62×4

7. 16.3×7

Directions: Please estimate the following products by rounding.

8. 1.8×3

9. 2.6×4

10. 6.7×9

11. 2.3×8

12. 1.5×7

Answer Key

1. 17.5
2. 20.7
3. 124.8
4. 22.5
5. 24.9
6. 58.48
7. 114.1
8. 6
9. 12
10. 63
11. 16
12. 14

The Distributive Property

Goal

The goals of this lesson can be found in the following objectives.

- Write numerical expressions for the product of a number and a sum
- Identify and apply the Distributive Property to evaluate numerical expressions
- Evaluate products using mental math.
- Apply the Distributive Property to evaluate formulas using decimal quantities.

Common Errors

A common error when working with the Distributive Property is that students forget to multiply the term outside the parentheses to both of the terms inside the parentheses. You can help students to remember this by drawing arrows from the term outside the parentheses to the two terms inside the parentheses. Look at the following example.

Example: $\overrightarrow{4(6+8)} = 4(6)+4(8)$

Now students may say, "Well I can just add 6 and 8 and then multiply fourteen times four." This is another way of solving the problem. However, this will only work with numerical expressions. The students haven't been introduced to the Distributive Property with variable expressions yet, so they don't understand the importance of distributing the term outside the parentheses.

While this is the case, the students should understand the procedure of distribution even if they choose to solve the problem using order of operations.

When working with area and decimals, review the formula for finding the area of a rectangle and for finding the area of a square. This is a good time to review multiplying decimals by whole numbers from the previous lesson. Remind students to add a decimal point into the product and to count decimal places from right to left.

Additional Examples

Here are some websites with information/games to help students practice working with the Distributive Property.

http://www.khanacademy.org/video/the-distributive-property?playlist=Developmental+Math –This is a Khan Academy video on the Distributive Property.

http://www.onlinemathlearning.com/the-distributive-property.html –This website has games, activities, videos and teaching tools for grade 6 with the Distributive Property.

Multiplying Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Multiply decimals by decimals using area models (hundredths grid).
- Place the decimal point in the product and confirm by estimation.
- Multiply decimals up to a given thousandths place.
- Solve real-world problems involving area of rectangles with decimal dimensions.

Common Errors

One of the tools that you can use to help students prevent errors is to actually photocopy and cut out hundreds grids for them to work with. In this way, they can cut and color all of the different values and you can check them. Students often enjoy this way of working. It helps several different learning styles as well. When multiplying decimals, they can overlap colors as is done in the text. Using the actual model for the students to work alongside the text will help them to understand multiplying decimals on a new level.

http://www.apples4theteacher.com/math/games/100-number-chart-one.html –This website has many different games using hundreds grids to help students become accustomed to using the tool.

http://www.superteacherworksheets.com/hundreds-chart.html –Here are printable hundreds chart to use with the students.

A common error is that students forget to use zeros as placeholders. Review regular multiplication and remind students of the steps. Then you can go to multiplication of decimals. When multiplying decimals, remind students to count decimal places from right to left, not left to right. We do so many things in school from left to right, that many students struggle with the change from right to left. You will need to remind them of this change many times. You can also use arrows on the board and a model of a problem to help students with this step.

Review the formula for area when working on the area problems with decimals. Do an example without decimals first.

For example: Find the area of a rectangle with a length of 6 feet and a width of 4 feet.

Area $= 24\ sq.ft$

Then move on to measurements with decimals. This should help students to make the transition from whole number measurements to decimal measurements.

Additional Examples

Directions: Multiply the following decimals. Remember to count decimal places in the product from right to left.

1. $3.4 \times 2.4 =$ _____
2. $6.7 \times 1.2 =$ _____
3. $4.5 \times 7.8 =$ _____
4. $5.6 \times 9.8 =$ _____
5. $1.23 \times 6.7 =$ _____
6. $2.34 \times 4.5 =$ _____
7. $.123 \times 45.6 =$ _____
8. $3.45 \times .124 =$ _____
9. Find the area of a rectangle with a length of 12.6 ft. and a width of 1.4 ft.
10. Find the area of a square with a side length of 15.5 inches.

Answer Key

1. 8.16
2. 8.04
3. 35.1
4. 54.88
5. 8.24
6. 10.53
7. 5.6088
8. .4278
9. 17.64 sq. ft.
10. 240.25 sq. in.

Dividing by Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Divide decimals by whole numbers.

- Find decimal quotients of whole numbers using additional zero placeholders.
- Divide decimals by whole numbers and round to a given place.
- Solve real-world problems involving the division of decimals by whole numbers.

Common Errors

A first common error is that when a division problem is written horizontally that students aren't sure which value to put in the division box and which to put outside the division box. You can teach them that the first number is the dividend and goes in the division box. The second value is the divisor and goes outside the division box.

$456 \div 2$ means that 456 goes in the division box and 2 goes outside it.

Students want to use remainders forever. You say, "Now that we can find decimals, you no longer will need remainders because we can figure out parts of a whole." Then you can use this as an introduction to zero placeholders. Using zero placeholders is tricky. You can remind students to use zeros until the answer doesn't have a remainder anymore. Then their work is done.

Lastly, be sure to review the rules of rounding decimal places. Students will be using this skill towards the end of the lesson.

Additional Examples

Here are a few websites with additional practice for students on dividing decimals.

http://www.aaamath.com/div66rx2.htm –This website has wonderful practice for students on the division of decimals. It gives an example, and then students work on solving problems.

http://classroom.jc-schools.net/basic/math-decim.html –This website has many, many games on using decimals. They are fun and interactive with different themes.

http://www.onlinemathlearning.com/decimal-games.html –This website also has many decimal games. There is a whole section of games where students practice operations with decimals.

Multiplying and Dividing Decimals by Powers of Ten

Goal

The goals of this lesson can be found in the following objectives.

- Use mental math to multiply decimals by whole number powers of ten.
- Use mental math to multiply decimals by decimal powers of ten.
- Use mental math to divide decimals by whole number powers of ten.
- Use mental math to divide decimals by decimal powers of ten.
- Write in scientific notation.

Common Errors

Many students find multiplying and dividing by powers of ten a lot of fun. They find patterns and have a great time using the patterns to figure out different values. When they do this, they can easily figure out the products and quotients.

Remind students that the number of zeros in the base ten number tells you how many places to move the decimal point. A common error is to only move the decimal one place no matter what the base ten number is. The students must look at the base ten number to determine how many places to move the decimal point.

A common error when multiplying by a base ten value is to move the decimal point the wrong direction. When multiplying numbers get larger, so we move the decimal point to the right. We move to the right when multiplying by a base ten whole number.

Now we have to help students to see that decimals are different. When we multiply by a base ten decimal, we move the decimal point to the left.

Then when moving on to division, again the base ten number tells us how many places to move the decimal point based on the number of zeros, but we move the decimal point to the left because the number gets smaller.

To help students, you can create a chart on the board.

$12 \times 10 = 120$ –one place to the right–one zero in whole number ten

$12 \times .10 = 1.2$ –one place to the left–one zero in decimal one–tenth

Always point out counting zeros and watching for whether the base ten is a whole number or a decimal.

http://www.aaamath.com/dec71ix2.htm –This website has an explanation and interactive games on working with decimals and scientific notation. This will be helpful when working on the last part of this lesson.

Additional Examples

Directions: Find each product.

1. $34 \times 10 =$ _____

2. $3.4 \times 100 =$ _____

3. $56 \times 1000 =$ _____

4. $56 \times .10 =$ _____

5. $789 \times .100 =$ _____

Directions: Find each quotient.

6. $79 \div 10 =$ _____

7. $789 \div 100 =$ _____

8. $78.9 \div .10 =$ _____

9. $78.9 \div .100 =$ _____

10. $78.9 \div .1000 =$ _____

Answer Key

1. 340
2. 340
3. 56,000
4. 5.6
5. 7.89
6. 7.9
7. 7.89
8. 789
9. 789
10. 789

Dividing by Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Divide decimals by decimals by rewriting divisors as whole numbers.
- Find quotients of decimals by using additional zero placeholders.
- Solve real-world problems involving division by decimals.

Common Errors

A common error for students is that they forget to move the decimal point in the divisor to make it a whole number. You can remind students that dividing by a whole number is easier than by a decimal, so we can change the divisor to a whole number and then divide.

A second common error connected is that the student then forgets to move the decimal point in the dividend. You can begin by talking about balance. What happens to one value, must happen to the other. If we move the decimal point in the divisor, then we must move it in the dividend as well.

Pay attention to the information in the text box. "Remember if you multiply the divisor by a power of ten, then you must also multiply the dividend by a power of ten."

This is really a lesson about practice. See additional examples.

Additional Examples

Directions: Find each quotient.

1. $33.5 \div 6.7$
2. $13.734 \div .6$
3. $37.107 \div 6.3$
4. $36.22 \div .2$
5. $7.38 \div .6$
6. $8.96 \div .2$
7. $5.95 \div .5$
8. $85.72 \div .02$
9. $94.74 \div .05$
10. $58.8 \div .04$

Answer Key

1. 5
2. 22.89
3. 5.89
4. 181.1
5. 12.3
6. 44.8
7. 11.9
8. 4,286
9. 95.7405
10. 1470

Metric Units of Mass and Capacity

Goal

The goals of this lesson can be found in the following objectives.

- Identify equivalence of metric units of mass.

- Identify equivalence of metric units of capacity.
- Choose appropriate metric units of mass or capacity for given measurement situations.
- Solve real-world problems involving metric measures of mass or capacity.

Common Errors

In Teaching Tips, it is recommended that students write down all of the units of mass and capacity in their notebooks. Be sure that students have done this by following the colored text boxes in the lesson. This also includes the equivalent measures for both mass and capacity. The most common error is mixing up units when these have not been committed to memory. Students should know the different measurement units in order from least to greatest and their equivalents too.

Additional Examples

These websites provide assistance with understanding the metric system.

http://www.gamequarium.com/metricsystem.html –This website has many fun games for students about the metric system of measurement.

http://www.thinkmetric.org.uk/ –This website provides students with an excellent introduction to the metric system.

Converting Metric Units

Goal

The goals of this lesson can be found in the following objectives.

- Convert metric units of length, mass and capacity using powers of ten.
- Compare and order given metric measurements of length, mass or capacity.
- Solve real-world problems involving conversion of metric measures of length, mass and capacity.

Common Errors

One of the biggest common errors when converting metric units is that students don't know the individual units and whether each unit is a measure of length, capacity or mass. To help with this, be sure that students have committed the different units to memory and that they know the equivalents.

Review multiplying and dividing by powers of ten and how to move the decimal point to do this. This skill students will be using when converting different measurement units.

Additional Examples

http://www.surfnetkids.com/metric_system.htm –This website provides a list of five of the best websites for working with students on the metric system.

2.5 Number Patterns and Fractions

This fifth chapter *Number Patterns and Fractions* covers prime and composite numbers and factorization. Students will also explore equivalent fractions, how to order fractions, and the relationship between fractions and decimals through methods of conversion. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Prime Factorization
- Greatest Common Factors
- Equivalent Fractions
- Least Common Multiple
- Ordering Fractions
- Mixed Numbers and Improper Fractions
- Changing Decimals to Fractions
- Changing Fractions to Decimals

Prime Factorization

Goal

The goals of this lesson can be found in the following objectives.

- Find factor pairs of given numbers.
- Use divisibility rules to find factors of given numbers.
- Classify given numbers as prime or composite.
- Write the prime factorization of given numbers using a factor tree.

Common Errors

The first common error exists because students don't understand factoring as a process of breaking down numbers into smaller parts. It is the first place to start so that you can help prevent errors later on. In later lessons, students will learn about factors and multiples. This is a common place where students become confused and often try to use factors and multiples interchangeably. You can help to correct this by beginning to always talk about factoring as breaking down numbers or simplifying numbers into their smaller parts.

Practicing times tables will also help students because errors are commonly made when students don't know their times tables by heart. Memorizing these tables will be key when working through this lesson.

The big error that students make with factor trees is that they stop too soon. To help with this, ask students to always work until they have gotten to the smallest prime numbers. Often these numbers are 2, 3, 5 or 7. If a student has not gotten the last row to some combination of these four numbers, then their work is not done. Ask students to write this down in their notebooks.

Always Factor UNTIL 2, 3, 5 or 7 combinations are all that is left.

Be sure that students understand that the operation between the prime factors is multiplication because factors are created by products.

Example: $2^3 \times 5^2$

Additional Examples

Directions: Factor the following to prime factors.

1. 72
2. 36
3. 21
4. 100
5. 144
6. 80
7. 99
8. 22
9. 45
10. 16

Answer Key

1. $3^2 \times 2^3$
2. $3^2 \times 2^2$
3. 3×7
4. $5^2 \times 2^2$
5. $3^2 \times 2^4$
6. $2^4 \times 5$
7. $3^2 \times 11$
8. 2×11
9. $3^2 \times 5$
10. 2^4

Greatest Common Factors

Goal

The goals of this lesson can be found in the following objectives.

- Find the greatest common factor of two or more numbers using lists.
- Find the greatest common factor of two or more numbers using factor trees.
- Solve real-world problems involving greatest common factors.

Common Errors

When beginning this lesson, be sure that the students understand factors as numbers multiplied together to create a product. If they don't understand this and know this right away, then be sure that they understand factors before beginning greatest common factors. Reviewing key words is one of the easiest ways to avoid errors.

If there are many students that struggled with prime factorization, then you can begin with common factors and then move on to greatest common factors. When finding common factors, students will make a list of several factors common to one or more numbers. Then they can find the greatest common factor from the list. This will help them to understand greatest common factors.

One of the examples in the book asks the students to find a greatest common factor of two numbers. Look at what is written in the text.

Here is a tricky one because there is more than one common factor. We have both five and two as common factors.

When you have more than one common factor, we multiply the common factors to find the GCF.

Spend some time with this example. You want the students to understand that if there is more than one common factor that you can multiply the common factors together and this will provide the greatest common factor.

Additional Examples

Here are some websites that provide students with additional practice on greatest common factors.

http://www.basic-mathematics.com/greatest-common-factor-game.html –This is an interactive game that is very basic where students are timed to see how quickly they can identify greatest common factors.

http://www.mathplayground.com/factortrees.html –This is a math playground game on prime factorization. It is very interactive and fun.

http://www.sheppardsoftware.com/mathgames/fractions/GreatestCommonFactor.htm –This is a fun game on greatest common factors. It also has three different levels of difficulty for students.

Equivalent Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Write fractions equivalent to a given fraction.
- Write given fractions in simplest form.
- Describe real-world fractional portions by writing in simplest form.

Common Errors

When beginning this lesson, be sure that the students understand the word "equivalent" as being equal. The form of the fraction may look different, but it is the same quantity. This is a key concept when working through this lesson. It may require several reminders for students to remember this definition.

Students make errors with fractions when they don't understand the numerator and the denominator. Students will need to know that the denominator tells you how many parts the whole has been divided into and the numerator tells you how many parts you have.

To create equal or equivalent fractions, students will need to know the rule. The rule is written in the green text box. They must multiply both the numerator and denominator by the same number. This is where many students will make an error. They will either forget to multiply both the numerator and the denominator or they will multiply by different numbers.

"Same number multiplied to both top and bottom of a fraction creates an equal fraction."

Example: $\frac{1}{2} \times 5 = \frac{5}{10}$

$$\frac{1}{2} = \frac{5}{10}$$

These are equivalent fractions, and this is a true statement.

A common error is when to divide and when to multiply when changing the form of a fraction. Equivalent fractions can be made by multiplying or dividing, but simplifying is always dividing. Students will need a lot of practice simplifying fractions.

Another common error is that students simplify a fraction once, but it still isn't in its simplest form. You can encourage the students to always double check the numerator and the denominator to see if it can be simplified again. But remind students to always check the numerator and denominator to see if there is a GCF to use to simplify.

Additional Examples

Directions: Simplify each fraction. Write simplest form if the fraction is in simplest form.

1. $\frac{30}{90}$
2. $\frac{8}{15}$
3. $\frac{12}{20}$
4. $\frac{31}{60}$
5. $\frac{22}{40}$
6. $\frac{28}{70}$
7. $\frac{144}{200}$
8. $\frac{45}{100}$
9. $\frac{19}{21}$
10. $\frac{13}{26}$

Answer Key

1. $\frac{1}{3}$
2. Simplest form
3. $\frac{3}{5}$
4. Simplest form
5. $\frac{11}{20}$
6. $\frac{2}{5}$
7. $\frac{18}{25}$
8. $\frac{9}{20}$
9. Simplest form
10. $\frac{1}{2}$

Least Common Multiple

Goal

The goals of this lesson can be found in the following objectives.

- Find common multiples of different numbers.
- Find the least common multiple of given numbers using lists.
- Find the least common multiple of given numbers using prime factorization.
- Find two numbers given the greatest common factor and the least common multiple.

Common Errors

The first thing to point out to students as they work in this lesson is that factors are multiplied together to get a product, and that the actual product is a multiple of the factors. You want to help students to clarify between factors and multiples so that they don't get mixed up as they are working in this lesson.

Point out that “counting by” different numbers is the same thing as multiples.

Example: Multiples of 4 means you count by 4. You start with four and keep going....

4, 8, 12, 16, 20, 24, 28, 32, 36.....

Common multiples and common factors are often confused. Remind students that common factors are smaller than the numbers that they start with because with factors we are breaking down numbers. Common multiples are larger than the numbers that they start with because we are “counting by” numbers.

Least common multiples are LARGER than the numbers that you start with. A common error with least common multiples is that students begin to look for factors. Let’s look at an example.

Example: Find the least common multiple of 6 and 8.

Many students will write 2 as the least common multiple. This is the least common factor which isn’t even something that we teach. If you get an answer like this, then you can be sure that the students don’t understand the difference between a factor and a multiple. A multiple must be larger than the numbers that you start with.

Answer: 6, 12, 18, 24

8, 16, 24

The least common multiple of 6 and 8 is 24.

Additional Examples

Directions: Find the LCM of each pair.

1. 10 and 8

2. 6 and 10

3. 10 and 15

4. 7 and 4

5. 12 and 8

Directions: Find the GCF of each pair.

6. 12 and 16

7. 9 and 12

8. 12 and 24

9. 36 and 45

10. 24 and 48

Answer Key

1. 40
2. 30
3. 30
4. 28
5. 24
6. 4
7. 3
8. 6
9. 9
10. 24

Ordering Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Compare fractions using lowest common denominator.
- Order fractions using lowest common denominator.
- Describe real-world portion or measurement situations comparing and ordering fractions.

Common Errors

One of the most common errors when comparing quantities is the correct use of the symbols. To help with this, you can say that the opening or "alligator" always "eats" the larger number. Use a whole number example before bringing this to the lesson with fractions.

Example: 34 >16

34 is being "eaten" by the symbol because it is the larger of the two quantities.

Now you are going to bring comparing to fractions. To avoid errors, explain to students that they will need to rename the fractions so that they have the same denominator before comparing them. Students will want to skip this step and will make errors as a result. Renaming is a necessary skill when working with fractions. Students will need to do this to accurately compare the fractions.

Ordering fractions requires the same renaming. Teach students to rewrite all of the fractions to have common denominators so that they can be compared easily.

A note: students will resist this because it is more work. A common error you will see is that they will choose $\frac{1}{3}$ as smaller than $\frac{1}{8}$ because of the denominators. This is not the case, so you will have to reteach renaming to common denominators if you see this. It is a sign that students are skipping steps and just comparing based on number not on the actual size of the fraction.

Additional Examples

Here are some websites that will help students with additional fraction practice.

http://www.gamequarium.com/fractions.html –This website has excellent games for students who are learning to compare and order fractions.

http://www.okaloosa.k12.fl.us/oakhill/fractions.html –This website is from a Florida school district which has a lot of information on fractions.

http://www.khanacademy.org/video/ordering-fractions?playlist=Pre-algebra –This is a Khan Academy video on ordering fractions.

Mixed Numbers and Improper Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Measure lengths to a fraction of an inch.
- Rewrite mixed numbers as improper fractions.
- Rewrite improper fractions as mixed numbers.
- Compare and order mixed numbers and improper fractions.

Common Errors

A common error when measuring lengths to a fraction of an inch is a confusion about the measurement of each line on a ruler. To help students with this, hand out rulers and review all of the different fractions of an inch. It is useful to have a larger ruler drawn on the board or put on the bulletin board for students to use as a reference.

http://www.vendian.org/mncharity/dir3/paper_rulers/ –This website has printable paper rulers for students to use as a reference.

http://www.rsinnovative.com/rulergame/ –This website has a ruler game which can help students learn how to read a ruler.

http://www.freeprintable.com/free-printables/ruler+with+all+fractions+labeled –This website has a printable ruler with all of the fractions labeled on it.

Avoid common errors by encouraging students to pay close attention to the information in the colored text boxes. In this lesson, there are boxes that have definitions for improper fractions and mixed numbers. Go over these definitions as they are presented to the students during the lesson.

A common error is made when students don't simplify the fraction part of the mixed numbers. Always remind students that fractions or fraction parts of mixed numbers must be written in simplest form.

Additional Examples

Directions: Write each mixed number as an improper fraction.

1. $3\frac{1}{3}$
2. $6\frac{2}{5}$
3. $7\frac{1}{2}$
4. $9\frac{3}{8}$
5. $5\frac{7}{12}$

Directions: Write each improper fraction as a mixed number. Be sure the fraction is in simplest form.

6. $\frac{12}{7}$
7. $\frac{32}{6}$
8. $\frac{48}{9}$
9. $\frac{51}{2}$
10. $\frac{18}{3}$

Answer Key

1. $\frac{10}{3}$
2. $\frac{32}{5}$
3. $\frac{15}{2}$
4. $\frac{75}{8}$
5. $\frac{67}{12}$
6. $1\frac{5}{7}$
7. $5\frac{1}{3}$
8. $5\frac{1}{3}$
9. $25\frac{1}{2}$
10. 6

Changing Decimals to Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Write decimals as fractions.
- Write decimals as mixed numbers.
- Write decimals as equivalent fractions and mixed numbers.
- Describe real-world portion or measurement situations by writing decimals as fractions.

Common Errors

This lesson combines decimals and fractions and teaches the students how to convert between the two. To begin, be sure that the students understand place value and how to read decimals and fractions. This review will help to prevent errors as you work through the lesson.

Begin this lesson by having students write decimals in tenths and hundredths into fractions with the correct denominator. Review simplifying. Once students understand how to write the decimal in fraction form, students will need to move on to the next step which is simplifying. All fractions must be simplified or written in simplest form. Simplifying is the most challenging skill in the lesson.

Additional Examples

Here are a few websites that will help students as they work through converting fractions and decimals.

http://www.mathsisfun.com/decimal-fraction-percentage.html –This website works on helping students to understand the connections between fractions, decimals and percentages.

http://www.aaamath.com/dec.htm –This website has many games and tutorials on working with decimals. You can find options for helping students to understanding converting decimals and fractions.

Changing Fractions to Decimals

Goal

The goals of this lesson can be found in the following objectives.

- Write fractions as decimals.
- Write mixed numbers as decimals.
- Write fractions and mixed numbers as repeating decimals.
- Compare and Order decimals and fractions.

Common Errors

The content of this lesson takes the previous lesson's content to the next level. In the last lesson we worked on converting decimals to fractions, now we are going to work on changing fractions to decimals.

A common error when doing this is that the students forget to create equivalent fractions with a base ten denominator. This is the first step in working with a fraction. Let's look at an example.

Example: $\frac{2}{5}$

To change this fraction to a decimal, we must find an equal fraction with a denominator of 10 or 100 etc. Choose one that makes sense. In this case, we can use 10.

$$\frac{2}{5} = \frac{4}{10} = .4$$

Notice that we changed to an equal fraction with a base ten denominator. From there converting to a decimal was very simple. The step of the equal fraction is not one that can be skipped. This is where many errors are made.

Additional Examples

These websites provide additional practice and explanation.

http://www.coolmath.com/prealgebra/02-decimals/12-decimals-repeating-01.htm –This is an excellent lesson on repeating decimals. It shows how to find them and identify them.

http://www.math.com/tables/general/arithmetic/fradec.htm –This website has a fraction to decimal conversion table. It could be used to create a homework assignment.

http://www.toonuniversity.com/flash.asp?err=198 –This website has a game where you compare fractions and decimals.

2.6 Addition and Subtraction of Fractions

This sixth chapter *Addition and Subtraction of Fractions* further develops a student's capability with fractions by introducing methods of addition and subtraction. Also covered are box-and-whisker plots, solving problems using diagrams, and basic problems involving time. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Fraction Estimation
- Adding and Subtracting Fractions with Like Denominators
- Adding and Subtracting Fractions with Different Denominators
- Adding and Subtracting Mixed Numbers
- Subtracting Mixed Numbers by Renaming
- Elapsed Time
- Box-and-Whisker Plots
- Problem-Solving Strategy-Draw a Diagram

Fraction Estimation

Goal

The goals of this lesson can be found in the following objectives.

- Round fractions to the nearest half.
- Round mixed numbers to the nearest whole number.
- Estimate sums and differences of fractions and mixed numbers.
- Estimate sums and differences involving real-world fractional amounts.

Common Errors

A common error in this lesson is that students will try to round to different fractions and whole numbers as they did when rounding whole numbers. In this lesson, when rounding fractions, they will be rounding to zero or to $\frac{1}{2}$ or to 1. Those are the only three options. It will seem funny to students to round down to zero. You will need to explain that when such a small fraction of a whole is represented that it is almost as if there isn't any part left at all.

Example: $\frac{1}{33}$

This is such a small part of a whole that it is closest to zero. The whole is divided into 33 parts, and then one part out of 33 is represented.

Now when rounding mixed numbers, we are rounding to the nearest whole. Students will be looking at the fraction and determining if it is closest to the number already represented or to the next number in the series.

Example: $6\frac{5}{6}$

Students can look at this mixed number and determine that it is closest to 7. Because five–sixths is almost another whole, we can round up to 7.

After working with rounding, students will work on estimating sums. The key to estimating sums is to use what they have learned rounding fractions and mixed numbers to figure out the accurate sum or difference. This is a first step to adding and subtracting fractions.

Additional Examples

Directions: Round each fraction.

1. $\frac{1}{12}$
2. $\frac{3}{7}$
3. $\frac{2}{15}$
4. $\frac{8}{11}$
5. $\frac{9}{10}$

Directions: Round each mixed number.

6. $4\frac{2}{3}$
7. $9\frac{1}{13}$
8. $12\frac{5}{6}$
9. $16\frac{2}{11}$
10. $8\frac{7}{9}$

Answer Key

1. 0
2. $\frac{1}{2}$
3. 0
4. 1
5. 1
6. 5
7. 9
8. 13
9. 16
10. 9

Adding and Subtracting Fractions with Like Denominators

Goal

The goals of this lesson can be found in the following objectives.

- Add fractions with like denominators.
- Subtract fractions with like denominators.
- Evaluate given numerical expressions involving sums and differences of fractions with common denominators.
- Solve real-world problems involving sums and differences of fractions with common denominators.

Common Errors

Adding and subtracting like denominators is not often a place where students make errors. They find it easy to simply work with the numerators and often don't give the denominators a second thought. This is why they do tend to make errors when working with unlike denominators. One of the ways that you can help with this is to

consistently remind students that you can only add and subtract this simply because the denominators are the same. You can say something like, "Notice that we only work with the numerators because the denominators are the same. If the denominators were different, then we would need to do something different."

Students sometimes forget to simplify. You must remind students that they will always need to simplify or write their final answer in simplest form.

When working with three fractions in a numerical expression, remind students to add or subtract fractions in order from left to right. That way they won't forget to add or subtract one of the fractions.

Additional Examples

Here are a few websites with additional practice for the students.

http://www.homeschoolmath.net/online/fractions.php#add –This website has a bunch of different interactive games that students can play where they will add and subtract fractions. Some work with like and unlike denominators. There are also games that practice simplifying.

http://www.visualfractions.com/ –This website has an excellent tutorial where students are taught how to work with fractions in a visual way. The tutorial includes work with adding and subtracting fractions.

Adding and Subtracting Fractions with Different Denominators

Goal

The goals of this lesson can be found in the following objectives.

- Add fractions with different denominators.
- Subtract fractions with different denominators.
- Evaluate given numerical expressions involving sums and differences of fractions with different denominators.
- Solve real-world problems involving sums and differences of fractions with different denominators.

Common Errors

When beginning this lesson, review adding and subtracting fractions with like denominators. Because there are several common errors made with unlike denominators, it is important to take this step. You can also help to avoid errors by reviewing the vocabulary associated with unlike denominators.

Review
least common multiple and connect it with lowest common denominator.

Review
renaming fractions because students will need to rename fractions with a common denominator.

A common error when renaming fractions is that students forget to multiply the numerator when the denominator has been renamed. You can teach them that what they do to the denominator they must also do to the numerator. This is just the way that it is. Keep repeating and reviewing this as the students work their way through the lesson.

Another common error is that students will forget to simplify their answer. They have done a lot of work by renaming the fractions and then adding them, but remind students to always check that their answer is in simplest form.

Evaluating numerical expressions with more than two fractions can be tricky for students. It is important that the students have a good grasp on working with unlike denominators before moving to this lesson. When students can add and subtract two fractions with different denominators with ease, then they are ready to move on to working with three fractions. Rushing to this step can cause students to make unnecessary errors.

Additional Examples

There are many websites that help students learn how to add and subtract fractions with different denominators.

http://www.visualfractions.com/ –This website has an excellent tutorial to teach students how to add and subtract fractions with different denominators.

http://www.aaamath.com/fra66kx2.htm –This is an interactive lesson on adding fractions with different denominators.

http://www.aaamath.com/fra66lx2.htm –This is an interactive lesson on subtracting fractions with different denominators.

http://www.superteacherworksheets.com/fractions-adding.html –This website provides teachers with worksheets on adding and subtracting fractions with and without common denominators. Answer keys are provided.

Adding and Subtracting Mixed Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Add mixed numbers.
- Subtract mixed numbers without renaming.
- Evaluate given numerical expressions involving sums and differences of mixed numbers.
- Solve real-world problems involving sums and differences of mixed numbers.

Common Errors

Begin this lesson by reviewing mixed numbers and how to identify a mixed number. You want the students to be thinking in terms of wholes and parts before we begin adding and subtracting mixed numbers.

A common error is that students will add the wholes before the parts. As is taught in the lesson, students should become accustomed to adding the fraction parts of the mixed numbers BEFORE the wholes. This helps them if they add the fractions and the fractions are equal to a whole. When this happens, then the students can add the whole to the other wholes in the problem. It will help prevent errors later on.

Begin with problems that have common denominators in the fractions and then move to unlike denominators. Be sure that students are adding and subtracting mixed numbers with common denominators with ease before you move on to uncommon denominators. We want the students to be able to proceed to this step without a lot of difficulty. Taking your time to be sure of student understanding is a definite way to prevent errors.

Review students in how to convert an improper fraction to a mixed number. Answers that are left improper are incorrect because they are not in simplest form. Remind students that all answers must be in simplest form.

Additional Examples

Here are a few websites on adding and subtracting mixed numbers.

http://www.mathsisfun.com/numbers/fractions-mixed-addition.html –This website has a visual tutorial for students on how to add and subtract mixed numbers. It uses pizza in the visuals.

http://www.math-drills.com/fractions.shtml –This is a website with free worksheets for teachers to use with their students. There are many fraction worksheets.

http://www.mathplayground.com/fractions_mixed.html –This website has a game for students to practice converting improper fractions to mixed numbers. It is an excellent review game for students.

http://www.aaamath.com/fra66hx2.htm –This website has a review of simplifying fractions for the students.

Subtracting Mixed Numbers by Renaming

Goal

The goals of this lesson can be found in the following objectives.

- Subtract mixed numbers with renaming.
- Evaluate numerical expressions involving differences of mixed numbers requiring renaming.
- Solve real-world problems involving differences of mixed numbers requiring renaming.

Common Errors

When beginning this lesson, review borrowing when subtracting with whole numbers. Look at this example.

Example: $300 - 19$

When presenting this problem, go through the steps of borrowing tens and then regrouping so that subtraction is possible. Once you have reviewed this problem, you can make the connection to working with mixed numbers.

"Sometimes, you will need to borrow to subtract when working with mixed numbers too. We call this renaming."

Example: $3\frac{1}{3} - \frac{2}{3}$

This example has common denominators, but you can show students that we can't subtract two-thirds from one-third. Because of this, we need to borrow a whole from the 3. When we do that we make the whole 2 and take the borrowed whole and turn it into fraction form.

$1 = \frac{3}{3}$

A common error is that students then forget to add this whole to the already existing fraction. We have to add it even though it will create an improper fraction. This is alright because we are going to subtract.

$$\frac{3}{3} + \frac{1}{3} = \frac{4}{3}$$

Now we can subtract with ease. Here is what the new problem looks like.

$$2\frac{4}{3} - \frac{1}{3} = 2\frac{3}{3} = 3$$

Remind students that while this is extra work, that it must be done in this way or subtraction is not possible. A key to decreasing errors is to encourage students to follow the steps.

Additional Examples

Directions: Subtract the following mixed numbers. Be sure your answer is in simplest form.

1. $6\frac{1}{8} - \frac{5}{8}$
2. $4\frac{2}{6} - \frac{5}{6}$
3. $7\frac{1}{2} - \frac{1}{2}$
4. $2\frac{1}{9} - 1\frac{7}{9}$
5. $8\frac{3}{12} - 5\frac{10}{12}$
6. $10\frac{3}{10} - 5\frac{9}{10}$
7. $5\frac{1}{8} - 3\frac{7}{8}$
8. $11\frac{3}{11} - 9\frac{9}{11}$

9. $8 - \frac{3}{5}$
10. $10 - \frac{6}{7}$

Answer Key

1. $5\frac{1}{2}$
2. $3\frac{1}{2}$
3. 7
4. $\frac{1}{3}$
5. $2\frac{5}{12}$
6. $4\frac{2}{5}$
7. $1\frac{1}{4}$
8. $1\frac{1}{2}$
9. $7\frac{2}{5}$
10. $9\frac{1}{7}$

Elapsed Time

Goal

The goals of this lesson can be found in the following objectives.

- Add and subtract measures of time.
- Find elapsed time between given start and stop times.
- Solve real-world problems involving elapsed time.

Common Errors

Common errors are made when students don't know the units of time and their equivalents. To help prevent errors, be sure that students have memorized the text box at the beginning of the lesson and the middle of the lesson. Fractional units of time should be known as well.

Additional Examples

Here are few websites that will help students to practice elapsed time.

http://www.shodor.org/interactivate/activities/ElapsedTime/ –This is an interactive website where students can play a game to help them practice calculating elapsed time.

http://www.superteacherworksheets.com/elapsed-time.html –This website has several worksheets on elapsed time. The worksheets are organized according to level so that you can select an appropriate level for the students.

Box-and-Whisker Plots

Goal

The goals of this lesson can be found in the following objectives.

- Order a set of data to find the median, quartiles and extremes.
- Draw a box-and-whisker plot to represent given data.
- Identify the median, quartiles, and extremes given a box-and-whisker plot.

- Compare and Interpret double box-and-whisker plots of real-world data.

Common Errors

In the blue text box in the beginning of this lesson, steps are written to help students to create box-and-whisker plots. Teaching students not to skip steps is key to helping students avoid errors. Notice that the text box instructs students of the steps to complete before drawing the box-and-whisker plot.

Be sure that students use rulers so that the lines are straight and the intervals are even. When this is the case, students won't make as many errors in their work.

Additional Examples

Here are a few websites on box-and-whisker plots.

http://www.mrnussbaum.com/graph/bw.htm –This is an interactive website for practicing box-and-whisker plots.

http://www.onlinemathlearning.com/box-whisker-plot.html –This website has videos, games and songs about box-and-whisker plots. It is geared for grade 8, but could be used for students who are needing additional challenge.

Problem-Solving Strategy-Draw a Diagram

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Draw a diagram.
- Plan and Complete alternative approaches to solving problems.
- Solve real-world problems using selected strategies as part of a plan.

Common Errors

The most common error students make when problem solving is that they don't take the time to read the problem carefully. Therefore, they aren't sure which strategy is needed for finding a solution. We can help them with this by encouraging students to read through each problem so that solutions are found through different strategies.

This lesson focuses on drawing a diagram. When something visual is mentioned, like fractions, this is a key for students to let them know that drawing a picture or diagram would be helpful when finding a solution.

Here are some other hints:

- Fractions
- Problems with geometric shapes
- Problems involving statistics

2.7 Multiplication and Division of Fractions

This seventh chapter *Multiplication and Division of Fractions* covers the methods needed to multiply and divide fractions and mixed numbers. Students are also introduced to using and converting customary units. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Multiplying Fractions and Whole Numbers
- Multiplying Fractions
- Multiplying Mixed Numbers
- Dividing Fractions
- Dividing Mixed Numbers
- Customary Units of Weight and Capacity
- Converting Customary Units
- Problem-Solving Strategy: Choose an Operation

Multiplying Fractions and Whole Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Multiply fractions by whole numbers.
- Multiply whole numbers by fractions.
- Estimate products of whole numbers and fractions.
- Solve real-world problems involving products of whole numbers and fractions.

Common Errors

The first common error that students make when multiplying fractions and whole numbers is that they turn the whole number into a fraction with the whole number as the denominator instead of a denominator of 1.

Example: 4 becomes $\frac{1}{4}$ instead of $\frac{4}{1}$

This can show you a lack of understanding of fractions and whole numbers. If a student confuses one–fourth with four, then he/she doesn't understand parts and wholes. He/she is looking for the "how to do it" and that is a procedural understanding and not a conceptual understanding. This can be corrected through the following exercise.

Ask the student who has made this error to draw a picture of 4 and of $\frac{1}{4}$

Then have them compare the two-how are they different? How are they the same? Thinking in this way should help the student move from a procedural understanding to a conceptual one.

Working with one-third, one-fourth or one-half of a number relies on a students conceptual understanding. The student has to be able to think about dividing up a quantity into pieces. If students struggle with this step, then

go back to drawing out fractions and showing the concept of one-half, one-third, one-fourth. You can also use the following example.

"One–half of our class likes chocolate ice cream. How many is that?"

Students will automatically move to the conceptual. Then you can point it out to them so that they can make the connection that they divided by 2.

Additional Examples

Directions: Multiply each fraction and whole number. Be sure that your answer is in simplest form.

1. $6 \times \frac{1}{3}$
2. $15 \times \frac{2}{3}$
3. $18 \times \frac{1}{6}$
4. $21 \times \frac{1}{3}$
5. $9 \times \frac{3}{5}$
6. $10 \times \frac{2}{5}$
7. What is one–half of 16?
8. What is one–fourth of 16?
9. What is one–third of 90?
10. What is one–third of 120?

Answer Key

1. 2
2. 10
3. 3
4. 7
5. $5\frac{2}{5}$
6. 4
7. 8
8. 4
9. 30
10. 40

Multiplying Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Multiply two fractions.
- Multiply three fractions.
- Evaluate numerical and algebraic expressions involving products of fractions.
- Solve real-world problems involving products of fractions.

Common Errors

When students begin multiplying fractions, the steps are so straight–forward that they don't have difficulties. In fact, the idea of multiplying numerators by numerators and denominators by denominators makes sense to the students and they are able to complete this process with ease.

The challenge comes when cross–simplifying is introduced. Students can simplify before multiplying by looking at the numbers on the diagonals. The tricky part comes when after simplifying, they need to multiply. A common error is that students will forget to multiply across horizontally and will instead multiply on the diagonals because that is how they just simplified. You will need to remind them that:

1. Simplify on the diagonals
2. Multiply horizontally or across

Remind students that if they forget to simplify first, that it is alright as long as they simplify at the end after multiplying. Students must always leave their product in simplest form.

Additional Examples

Directions: Simplify first and then multiply.

1. $\frac{3}{6} \times \frac{4}{6}$
2. $\frac{3}{4} \times \frac{1}{3}$
3. $\frac{6}{7} \times \frac{3}{9}$
4. $\frac{5}{6} \times \frac{6}{7}$
5. $\frac{4}{5} \times \frac{10}{15}$
6. $\frac{9}{10} \times \frac{1}{3}$
7. $\frac{8}{9} \times \frac{3}{6}$
8. $\frac{1}{2} \times \frac{4}{5}$
9. $\frac{3}{7} \times \frac{14}{21}$
10. $\frac{8}{9} \times \frac{3}{4}$

Answer Key

1. $\frac{1}{3}$
2. $\frac{1}{4}$
3. $\frac{2}{7}$
4. $\frac{5}{7}$
5. $\frac{8}{15}$
6. $\frac{3}{10}$
7. $\frac{4}{9}$
8. $\frac{2}{5}$
9. $\frac{2}{7}$
10. $\frac{2}{3}$

Multiplying Mixed Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Multiply mixed numbers.
- Evaluate numerical and algebraic expressions involving products of mixed numbers.
- Solve real-world problems involving products of mixed numbers.

Common Errors

In the last lesson, students learned how to multiply two or more fractions together. In this lesson, students will learn how to multiply mixed numbers. This does involve an additional step before they can simplify and multiply. It is a good idea to begin this lesson with a review of the material from the previous one. Then you can build on multiplying fractions by adding mixed numbers.

When working on this lesson, be sure to have students convert mixed numbers to improper fractions before multiplying. Avoid errors by not assuming that the students remember how to convert mixed numbers to improper fractions. It is always helpful to review this process first.

Be sure that students understand and follow these steps:

1. Convert mixed numbers to improper fractions.
2. Simplify on the diagonals.
3. Multiply horizontally for a product.
4. Be sure that the product is in simplest form.

Following steps is one way to avoid common errors.

Additional Examples

Directions: Multiply the following mixed numbers. Be sure that your answer is in simplest form.

1. $3\frac{1}{2} \times 6$
2. $4\frac{1}{3} \times 2$
3. $8\frac{1}{2} \times 9$
4. $6\frac{1}{2} \times 2\frac{1}{3}$
5. $8\frac{1}{2} \times 6$
6. $4\frac{1}{3} \times 2\frac{1}{2}$
7. $2\frac{4}{5} \times 1\frac{1}{3}$
8. $1\frac{1}{2} \times 7\frac{1}{3}$
9. $8\frac{1}{4} \times 2\frac{1}{3}$
10. $6\frac{4}{7} \times 8\frac{1}{3}$

Answer Key

1. 21
2. $8\frac{2}{3}$
3. $76\frac{1}{2}$
4. $15\frac{1}{16}$
5. 51
6. $10\frac{5}{6}$
7. $3\frac{11}{15}$
8. 11
9. $19\frac{1}{4}$
10. $54\frac{16}{21}$

Dividing Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Identify and write reciprocal fractions.

- Divide a fraction by a whole number.
- Divide a whole number by a fraction.
- Divide a fraction by a fraction.
- Solve real-world problems involving quotients of fractions.

Common Errors

The most common error for students when dividing fractions is that they forget to change the second fraction to it's reciprocal. They often remember to change the operation from division to multiplication, but they forget that the second part of the process is to change the second fraction to it's reciprocal. Here are the steps that students must follow.

1. Change the operation to multiplication.
2. Change the second fraction to it's reciprocal.

You can also remind students that once they have changed the division problem to multiplication and changed the second fraction to it's reciprocal that they can simplify first and then multiply or multiply and then simplify. The key is that the answer must be in simplest form.

Often the answer in a division problem is an improper fraction. A common error that students make is that they forget to change the improper fraction to a mixed number. You will have to tell them that an improper fraction is not in simplest form. For the answer to be in simplest form, they must change the improper fraction to a mixed number.

Additional Examples

Directions: Divide each pair of fractions. Be sure that your answer is in simplest form.

1. $\frac{1}{2} \div \frac{1}{3}$
2. $\frac{4}{5} \div \frac{1}{2}$
3. $\frac{6}{7} \div \frac{2}{3}$
4. $\frac{5}{9} \div \frac{3}{4}$
5. $\frac{6}{10} \div \frac{1}{2}$
6. $\frac{10}{12} \div \frac{3}{4}$
7. $\frac{16}{18} \div \frac{1}{2}$
8. $\frac{14}{16} \div \frac{2}{3}$
9. $\frac{18}{20} \div \frac{2}{5}$
10. $\frac{20}{21} \div \frac{3}{7}$

Answer Key

1. $1\frac{1}{2}$
2. $1\frac{3}{5}$
3. $1\frac{2}{7}$
4. $\frac{20}{27}$
5. $1\frac{1}{5}$
6. $1\frac{1}{9}$
7. $1\frac{7}{9}$
8. $1\frac{5}{16}$
9. $2\frac{1}{4}$
10. $2\frac{2}{9}$

Dividing Mixed Numbers

Goal

The goals of this lesson can be found in the following objectives.

- Divide a mixed number by a fraction.
- Divide a mixed number by a mixed number.
- Solve real-world problems involving quotients of mixed numbers.

Common Errors

When dividing mixed numbers, we are going to be combining several different processes together. Here is a list of the steps that we are combining.

1. Convert mixed numbers to improper fractions.
2. Change division to multiplication.
3. Change the second fraction to its reciprocal
4. Cross simplify
5. Multiply horizontally
6. Write the answer in simplest form

You can make a list of these steps on the board to help the students to keep track of them. The most common error with division of fractions and mixed numbers is that students forget the steps or skip some of them to make things easier. Only skipping steps doesn't make things easier because students have to redo work that they have already done.

Review each step with the students and review anything that is unclear before letting the students work on their own.

Additional Examples

Here are a few websites to help students with dividing mixed numbers.

http://www.aaamath.com/fra.html –This website has many lessons on fractions including dividing fractions and working with mixed numbers.

http://www.math.com/school/subject1/lessons/S1U4L8GL.html –Here is an interactive tutorial on dividing mixed numbers.

http://www.khanacademy.org/video/dividing-mixed-numbers?playlist=Developmental+Math –This is a Khan Academy video on how to divide mixed numbers.

http://edhelper.com/fractions.htm –This is a website with many free worksheets on fractions. You can choose the grade, level and topic of the worksheet.

Customary Units of Weight and Capacity

Goal

The goals of this lesson can be found in the following objectives.

- Identify equivalence of customary units of weight.
- Identify equivalence of customary units of capacity.
- Choose appropriate customary units of weight or capacity for given measurement situations.

- Solve real-world problems involving customary measures of weight or capacity.

Common Errors

A key to avoiding common errors in equivalence is to remember the conversion steps that were introduced in earlier chapters. Students can multiply to go from a larger unit to a smaller unit and divide to go from a smaller unit to a larger unit.

A common error can occur when students convert units and the answer is not even. All of a sudden, they can forget everything that they have already learned about fractions and decimals. This happens because the students don't connect measurement to fractions and decimals. Be sure to point out that there are many times in life when measurements are not whole numbers. When this happens, they will need to write a fraction or decimal to represent the measure.

Additional Examples

Here are a few websites to help with units of weight and capacity.

http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/6_Weight_and_Capacity/index.html –This website has a wonderful video on understanding weight and capacity.

http://www.mathplayground.com/wangdoodles.html –This is a math playground game that helps students to understand weight and equivalent measures of weight.

Converting Customary Units

Goal

The goals of this lesson can be found in the following objectives.

- Convert customary units of measure using multiplication.
- Convert customary units of measure using division.
- Add and subtract customary units of measure.
- Solve real-world problems involving customary units of measure.

Common Errors

This lesson builds on the previous lesson by teaching students how to convert customary units of measurement.

Remember the two key rules for conversion: when converting from a larger unit to a smaller unit, we multiply. When converting from a smaller unit to a larger unit, we multiply. To be able to work with these rules, students will need to know the units from smallest to largest by heart. Errors are made when students don't know how to compare the different units of measurement.

When adding and subtracting customary units of measurement, a common error can be made if the students don't select the correct unit. Review each of the different units of measurement and which ones can best be used for conversions.

Additional Examples

The following websites offer assistance on converting customary units of measurement.

http://illuminations.nctm.org/LessonDetail.aspx?id=U148 –This is a great website from NCTM on measurement both customary and metric. There are several different options for teachers to use and different lessons are illustrated.

Problem-Solving Strategy: Choose an Operation

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Choose an operation.
- Plan and compare alternative approaches to solving problems.
- Solve real-world problems using selected strategies as part of a plan.

Common Errors

One of the keys of successful problem solving is choosing the correct operation to solve a problem. If the correct operation is not chosen, then the student is often frustrated and unable to find a solution.

To choose the correct operation, students must learn the key words associated with each operation and commit these words to memory. Here are the notes from the lesson.

Addition –a problem where two quantities are being combined. Key words are sum, total, in all, combined, altogether.

Subtraction –a problem where two quantities are being compared or one quantity is being removed from another quantity. Key words are left, left over, take away, less than, difference, more, less, more than, less than, compare, greater than, increased or decreased by.

Multiplication –word problems using repeated addition or groups. Key words are a lot like addition-pay attention to the repeating-that will help you know that you need to multiply. Key words are total, in all, each, every, per, how much, at this rate and of.

Division –Division problems generally involve a situation in which a single quantity is split up into many equal-sized parts. Key words are split, divide, shared, equal size, average, groups, per.

Once students have learned the words associated with each operation, they can learn to underline these words in the actual problem. This is a way to prevent the common error of selecting an operation that will not help to solve the problem.

Additional Examples

http://www.teachervision.fen.com/math-operations/problem-solving/48933.html –This website has a lesson plan to help teachers with the strategy of choosing an operation. It also provides teachers with key questions to ask students.

2.8 Ratios, Proportions and Percents

This eighth chapter *Ratios, Proportions, and Percents* introduces students to ratios and rates, basic uses of proportions including understanding scale drawings, and percents. Also explored is the relationship among percents, decimals, and fractions. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter:

- Ratios
- Rates
- Solving Proportions
- Proportions and Scale Drawings
- Understanding Percent
- Percents, Decimals and Fractions
- Finding a Percent of a Number
- Problem-Solving Strategy: Use a Proportion

Ratios

Goal

The goals of this lesson can be found in the following objectives.

- Identify and write different forms of equivalent ratios.
- Write ratios in simplest form.
- Write and compare ratios in decimal form.
- Solve real-world problems involving ratios.

Common Errors

A common error when working with ratios is that students become used to writing a ratio in only one way. For example, they become accustomed to only writing ratios in fraction form, so that when a ratio is written in one of the other forms, the student isn't sure how to work with the ratio. Look at this example.

Example: $\frac{4}{5}$ can be written as 4:5 or 4 to 5

The numerator or top number of the ratio is also the first number in the other two ways of writing the same ratio. Students should be able to use all three forms of a ratio with ease and interchangeably.

Ratios can also be simplified. A common error of students is that if a ratio is not written in fraction form, then they forget to simplify it.

Example: 6:10

This is a ratio, but it is not in simplest form. However because the form of the ratio is not fraction form, students make an error by often leaving it alone. To help correct this error, students can change all ratios to fraction form and then simplify them.

6:10 becomes $\frac{6}{10}$ which simplified to $\frac{3}{5}$ or 3:5.

This is the final answer.

A common error is made when students have to convert ratios to decimal form. Avoid errors by reviewing the steps before students work independently.

Additional Examples

Here are a few websites with information on ratios.

http://www.aaamath.com/grade7.htm –This website has several games and tutorials on working with ratios. Even though the websites says seventh grade, the games and tutorials are appropriate for sixth grade as well.

http://www.mathworksheets4kids.com/ratio.html –This website has many different worksheets for teachers to use with their students. There are worksheets on identifying ratios and writing ratios as well as worksheets on simplifying ratios.

Rates

Goal

The goals of this lesson can be found in the following objectives.

- Identify and write equivalent rates.
- Write and compare unit rates.
- Solve real-world problems involving rates.

Common Errors

Students can begin by understanding that a rate is a type of ratio. A common misunderstanding is that a rate has to be compared to one, but this is a unit rate and not a regular rate. There can be both.

For example: 5 tickets for $10.00 is a rate

The unit rate would be the cost for one ticket which would be $2.00.This is a simple example of how a rate varies from a unit rate.

A common error is in identifying what is being compared by the rate. Sometimes, students aren't sure what is being compared.

For example: There are five chickens for $15.00.

In this example, we are comparing the chickens to the cost. We aren't comparing chickens to eggs or to people. We are comparing chickens and costs. This is our comparison.

$$\frac{Chickens}{Cost}$$

Here is our rate. Now we can fill in the given numbers.

$$\frac{5}{\$15.00}$$

This is the rate.

Now if we wanted the unit rate, we would be looking for the cost of one chicken. To figure this out, we write a comparison using 1.

$$\frac{5}{\$15.00} = \frac{1}{\$3.00}$$

If 5 chickens costs $15.00, then 1 chicken costs $3.00.

Additional Examples

The following websites will provide additional lessons or worksheets on rates and unit rates.

http://www.aaamath.com/rat-unit-rate.htm –This is an interactive lesson on unit rates.

http://www.mathsisfun.com/measure/unit-price-game.html –This website explains how to identify and define a unit rate. Then it has a game for students to play on how to use unit rates.

http://edhelper.com/ratios.htm –This website has a list of worksheets for teachers on ratios, proportions and unit rates.

http://tutor-usa.com/free/pre-algebra/worksheet/ratios-rates-and-unit-rates –This free pre-algebra worksheet is on ratios, rates and unit rates. It includes an answer key.

Solving Proportions

Goal

The goals of this lesson can be found in the following objectives.

- Use cross products to identify proportions.
- Solve for the unknown part of given proportions using mental math.
- Solve for the unknown part of given proportions using related equations.
- Solve real-world problems involving proportions.

Common Errors

One of the first common errors that students might make is to multiply horizontally because ratios are written in fraction form. We talk about testing to see if two ratios form a proportion by using cross-products. If a student is confused, you will see him/her multiply across to see if the two ratios form a proportion. Then the student will say no because the product of the denominators is not equal to the product of the numerators. However, this is not an accurate test of a proportion.

To help with this issue, you can explain to students that you are not looking for a product. You are "doing a test". The way that the test works is to multiply cross–products and if the answers are the same, then the two ratios form a proportion.

When setting up a proportion from a word problem, remind students that they need to watch how they set up the problem. Look at this example from the text.

Example: ***Notice that we put the same unit in the numerator of both ratios and the same unit in the denominator of both units.***

$\frac{laps}{minutes} = \frac{laps}{minutes}$

This is the actual example from the text with the direction preceding the proportion. Students must always match the units so that they are the same. In this example, the units "laps" are both in the numerator and the "minutes" are in the denominator. This is the best way to ensure that the proper item is found in the solution.

Additional Examples

The text has many examples for solving proportions. Here are a few websites with additional tutorials, games and worksheets for learning to solve proportions.

http://edhelper.com/ratios.htm –This website allows teachers the opportunity to choose which worksheets they wish to create. There are several options for proportions.

http://www.arcademicskillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html –This is a great online game where students ride dirt bikes and solve proportions. Their work is timed so competition can become a part of the game.

http://www.math-videos-online.com/proportion-word-problem.html –This website has a tutorial on how to solve a word problem using a proportion. It includes video.

Proportions and Scale Drawings

Goal

The goals of this lesson can be found in the following objectives.

- Given scale ratios, find actual or scale dimensions using proportions.
- Given actual and scale dimensions, find scale ratios using proportions.
- Solve real-world problems involving scale drawings and maps using proportions.

Common Errors

This lesson introduces students to another application for proportions. We have seen proportions used to solve problems involving time and unit rates, now we are going to use proportions to solve problems involving scale measurement and distances on a map. Students will need to know the basics of solving proportions to be successful in this lesson. Be sure to take a few minutes at the beginning of class to review how to solve a proportion by using cross-products.

A common error can be the set up of the proportion. Remind students that if they compare inches to feet in one ratio that the other ratio must also compare inches to feet. Look at this example from the text.

$\frac{1\ inch}{4\ feet} = \frac{x\ inches}{16\ feet}$

The scale here is 1 inch = 4 feet. Then the proportion also compares inches to feet. We know the number of feet we are trying to measure, but we don't know the inches. Therefore, we can write the proportion in this way and use cross-products to solve for the missing information.

Here is another example for the students.

Example: *One inch is equal to five meters. How many inches will show us a distance of 45 meters?*

This is a good example for learning to set up a proportion because the information presented is very straightforward and does not have a ton of detail mixed into it. The first thing students must do is write a ratio to show the scale.

$1\ inch = 5\ meters$

$$\frac{1\ inch}{5\ meters}$$

Next we can fill in the other ratio to show the proportion. We use a variable for the unknown part of the proportion. In this case, we are looking for inches.

$$\frac{1\ inch}{5\ meters} = \frac{x\ inches}{45\ meters}$$

When we cross-multiply and solve, we learn that the map will show 9 inches for 45 meters.

Additional Examples

Here are a few websites with additional resources for teaching scale and proportion.

http://digitallesson.com/humongous-hero-proportion-and-scale-project –This is an excellent project for students on scale and proportion. It was designed for middle school students and is a multi-day project. It is engaging and interesting for the students.

http://www.basic-mathematics.com/scale-drawings.html –This website provides students with a tutorial on using scale and proportions to complete scale drawings.

http://math4children.com/Topics/Percentages/index.html –This website has resources that are printable, games and other lessons on scale and proportion as well as other related topics.

Understanding Percent

Goal

The goals of this lesson can be found in the following objectives.

- Identify equivalent ratios as fractions, decimals and percents.
- Write percents as decimals.
- Write percents as fractions.
- Read and interpret circle graphs of real-world data.

Common Errors

This lesson is going to connect fractions and decimals, both previously learned skills, with the new topic of percents. For this to make sense, it would be best to begin this lesson with a review of fractions and decimals. Be sure to make the connection between the two by using equivalents.

For example: $\frac{25}{100} = .25$

$$\frac{5}{10} = .5$$

You can start with a review when the fraction is written with a base ten denominator. Then move to when the fraction has a different denominator.

For example: $\frac{3}{4} = .75$

Review dividing the numerator by the denominator to find the decimal answer.

$$4\overline{)3.00}^{.75}$$

This work should be a review for the students. If students need more practice, see the section in additional examples.

Next you can introduce percents. When students understand that fractions, decimals and percents are equivalent, meaning that this is a different way of writing the same thing, then you can really build on an understanding of percents.

Additional Examples

Directions: Convert each fraction to a decimal.

1. $\frac{1}{4}$
2. $\frac{2}{5}$
3. $\frac{3}{5}$
4. $\frac{3}{4}$
5. $\frac{2}{4}$
6. $\frac{1}{8}$

Directions: Write each decimal as a percent.

7. .16
8. .33
9. .19
10. .36

Directions: Write each fraction as a percent.

11. $\frac{1}{2}$
12. $\frac{1}{4}$
13. $\frac{3}{5}$
14. $\frac{6}{10}$

Answer Key

1. .25
2. .40
3. .60
4. .75
5. .50
6. .125
7. 16%
8. 33%
9. 19%
10. 36%
11. 50%
12. 25%
13. 60%
14. 60%

Percents, Decimals and Fractions

Goal

The goals of this lesson can be found in the following objectives.

- Write fractions as percents.
- Write decimals as percents.

- Compare and order fractions, decimals and percents.
- Write fractional or decimal real-world data as percents.

Common Errors

When working with converting between fractions, decimals and percents, most students will not have any difficulty. They find the conversions easy and many can do the conversions in their heads. This is excellent.

When the conversions are not equal, students can tend to have more difficulties. Look at this example from the text.

Example: One special fraction to work with is one-third. To convert one-third to a percent is a little tricky because 3 does not divide evenly into 100. Take a look.

$\frac{1}{3} = \frac{}{100}$

When completing this problem, we end up with a repeating decimal or the 3's just continue on and on and on.

.33333333 etc.

To work with this fraction, we call it $33\frac{1}{3}\%$.

This is one of those special conversions that students will need to commit to memory. Remind them that it is possible to have a part of a percent.

A common error can happen when converting a decimal to a percent when the decimal is written in tenths and not hundredths. Take a look at this example.

Example: Write .5 as a percent.

Many students will write 5%. But it isn't five percent because this is tenths not hundredths. This is a common error. You must remind students that they need to move the decimal point two places and not one.

.5 = 50%

This is the correct answer.

The key to comparing fractions, decimals and percents is to be sure that the numbers given are all in the same form. You want all fractions, all decimals or all percents. This will involve converting one or more of the values so that they can be easily compared. Mistakes are made when students don't do this step. Here is a note on this from the text.

To compare fractions, decimals and percents, we have to think of them in the same form. If we are comparing a fraction and a percent, we have to write both of them either as fractions or percents so that we can figure out which is greater.

Additional Examples

Here are a few website with additional resources for teaching fractions, decimals and percents.

http://www.purplemath.com/modules/percents.htm –This website has a tutorial on converting between fractions, decimals and percents.

http://www.math-play.com/Fractions-Decimals-Percents-Jeopardy/fractions-decimals-percents-jeopardy.html –This is a Jeopardy style game on fractions, decimals and percents.

Finding a Percent of a Number

Goal

The goals of this lesson can be found in the following objectives.

- Find a percent of a number.
- Find prices involving discounts.
- Find prices involving sales tax.
- Estimate tips.
- Find amounts involving simple interest.

Common Errors

In this lesson, students will be finding the percent of a number and will then be applying this information to real–life skills involving discounts, sales tax, tips and simple interest. First, we can look at common errors in finding the percent of a number.

Students can forget which operation is used in finding the percent of a number. To do this, we use multiplication. Multiplication is the key operation for finding the percent of a number. We can't multiply a percent–at least we can't when it is in percent form. Therefore, students will need to convert the percent to a decimal so that they can work with the correct form of the value.

Some students will be more comfortable using a proportion. That is fine too. Students can use the fraction form of the percent to find the answer. The key thing to reinforce is that the form of the percent will need to change to a fraction or a decimal so that it can be used to find the missing number.

A common error for working with discounts, sales tax, interest and tips is that sometimes we add the answer to the original value and sometimes we subtract it. Students will need to remember the following rules.

Discounts –a savings, so we subtract

Sales Tax –an additional fee, so we add

Interest –an additional reward so we add

Tips –an additional fee for service, so we add

Once students remember whether or not they add or subtract the additional amount, they will find success.

Additional Examples

The following websites provide students with games, tutorials or worksheets on finding the percent of a number.

http://www.mathplayground.com/balloon_invaders_percent.html –This is a game on mathplayground.com so that students will practice finding the percent of a number.

http://www.mathplayground.com/percent_shopping.html –This is a percent shopping game on mathplayground.com where students practice using percents in a real–world context.

http://www.mathgoodies.com/lessons/ –This website has worksheets on percent applications like discounts, sales tax, tips etc.

Problem-Solving Strategy: Use a Proportion

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Use a Proportion.
- Plan and compare alternative approaches to solving problems.
- Solve real-world problems involving rates, proportions, or percents using selected strategies as part of a plan.

Common Errors

The first way to avoid errors is to be sure that the students understand that proportions can only be used in certain situations. Proportions are equal ratios, so there is comparing involved. We compare quantities, then we can use a proportion. When there are problems that involve comparing, then we can use a proportion.

This is stated clearly in the text, but it should be reviewed with the students. When they have a problem where it seems like a proportion would be the best strategy, then the students will need to figure out what is being compared. They can write this out.

Example: Comparing speed and distance with speed and distance

Comparing height and speed with height and speed.

If these comparisons aren't presented in the problem, then a proportion can't be used to solve the problem. The book "If You Hopped Like a Frog" by David Schwartz is a great book with many problem solving examples for using proportions to solve problems.

Additional Examples

Here are a few websites which provide additional practice for students.

http://www.algebralab.org/Word/Word.aspx?file=Algebra_Proportions.xml –This website begins with a tutorial and then provides students with interactive examples to practice using proportions when problem solving. It would be an excellent resource for an advanced student.

http://www.youtube.com/watch?v=VikfwNpnZ10 –This is a youtube video from yourteacher.com which is a tutorial on how to use proportions when problem solving.

http://illuminations.nctm.org/LessonDetail.aspx?id=L284 –This is an illuminations worksheet/lesson from NCTM which focuses on proportions and problem solving.

2.9 Geometric Figures

This ninth chapter *Geometric Figures* covers basic principles of geometry. Students are introduced to angles, triangles, quadrilaterals, and polygons. Also explored are congruent and similar figures, line symmetry, and strategies for using Venn Diagrams to solve problem. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Introduction to Geometry
- Classifying Angles
- Classifying Triangles
- Classifying Quadrilaterals
- Classifying Polygons
- Congruent and Similar Figures
- Line Symmetry
- Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Introduction to Geometry

Goal

The goals of this lesson can be found in the following objectives.

- Identify points, rays, lines and segments using words and symbols.
- Identify intersecting and parallel lines.
- Identify angles by vertex and ray.
- Draw angles using a protractor.

Common Errors

One of the most common errors that students make when learning geometry is that they don't take the time to learn the definitions and the symbols. As a result, they aren't sure of the meaning of the different geometric terms and without this understanding and knowledge they aren't able to accurately complete the required tasks. It is recommended that students keep a geometry picture dictionary. The only thing in this notebook is the terms that have been covered, with definitions and symbols. This can be used a reference tool or as a study tool for students.

Errors using protractors are common. There are several things to watch for when working with a protractor. Students need to be sure that the bottom line of the angle matches with the bottom line on the protractor. If this is not the case, then the measure of the angle won't be accurate. Secondly, students become confused when there are two numbers on top of each on the protractor. One is the measure for an acute angle and one is the measure for an obtuse angle. This is where you can tell if a student knows the definition of acute or obtuse. By looking at the angle, students can distinguish whether the angle is going to be less than 90 or more than 90. This indicates which measurement is the accurate measurement on the protractor. Errors made here are caused by one of these reasons.

1. Students don't know the difference between an acute or obtuse angle by looking at it –this could be caused by not knowing the definitions.

2. Students haven't lined up the bottom ray with the bottom line on the protractor.

Additional Examples

Here are some websites that provide additional practice for students.

http://classroom.jc-schools.net/basic/mathgeom.html –This website has a ton of practice games on it for students to enjoy working on basic geometry. You can choose which games you want the students to play.

http://www.mathsisfun.com/geometry/protractor-using.html –This is a great animation on how to use a ruler and a protractor.

http://www.kidport.com/grade6/math/measuregeo/measuringangles.htm –This website has an interactive activity on measuring different angles.

http://www.superteacherworksheets.com/angles.html –This is a great website with many worksheets for teachers to use with their students.

Classifying Angles

Goal

The goals of this lesson can be found in the following objectives.

- Classify angles as acute, right, obtuse or straight.
- Classify angle pairs as supplementary or complementary
- Use real-world angle pair diagrams to find unknown angle measures.

Common Errors

The first common error is made when students haven't committed the definitions of the geometry terms to memory. Because of this, they aren't able to correctly identify them. It is recommended that students keep a separate geometry dictionary to help them to learn the different terms.

A common error exists with complementary and supplementary angles when students forget that these are angle pairs. A pair is two angles. To identify supplementary or complementary angles, we are looking for pairs of angles.

To find unknown measures, students will need to use inverse operations. Since angle pairs have an angle sum, students will need to use subtraction to figure out the measure of the missing angle. Reminding students that the inverse of addition is subtraction will help them to find the correct missing angle measure.

Additional Examples

The following websites will provide students with additional practice on complementary and supplementary angles.

http://www.mathsisfun.com/geometry/complementary-angles.html –This website has a tutorial on identifying complementary angles.

http://www.mathsisfun.com/geometry/supplementary-angles.html –This website has a tutorial on supplementary angles.

http://www.math-aids.com/Geometry/Angles/ –This website has a link to many, many worksheets for students to practice complementary and supplementary angles as well as many other math topics.

Classifying Triangles

Goal

The goals of this lesson can be found in the following objectives.

- Classify triangles by angles.
- Classify triangles by sides.
- Draw specified triangles using a ruler and a protractor.
- Find unknown angle measures in given triangles.

Common Errors

A common error is made when students confuse identifying triangles according to angles or identifying angles according to side lengths. You can point out to students that the same words that are used to describe angles: acute, obtuse etc are used to describe the angles of a triangle too. When looking at a triangle, you want the students to be looking at the space created between the line segments and not the lines themselves. This will direct them to looking at angle measures.

The length of the sides is another way to classify triangles. When looking at these different lengths, students will see that there are ways to classify angles based on the side lengths. Knowing these definitions and terms is necessary for student success.

When teaching students to draw triangles using a protractor and a ruler, you can help them by having them turn the paper depending on which angle of the triangle they are drawing. Be sure to review how to use a protractor so that students are reminded. This repetition will help them to draw each triangle accurately.

Additional Examples

The following websites provide students with additional practice on working with triangles.

http://www.mathsisfun.com/geometry/triangles-interactive.html –This website has an interactive game where students can change the triangle by altering the angles measures or side lengths.

http://www.mathwarehouse.com/geometry/triangles/triangle-types.php –This website has a tutorial and interactive questions on identifying triangles.

http://www.helpingwithmath.com/by_subject/geometry/geo_worksheets.htm –This website has a list of worksheets that can be used to help students identify triangles.

Classifying Quadrilaterals

Goal

The goals of this lesson can be found in the following objectives.

- Classify quadrilaterals by angles
- Classify quadrilaterals by sides
- Draw specified quadrilaterals using ruler and protractor
- Find unknown angle measures in given quadrilaterals

Common Errors

This lesson builds on the previous lesson by requiring the students to identify quadrilaterals based on sides and angles. The same classifying tools from the previous lesson on triangles will be used in this lesson on quadrilaterals.

Using a separate notebook for definitions and descriptions of all of the different quadrilaterals will be helpful for students to use a reference and study tool.

Finding missing angles can be tricky for students because they have to start with the sum and work backwards. You may have seen difficulty in this area when the students were working with triangles. Because they start with the sum of 180° and work backwards to find the missing pieces, students can become confused. This may happen again when working with quadrilaterals. You can help students with this process by consistently pointing out that the sum of angle measures is 360°. This is the starting point and then we work backwards to find the measure of a missing angle.

Example: $54+120+16+x=360$

$x=170^\circ$

This is the measure of the missing angle.

We started with the sum of the angle measures and then used mental arithmetic to work backwards and find the measure of the missing angle. Working in this way also requires students to use an inverse of addition, subtraction. Beginning to think in terms of inverse operations will help students as they begin approaching algebra.

Additional Examples

The following websites will provide students with additional practice.

http://www.swarthmore.edu/SocSci/Education/Portfolios/rwillem1/3WeekLessonPlan.html –This is a three week lesson plan for quadrilaterals and polygons. It could be used as a supplemental unit for students when working with geometry.

http://www.woodlands-junior.kent.sch.uk/maths/shape.htm –This is a great website with all kinds of interactive games for students who are learning geometry.

http://www.superteacherworksheets.com/geometry.html –This website has many, many worksheets that will provide students with additional practice.

http://www.studyzone.org/mtestprep/math8/a/missing_angles7l.cfm –This website has an interactive lesson on finding the missing angle of a quadrilateral. Then it provides students with practice problems.

Classifying Polygons

Goal

The goals of this lesson can be found in the following objectives.

- Classify polygons.
- Distinguish between regular and irregular polygons.
- Relate sides of polygons to the sum of the interior angles.
- Relate sides of polygons to the number of diagonals form a vertex.

Common Errors

This lesson is all about defining and identifying different figures. The polygons in this chapter have to be learned. Students should use an additional notebook and practice drawing and identifying each type of polygon by the number of sides that it has. Once students have learned how to identify each type of polygon and committed this information to memory, then you can look at angle measures.

The difference between a regular and an irregular polygon is one that is confusing to students. The key is to point out to the students is that whether a figure is regular or irregular depends on the side lengths.

Another important point is that the name of the polygon does not change. If it is a hexagon, then it is always a hexagon. It is either a regular hexagon or an irregular hexagon, but the fact that it is a six sided figure does not change.

Using the formula will help students to avoid errors when calculating the sum of the interior angles of each different polygon. Here is the formula as it is presented in the text.

$x =$ **number of sides**

$(x-2)180 =$ **sum of the interior angles**

Additional Examples

Directions: Find the sum of the interior angles of each named polygon.

1. Square
2. Triangle
3. Nonagon
4. Decagon
5. Hexagon
6. Heptagon
7. Octagon
8. Quadrilateral

Answer Key

1. 360°
2. 180°
3. 1260°
4. 1440°
5. 720°
6. 900°
7. 1080°
8. 360°

Congruent and Similar Figures

Goal

The goals of this lesson can be found in the following objectives.

- Identify given triangles as similar, congruent or neither.
- Identify corresponding parts of congruent figures.
- Identify corresponding parts of similar figures.
- Find unknown measures of corresponding parts of similar figures.
- Use similar figures to measure indirectly.

Common Errors

The word "corresponding" is not a word that is part of normal everyday speech. As a result, when used in geometry, it is often a word that students have difficulty using and remembering its definition is equally challenging. If you can teach the students that "corresponding" means "matching" that will help them a great deal. Then when they look at two figures, they will be able to distinguish which sides are corresponding or matching and which angles are corresponding or matching.

Students should include the symbols for similar and congruent in their notebooks. The definitions should be included as well. Knowing this information is a way for students to prevent errors.

Before working on section IV, it is recommended that you review proportions. At the end of the last chapter, students used proportions as a strategy for problem solving. This strategy was used when things were being compared. Then the students used cross-products to solve for the missing value. The same procedure is going to be used here with similar figures. The students will be comparing side lengths. The side lengths form a proportion and cross-products are used to solve for missing side lengths. Reviewing this content before applying it to similar figures will help the students to make accurate connections and find correct answers.

Additional Examples

The following websites provide students with additional practice.

www.teachertube.com/viewVideo.php?video_id=122169 –This is a teachertube video on similar triangles, proportions and indirect measurement.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L515 –This website has a project from NCTM on using proportions to solve shadow problems. This would be an excellent supplement to this lesson.

http://www.uen.org/Lessonplan/preview.cgi?LPid=23525 –This website has lesson plans and worksheets on similar triangles and proportions. It has several options for student practice.

Line Symmetry

Goal

The goals of this lesson can be found in the following objectives.

- Identify lines of symmetry in figures and objects
- Draw figures with specified symmetry
- Recognize congruence in mirror images

Common Errors

The big challenge for students when working with line symmetry is in understanding all of the different types of symmetry. Using a notebook to help keep each type of symmetry clear will help students to avoid errors.

Additional Examples

These websites provide students with additional practice.

http://www.greatmathsgames.com/Symmetry/ –This website has fascinating discussions and examples of symmetry. There are several different investigations that students can participate in.

http://www.woodlands-junior.kent.sch.uk/maths/shape.htm –This website has interactive games on symmetry and builds on student understanding of symmetry.

http://www.superteacherworksheets.com/symmetry.html –This website has many worksheets on line symmetry.

Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Look for a Pattern.
- Develop and use the strategy: Use a Venn Diagram.
- Plan and Compare alternative approaches to solving problems.
- Solve real-world problems using selected strategies as a part of a plan.

Common Errors

A common error for students is that they often don't understand the purpose of a Venn Diagram. Looking for and finding a pattern makes a lot more sense to them, but Venn Diagrams can be confusing and students often don't understand their purpose.

A Venn Diagram is used to show common numbers or common values between two different values. In other words, something has to be overlapping in the given values for a Venn Diagram to make sense. Students will need additional practice when understanding Venn Diagrams. You can use the websites in the next section to help students with Venn Diagrams.

Additional Examples

http://www.crickweb.co.uk/ks2numeracy-shape-and-weight.html#triangles –This website has a triangle sorting interactive game that uses a Venn Diagram.

http://www.mathworksheets4kids.com/venn-diagram.html –This website has a vast collection of Venn Diagram worksheets.

http://www.education.com/worksheets/patterns/ –This worksheet has middle school worksheets on patterns.

2.10 Geometry and Measurement

This tenth chapter *Geometry and Measurement* further covers geometric principles through methods for finding the area of parallelograms, triangles, and circles. The student is also introduced to the concepts of solid figures, surface areas, and volumes of prisms and cylinders. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Area of Parallelograms
- Area of Triangles
- Circumference of Circles
- Area of Circles
- Classifying Solid Figures
- Surface Area and Volume of Prisms
- Surface Area and Volume of Cylinders
- Problem-Solving Strategy: Solve a Simpler Problem

Area of Parallelograms

Goal

The goals of this lesson can be found in the following objectives.

- Recognize the area of a parallelogram as the area of a related rectangle.
- Find the area of parallelograms given base and height.
- Find unknown dimensions of parallelograms given area and another dimension.
- Estimate actual areas approximated by parallelograms in scale drawings.
- Solve real world problems involving area of parallelograms.

Common Errors

Most students understand area as the measure of space inside of the perimeter. If formulas are provided, most can use the formula to find the correct measure of the area of the given figure. Where students often make an error is in the labeling of the units. When working with perimeter, students only need to use the unit to label the perimeter. This is because the operation used to find perimeter is addition. When students are finding area, they need to use square units because $units \times units$ is equal to square units. Students often have difficulty remembering this.

There are two places where a mistake can be made. The first is in remembering to label the units, and the second is in remembering to include square units. You will need to review this with the students many times. When reviewing it, you can point out the connection between the operation of multiplication and the square units in the label.

When students are given the area of a figure and one other dimensions, they can sometimes become stuck trying to figure out whether they have been given the length or the width in a rectangle or the base or the height in a parallelogram. This is irrelevant. As long as the have been given the area and one dimension, they can work backwards and find the other dimension.

Take the time to review scale measurement before beginning the section on scale drawings and parallelograms. Many students will have forgotten and taking the time in the beginning will help to avoid reteaching in the middle of the lesson. Clarify any misunderstandings before beginning the lesson.

Additional Examples

These websites can be used by students for additional practice.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=21 –This website is created by NCTM. This link will connect students to an interactive investigation of area with rectangles, parallelograms and triangles.

http://www.mathgoodies.com/lessons/vol1/area_parallelogram.html –This website has a tutorial on finding the area of a parallelogram.

http://www.mathworksheets4kids.com/area.html –This website has a list of worksheet options for exploring area with different figures. It also has a worksheet option where students are given the area and need to work backwards to find the missing measurement of the figure.

Area of Triangles

Goal

The goals of this lesson can be found in the following objectives.

- Recognize the area of a triangle as half the area of a related parallelogram.
- Find the area of triangles given base and height.
- Find unknown dimensions of triangles given area and another dimension.
- Find areas of combined figures involving triangles.

Common Errors

This lesson will build on the information that students have already learned about area. Except this time, the figure will be a triangle and not a rectangle or a parallelogram. Students will need to be reminded that the operation connected with area is multiplication, so the units will be square units in the answer. Building this habit early will help students in the future.

A common error when working with triangles is that students don't realize that multiplying by $\frac{1}{2}$ is the same as dividing by 2. Students who aren't familiar with this will simply skip the $\frac{1}{2}$ part of the problem or will become stopped and won't be able to continue. Review this with them before beginning the independent work.

When working backwards, for example, students have been given the area and one dimension and are in search of the other dimension, be sure to remind the students not to forget the $\frac{1}{2}$ in their problem solving.

The last section of the lesson has combined figures in it. Students will need to use more than one formula to find the area of the figure. You can review that the word "combined" indicates that the two areas will be added together for a final area. There are a few steps to working on these problems. First, find the area of one figure, then find the area of the other, finally, add the two areas together. Remind the students that even though they are adding two areas, that the units are still square units because we are working with area.

Additional Examples

The following websites provide students with additional tutorials and practice.

http://www.mathgoodies.com/lessons/vol1/area_triangle.html –This website has an detailed tutorial on finding the area of a triangle.

http://www.aaamath.com/geo78_x6.htm –This is an interactive lesson on calculating the area of a triangle.

http://www.mathworksheets4kids.com/triangles.html –This website has a list of worksheets that will help students to practice finding the area of a triangle.

Circumference of Circles

Goal

The goals of this lesson can be found in the following objectives.

- Identify ratio of circumference to diameter as pi.
- Find the circumference of circles given diameter or radius.
- Find diameter or radius of circles given circumference.
- Solve real world problems involving circumference of circles.

Common Errors

There is a detailed description in this lesson about pi and about how to understand pi and how 3.14 was first discovered. However, many students will still have difficulty understanding pi and it's necessary relationship to the circle.

http://www.math.com/tables/constants/pi.htm#w –This is a teacher's resource page about pi and about the many different explorations/explanations you can give students about pi. This page is very detailed and is a teacher resource.

http://math.about.com/library/weekly/aa111002a.htm –This is a teacher resource from about.com on investigating pi.

You can help students to understand circumference by talking about it as the "perimeter" of the circle. The formula is an important part of a student's accurate work.

Be sure that students understand the difference between the radius and the diameter.

Additional Examples

These websites provide students with additional practice.

http://www.mathgoodies.com/lessons/vol2/circumference.html –This website has an interactive lesson and practice for students when calculating circumference.

http://www.superteacherworksheets.com/radius-diameter.html –This website has many worksheets on radius, diameter and on calculating circumference of a circle.

Area of Circles

Goal

The goals of this lesson can be found in the following objectives.

- Find areas of circles given radius or diameter.
- Find radius or diameter of circles given area.
- Find areas of combined figures involving parts of circles.
- Display real-world data using circle graphs.

Common Errors

In this lesson students are going to be working with the area of circles. They can begin this lesson by reviewing that when calculating the area, that they are figuring out the measurement in the center of the circle. Students will need to be familiar with the formula for area and be familiar with distinguishing between the formula for area of a circle and the formula for the circumference of a circle.

A common error is to mistake r^2 for radius $\times 2$. This is not the same thing. If you notice that this misunderstanding is present in the class, take the time to review exponents. This review will help to avoid reteaching in the middle of the lesson.

Remind students that once again we are figuring out the area of a figure, so our units are square units. This will be a change from circumference. Like perimeter, circumference is not calculated in square units. Area is always calculated in square units.

When given the area and one other dimension, remind students to use the formula to "work backwards". This working backwards will help the students to remember which operations to perform in what order so that they can find an accurate answer.

Additional Examples

The following websites will provide students with additional practice on the content from the lesson.

http://nces.ed.gov/nceskids/graphing/classic/ –On this website, students can practice designing and creating circle graphs. These graphs can also be printed.

http://www.aaamath.com/exp612x2.htm –This is an interactive lesson to help students practice finding the area of a circle.

http://www.mathworksheets4kids.com/circles.html –This website has many different worksheets to help students practice finding the area of a circle.

Classifying Solid Figures

Goal

The goals of this lesson can be found in the following objectives.

- Classify solid figures as prisms, cylinders, pyramids, cones or spheres.
- Identify faces, edges and vertices of solid figures.
- Select real-world examples of given solid figures.

Common Errors

One of the most common errors is that students don't take the time to learn the vocabulary associated with solid figures. Students need to take the time to learn how to identify each solid figure based on its characteristics. They also need to know how to define a face, edge and vertex. It is recommended that students use an additional notebook to keep track of these terms and that they are required to commit the definitions to memory.

Additional Examples

The following websites will provide students with additional practice in working with solid figures.

http://www.math-drills.com/geometry.shtml –This website has many free worksheets on solid figures.

http://www.gamequarium.org/dir/Gamequarium/Math/Geometry/Solid_Figures/ –This is a game to help students to learn the names of solid figures.

http://www.woodlands-junior.kent.sch.uk/maths/shape.htm –This website has many different interactive games that are connected to this lesson. An excellent resource for students.

http://www.mathsisfun.com/geometry/vertices-faces-edges.html –This website has a tutorial for students on identifying faces, edges and vertices of solid figures.

Surface Area and Volume of Prisms

Goal

The goals of this lesson can be found in the following objectives.

- Identify surface area of prisms as the sum of the areas of faces using nets.
- Find surface area of rectangular and triangular prisms using formulas.
- Identify volumes of prisms as the sum of volumes of layers of unit cubes.
- Find volumes of rectangular and triangular prisms using formulas.

Common Errors

Nets are often confusing for students. You would think it would help the students to take apart a figure and look at a three-dimensional figure in a two–dimensional way, however, many students become confused because they aren't able to visualize the figure as a whole once it has been drawn as a net. To help with this, it is recommended that you begin by building some solid figures, then take them apart so that the students have a structural understanding of the solid figure.

http://www.korthalsaltes.com/ –This website has many free patterns for creating solid figures out of paper.

Once the students have an understanding of the nets and the three–dimensional figures, then you can move on to surface area and volume of prisms. Be sure that you review the characteristics of a prism so that the students don't become confused. Surface area is explained and the outer covering and volume as the space contained inside a solid figure.

A common error for students when figuring out surface area is to forget to calculate all of the surfaces. Because of this, they will leave off one or two of the surfaces so that the total surface area is not accurate.

Students need to understand that surface area is made up of the combined areas of all of the faces of a solid. Many small areas have been added together.

Volume is cubic measurement. Because of this, the measurement of volume is cubic units. You an explain this best by showing students how a prism has height, width and length. This means that you multiply the three units together. That is why $inches^3$ or $units^3$ is the way that volume is measured. This is often forgotten by students.

Additional Examples

These websites provide tutorials and additional practice for students.

http://gwydir.demon.co.uk/jo/solid/info.htm –This website is a resource site for teachers on how to teach solid figures and their nets.

http://www.mathguide.com/lessons/SurfaceArea.html –This website has a lesson on how to teach surface area of common solid figures.

http://www.learner.org/interactives/geometry/area_surface.html –This is an interactive website that has resources so that students can play with surface area and volume.

http://www.mathworksheets4kids.com/surface-area.html –This website has many worksheets on surface area for student practice.

Surface Area and Volume of Cylinders

Goal

The goals of this lesson can be found in the following objectives.

- Identify surface area of cylinders as the sum of the areas of faces using nets.
- Find surface area of cylinders using formulas.
- Identify volume of cylinders as the sum of volumes of layers of unit cubes.
- Find volumes of cylinders using formulas.

Common Errors

The most common error that students make when working with cylinders is that they forget that there are two circular bases that need to be calculated. On the whole, most students will find cylinders much easier to work with as compared to prisms. There are many more sides in a prism and much more chance of an error. A cylinder has the two circular bases and the rectangular "wrapped" side and once these surfaces are calculated and added together, the surface area has been calculated. Remind students again that area is calculated in square units.

Again with volume, students need to think in terms of cubic units. Once students have identified the formula needed, they must complete the calculations and always label the volume of solid in terms of cubic units.

To be successful when working with surface area and volumes, the students must be very familiar with the formulas needed to calculate each measurement. It is recommended that the students have a separate notebook to keep this information so that they can refer back to it when working on solving problems.

Additional Examples

The following websites will provide students with additional practice.

http://www.mathworksheets4kids.com/surface-area.html –This website has many worksheets for figuring out the surface area of a cylinder.

http://www.aaamath.com/exp79x10.htm –This website has an interactive lesson for teaching the surface area of a cylinder.

http://www.learningwave.com/lwonline/geometry_section2/cylinder1.html –This website has a tutorial for teaching about how to find the volume of a cylinder.

http://www.mathwarehouse.com/solid-geometry/cylinder/formula-volume-of-cylinder.php# –This website has a short tutorial and then interactive problems for students to use to practice finding the volume of a cylinder.

Problem-Solving Strategy: Solve a Simpler Problem

Goal

The goals of this lesson can be found in the following objectives.

- Read and understand given problem situations.
- Develop and use the strategy: Solve a Simpler Problem.
- Plan and Compare Alternative Approaches to Solving Problems.
- Solve real-world problems using selected strategies as part of a plan.

Common Errors

It is very important when introducing strategies to be sure that the students understand when they should use which strategy in problem–solving. In this lesson, students are introduced to the strategy: solve a simpler problem. Many students will have a question about this strategy. Does this mean only do part of the problem? How do you simplify a problem? These questions will be common as this strategy is introduced.

When teaching this strategy, be sure that students understand that they will need to solve the whole problem and not just a part of it. Simplifying a problem doesn't mean getting rid of part of the problem. This strategy is about breaking down the problem so that each operation can be solved separately. If a problem has more than one step or more than one operation, then it would make sense for the students to break down the problem, solve each step separately and then find a common answer.

Additional Examples

http://library.thinkquest.org/25459/learning/problem/ –This website is full of problem solving resources for teachers to use with their students.

http://sss.uniondaleschools.org/www/uniondaleschools_sss/site/hosting/School/smithstreetschoolorg/solve_a_simpler_problem.htm –This website has an excellent example for students on how to work with the strategy: solve a simpler problem.

2.11 Integers

In this eleventh chapter, *Integers,* students will learn about positive and negative numbers, adding numbers, sums and differences of integers, multiplication, division, coordinates to name locations, transformations, and all about data displays. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Comparing Integers
- Adding Integers
- Subtracting Integers
- Multiplying Integers
- Dividing Integers
- The Coordinate Plane
- Transformations
- Surveys and Data Displays

Comparing Integers

Goal

The goals of this lesson can be found in the following objectives.

- Write integers representing situations of increase/decrease, profit/loss, above/below, etc.
- Identify opposites of given integers
- Compare and order integers on a number line.
- Compare and order positive and negative fractions and decimals.

Common Errors

One of the most common errors that students make when comparing quantities is that they mix up the symbols for greater than and less than. They think that the greater than symbol means less than and that the less than symbol means greater than. One of the ways that you can help students to keep these two symbols straight is to teach them that the "the alligator" meaning the opening of the symbol always eats the larger number.

Example: 6 >4 the opening eats the larger number. In this case, 6 is greater than 4.

When working with integers, it is essential that the students understand how to use these two symbols appropriately so that they aren't mixed up. Mixing up the symbols will always cause an incorrect answer.

Another common error with integers is to think that -33 is greater than -12. The 33 looks larger, but negative values aren't the same as positive so -33 is less than -12. You can use an example of owing money or of a negative value that is closest to the positive side is always greater than a value that is further away. This will need to consistently reinforced throughout this lesson.

Understanding which values are less than or greater than is also important when ordering integers. Students who

make many mistakes on this section don't have a good grasp of comparing integers. In a situation like this it would make more sense to go back and review what the students have already learned.

Additional Examples

The following websites can provide additional practice for students.

http://www.xpmath.com/forums/arcade.php?do=play&gameid=61 –This website has wonderful games on comparing integers.

http://www.mathgoodies.com/lessons/vol5/compare_order.html –This is an interactive lesson on comparing and ordering integers which uses real–world examples.

Directions: Compare the following integers using <, >or =.

1. -4 _____ -2
2. –12 _____ -66
3. 66 _____ 65
4. -44 _____ -40
5. -98 _____ -95
6. -80 _____ -79
7. 44 _____ 56
8. 16 _____ -16
9. -23 _____ 7
10. -32 _____ -38

Answer Key

1. <
2. >
3. >
4. <
5. <
6. <
7. <
8. >
9. <
10. >

Adding Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find sums of integers on a number line.
- Identify absolute values of integers.
- Add integers with the same sign.
- Add integers with different signs.

Common Errors

A common error connected to absolute value is that students will write an absolute value as negative. They want it to be a value just like a positive or negative number, however, an absolute value is the distance that a value is from

zero. We count this distance in units. Units can't be positive or negative, so an absolute value is never negative, it is always positive.

Example: $|-16| = |16|$

You can write a statement like this on the board. Students who don't understand absolute value will say that this is not a true statement. This is an excellent assessment tool for you to assess student understanding. Now that this incorrect statement has been made, you can go back and reteach absolute value as the number of units that a value is from zero. It doesn't matter if it is positive or negative, it is the number of units that counts.

You can remind students that adding integers with different signs is the same as finding differences. Students who have a difficult time with this can be taught it using gains and losses. You can always use money –students relate to money and understand how to work with money. This is an excellent way to teach adding different signs to students.

Additional Examples

Directions: Compare each pair of integers using <, >or =.

1. $|-8|$ ___ 8
2. -9___ -11
3. $|11|$ ___ 13
4. 16 ___ -16
5. -5 ___ -8
6. $|9|$ ___ $|-9|$
7. -14 ___ -18
8. 0 ___ -5
9. -22 ___ -26
10. -12 ___ -9

Answer Key

1. =
2. >
3. <
4. >
5. >
6. =
7. >
8. >
9. >
10. <

Subtracting Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find differences of integers on a number line.
- Subtract integers with the same sign.
- Subtract integers with different signs.
- Solve real-world problems involving sums and differences of integers.

Common Errors

One way of teaching subtracting integers is to use the inverse of addition. Students who have difficulty with the conceptual process of subtracting positive and negative numbers usually can follow the steps and this helps them make sense of it. When working in this way, two steps are followed.

1. Subtraction is changed to addition
2. The second number is made into it's opposite

Example $-5-6$ becomes $-5+-6$

This is also where a common error takes place. When the second number is a positive number, many times the students will forget to rewrite it as a negative number. Then simply leave it alone. This is not the case with subtracting a negative. Look at this example.

Example $-9--8$ becomes $-9+8$

Students rarely forget to make this change, but when the second number is positive, they often forget to make it it's opposite. Reviewing the steps and requiring students to memorize these steps will help them to remember.

When working with more than one operation in a number sentence, teach the students to work from left to right. This will help to avoid errors in arithmetic.

Additional Examples

These websites provide students with additional practice.

http://www.mathgoodies.com/lessons/vol5/subtraction.html –This website has an excellent lesson on subtracting integers by adding the opposite.

http://www.basic-mathematics.com/subtracting-integers-calculator.html –This website has a game where students can practice subtracting integers.

http://www.math-play.com/integers-game.html –This website has a pirate game on adding and subtracting integers. It is animated and engaging.

Multiplying Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find products of positive integers.
- Find products of positive and negative integers.
- Find products of negative integers.
- Evaluate numerical and algebraic expressions involving integer multiplication.

Common Errors

Multiplying integers relies on a student's ability to remember the rules and apply them. Students make errors when multiplying integers because they haven't committed the rules to memory. Once students have memorized the rules, it is necessary for them to practice applying them.

Students who do not have a good grasp of their multiplication tables will also struggle when it comes to multiplying integers. It is necessary for teachers to review these tables ongoingly. Students will need to apply these previously learned skills when multiplying integers.

Additional Examples

These websites provide students with additional practice and/or tutorials.

http://www.arcademicskillbuilders.com/games/integer-warp/ –This website has a very cute space ship game which requires that students multiply integers.

http://www.math-play.com/multiplying-and-dividing-integers-game.html –This website has a game on multiplying and dividing integers.

http://www.math-drills.com/integers.shtml –This website has many, many worksheets on integers and integer operations.

Dividing Integers

Goal

The goals of this lesson can be found in the following objectives.

- Find quotients of positive integers.
- Find quotients of positive and negative integers.
- Find quotients of negative integers.
- Evaluate numerical and algebraic expressions involving integer division.

Common Errors

Dividing integers relies on a student's ability to remember the rules and apply them. Students make errors when dividing integers because they haven't committed the rules to memory. Once students have memorized the rules, it is necessary for them to practice applying them.

Students who do not have a good grasp of their multiplication tables will also struggle when it comes to dividing integers. It is necessary for teachers to review these tables ongoingly. Because division is the inverse of multiplication, if a student does not know their times tables, then the student will have difficulty when dividing positive and negative numbers.

Additional Examples

The following websites provide students with additional practice.

http://www.math-drills.com/integers.shtml –This website has many worksheet available for students to practice dividing integers.

http://www.aaamath.com/div65_x2.htm –This website has an interactive lesson on dividing negative and positive integers.

http://www.math-play.com/multiplying-and-dividing-integers-game.html –This is a game on multiplying and dividing integers.

http://www.gamequarium.com/integers.html –This website has several fun games for working with integers.

The Coordinate Plane

Goal

The goals of this lesson can be found in the following objectives.

- Graph ordered pairs of integer coordinates as points in all four quadrants.

- Graph geometric figures given coordinates of vertices.
- Locate places on maps using integer coordinates.
- Describe paths between given points as integer translations.

Common Errors

Remind students that when plotting points, that they need to begin counting at the origin (0,0) to avoid making mistakes. When students don't begin at the origin, they are more likely to make a mistake in counting the value of the ordered pair.

A common error when graphing figures on the coordinate plane is for students to graph one point of a figure, then move to the other points without going back to the origin. As a result, the figure won't be accurate. Students need to be reminded to always begin at the origin even if the points will become connected later to form a figure.

Longitude and latitude can be confusing concepts for students. You will find additional resources for learning longitude and latitude in the next section.

Additional Examples

http://geography.about.com/cs/latitudelongitude/a/latlong.htm –This website from about.com has resources for teaching longitude and latitude to students.

http://www.math-play.com/Coordinate%20Plane%20Game/Coordinate%20Plane%20Game.html –This is a game about graphing on the coordinate plane.

http://www.mathplayground.com/locate_aliens.html –This is a fun game about aliens on the coordinate plane.

Transformations

Goal

The goals of this lesson can be found in the following objectives.

- Identify transformations in the coordinate plane as translations (slides), reflections (flips) and rotations (turns).
- Graph paired transformations of figures given coordinates of vertices.
- Describe transformations as $x-$ and $y-$coordinate changes.
- Identify equivalent transformations with different coordinate changes.

Common Errors

A common error is made when students don't understand the different types of transformations. It is recommended that students write down these types of transformations and define them so that they can use their notes for reference.

Common errors happen when graphing points on the coordinate plane because students will forget to always begin at the origin (0, 0). This can be reviewed with the students throughout the lesson to reinforce the way things are done.

When thinking about change as an ordered pair, many students may become confused. They will want to graph this point because that is what they have always done with ordered pairs. You will need to tell them that this is a switch from what they have been doing. Now they are going to use an ordered pair to describe what has happened to the figure that was first graphed. There has been some change from the original figure to the new figure. That change is going to be shown in an ordered pair.

Change of (5, 2)

This means that every vertex of the first figure moved horizontally to the right 5 units and vertically 2 units up.

Change of (-2, 2)

This means that every vertex of the first figure moved horizontally two units to the left and vertically two units up.

Notice that the positive x or y value determines the direction that each vertex moves.

Additional Examples

These websites provide students with additional practice.

http://www.mathsisfun.com/geometry/transformations.html –This website has an excellent tutorial on transformations and on working with transformations.

http://www.misterteacher.com/abc.html –This website has a tutorial on transformations, but also has an area where students can create their own transformations online.

http://www.mathsonline.co.uk/nonmembers/gamesroom/transform/transform.html –This website has a very cute game on transformations.

Surveys and Data Displays

Goal

The goals of this lesson can be found in the following objectives.

- Collect and organize real-world survey data.
- Choose an appropriate data display.
- Distinguish which data displays are more effective for a specific purpose.
- Analyze and interpret statistical survey data.

Common Errors

There aren't really any common errors when working with surveys and data displays but students should complete a survey before creating a data display. See the next section for resources.

Additional Examples

http://www.teach-nology.com/worksheets/math/survey/ –This website has a collection of great little surveys that you can conduct with your class.

http://greatmathsgames.com/data/item/28-graphing-42-questions.html –This website has some fun games on graphing, data displays and surveys.

http://mdk12.org/instruction/curriculum/mathematics/math_guidelines.html –This is a teacher resource page on creating exemplary data displays.

2.12 Equations and Functions; Probability

This final chapter, students will begin learning all about expressions and equations. They will also learn about solving equations, single-variable equations, graphing functions, probability, and how to calculate the probability of independent events. In this flexbook, titled Common Errors, you will be given examples of places where students might make mistakes or have difficulties. This is designed to help you to troubleshoot these areas when teaching the lesson. There have also been some additional examples provided.

Lessons

The following lessons are part of this chapter.

- Writing Expressions and Equations
- Solving Equations Using Addition and Subtraction
- Solving Equations Using Multiplication and Division
- Functions
- Graphing Functions
- Introduction to Probability
- Finding Outcomes
- Probability of Independent Events

Writing Expressions and Equations

Goal

The goals of this lesson can be found in the following objectives.

- Write addition and subtraction phrases as single-variable expressions.
- Write multiplication and division phrases as single-variable expressions.
- Write sentences as single-variable equations.
- Model real-world situations with simple equations.

Common Errors

A common mistake when writing expressions and equations is for students to use the two terms interchangeably. An expression is different from and equation. Students need to know and understand this difference.

An expression is a combination of variables, numbers and operations without an equals sign.

This is presented in the beginning of the lesson in the text. When students understand the difference between an expression and an equation, then they are able to write them with confidence.

This lesson is challenging to students with language based learning disabilities because they are required to use verbal language to express mathematical information. Use a notebook to help students to make lists of key words so that they can identify them when writing expressions and equations.

A common error is usually made when students see the words “less than”. They know that this means subtraction, but often confuse the order of the numbers.

Example: Six less than 12

Most students would write it as: $6 - 12$

But that is not what the sentence says. It says that we have six less than 12. So 12 should come first in the number sentence that we write.

The correct answer is $12-6$. This expression shows that we have six less than 12.

Here are the steps to help students as they work through this lesson. These are the steps that are written in the text.

When deciphering phrases, you will be looking for the same things as the last section.

1. Identify any numbers
2. Identify the operation involved
3. Identify the variable

Additional Examples

These websites will provide students with additional practice.

http://www.mathgoodies.com/lessons/vol7/expressions.html –This website has a tutorial and sample problems on writing expressions.

http://www.math.com/school/subject2/lessons/S2U1L3GL.html –This is an interactive website that has students practice writing expressions from verbal sentences.

http://www.superteacherworksheets.com/algebra-basic.html –This website has many worksheet choices for practicing writing expressions and equations.

Solving Equations Using Addition and Subtraction

Goal

The goals of this lesson can be found in the following objectives.

- Simplify sums and differences of single-variable expressions
- Solve single-variable addition equations
- Solve single-variable subtraction equations
- Model and solve real-world problems using addition or subtraction equations

Common Errors

A common error when combining like terms is that the students forget that the sign that is in front of the term goes with the term.

For example: $-8x-2x=-10x$

The sign in front of the term goes with the term in this example. Let's look at one that is a bit more complicated.

Example: $8x+5y-3x=5x+5y$

In this example, the $-3x$ is combined with the $8x$. The sign in front of the $-3x$ indicates how it can be combined or simplified with the other terms in the sentence.

Be sure that students understand that an inverse operation is a fancy way of saying an opposite operation. Here are the operations and their inverses that students should know.

Addition inverse subtraction

Multiplication inverse division

Knowing these rules will help students to work through solving equations.

Additional Examples

These websites will help to provide students with additional practice.

http://www.crctlessons.com/one-step-equations-with-addition.html –This is a video and tutorial on solving one-step equations with addition and subtraction.

http://hotmath.com/help/gt/genericalg1/section_1_1.html –This is a page of practice problems on one-step equations and also on inverse operations.

Directions: Write each as its inverse.

1. Add ten.
2. Subtract twenty
3. Add five
4. Subtract nineteen
5. Add sixteen

Answer Key

1. Subtract 10
2. Add 20
3. Subtract 5
4. Add 19
5. Subtract 16

Solving Equations Using Multiplication and Division

Goal

The goals of this lesson can be found in the following objectives.

- Simplify products and quotients of single-variable expressions.
- Solve single-variable multiplication equations.
- Solve single-variable division equations.
- Model real-world problems using multiplication or division equations.

Common Errors

When students begin simplifying expressions that have division in them, they can make mistakes by forgetting to simplify the entire expressions.

Example: $\frac{4x}{2x} = 2x$

This answer is incorrect. In this example, the coefficient or number part of each term was simplified, but the variables could have been simplified and weren't. Therefore, this answer is incorrect.

The correct answer is 2, because four divided by two is two and the variable x divided by x is 1, so our answer is 2. Remind the students that any number or variable divided by itself is always one.

Students also forget that the fraction bar means division. Reviewing this at the beginning of the lesson will help to avoid reteaching in the middle of the lesson.

Single variable multiplication and division equations are easily solved for the unknown quantity by using the inverse operation.

Additional Examples

Directions: Simplify each expression.

1. $5(3x)$
2. $6x(2y)$
3. $4y(2z)$
4. $9x(9)$
5. $8a(9b)$
6. $\frac{9xy}{3x}$
7. $\frac{2x}{2xy}$
8. $\frac{10a}{5a}$
9. $\frac{3a}{6}$
10. $\frac{15x}{3y}$

Answer Key

1. $15x$
2. $12xy$
3. $8yz$
4. $81x$
5. $72ab$
6. $3y$
7. $\frac{1}{y}$
8. 2
9. $\frac{a}{2}$
10. $\frac{5x}{y}$

Functions

Goal

The goals of this lesson can be found in the following objectives.

- Write an expression for an input-output table.
- Evaluate a given function rule for an input-output table.
- Write a function rule from an input-output table.
- Model and solve real-world problems involving patterns of change.

Common Errors

Student success in this section depends on a student's ability to decipher rules from patterns. When working with input/output tables, the students will need to write an expression that describes a pattern. Any work that you can do with students on deciphering patterns will help students to be prepared for functions.

http://www.mathsisfun.com/pascals-triangle.html –This website looks at the patterns found in Pascal's triangle. This is an excellent resource for working with patterns.

Remind students that the rule must work for all of the values in the table. This is a place where students might make an error. They begin by checking the rule with the first or second value and then decide that it works. If they don't check the rule with each value in the table, then they can't be certain that the rule is correct. This is a common error.

Review the following hints for writing function rules with the students. This is written clearly in the text.

Hints for Writing Function Rules

1. Decipher the pattern of the function. What happened to the input to get the output?
2. This is the rule for the function.
3. Write the rule as an expression.

Additional Examples

These websites provide tutorials and extra practice for students.

http://www.shodor.org/interactivate/activities/ –This is an interactive website on patterns and functions.

http://www.linkslearning.org/Teachers/1_math/7_math_LINKS/index.html –This website has a lesson plan for working with functions. It is a resource for teachers.

http://www.mathplayground.com/functionmachine.html –This is a function machine game from mathplayground.com.

Graphing Functions

Goal

The goals of this lesson can be found in the following objectives.

- Graph linear functions in the coordinate plane.
- Distinguish between linear and non-linear functions.
- Use function graphs to relate perimeter, area and volume to linear dimensions of objects.
- Model and solve real-world problems involving patterns of change with multiple representations of functions.

Common Errors

A common error that students make when graphing functions is to mistake the input/output values with the x and y values. For the graph to make sense, the input values connect with the x value and the output connect with the y value. Using this information helps students write ordered pairs that they use when graphing.

Students should be sure that the intervals on their graphs are the same. If they have different sized intervals, then a graph that should be linear might not look linear. The graph of a linear function should form a straight line.

Additional Examples

The following websites provide additional practice for students.

http://www.mathwire.com/games/algebragames.html –This website has a game on input/output tables and functions. There are several interactive games on this website.

http://teams.lacoe.edu/documentation/classrooms/amy/algebra/5-6/activities/functionmachine/functionmachine5_6.html –This website has another game on function machines.

Introduction to Probability

Goal

The goals of this lesson can be found in the following objectives.

- Recognize the probability of an event as the ratio of favorable outcomes to possible outcomes.
- Describe probabilities of events as fractions, decimals or percents.
- Find the probabilities of complementary events.

- Predict whether specified events are impossible, unlikely, likely or certain.

Common Errors

A first common error could occur in writing the probability ratio incorrectly. Remind students that it is favorable outcomes to possible outcomes.

A common error could also happen when students go from the fraction form of a probability to the percent or decimal form. Students can make errors in these conversions. Be sure to review converting between fractions, decimals and percents before beginning this lesson.

Additional Examples

The following websites will provide students with additional practice.

http://www.mathgoodies.com/lessons/vol6/intro_probability.html –This website has a tutorial and interactive lesson as an introduction to probability.

http://www.brightstorm.com/math/algebra-2/combinatorics/introduction-to-probability/ –This is a Brightstorm video that provides students with an introduction and overview of probability.

http://www.superteacherworksheets.com/probability.html –There are many probability worksheets on this website.

Finding Outcomes

Goal

The goals of this lesson can be found in the following objectives.

- Use tree diagrams to list all possible outcomes.
- Find all possible combinations.
- Find all possible permutations.
- Describe real-world situations involving combinations or permutations.

Common Errors

Tree diagrams must be organized. A common error is for students to be disorganized in their work. As a result their tree diagrams are a mess and the outcomes are difficult to calculate. For tree diagrams to be a success, the work must be organized with every possible choice accounted for. Some students may need a pre-organized template to work with until they learn to organize their work.

Searching in google can provide you with a template that can be adjusted and used as a worksheet for students who need help organizing their work.

A common error can also occur when students don't understand the difference between a permutation and a combination. It is recommended that students keep a separate notebook of terms and definitions to help them to keep track of vocabulary words. The notation for permutations and combinations can also be kept in this notebook.

Additional Examples

These websites provide students with additional practice.

http://math.youngzones.org/tree.html –This website has a brief diagram and tutorial on tree diagrams and calculating outcomes.

http://regentsprep.org/Regents/math/ALGEBRA/APR4/PracTre.htm –This website has advanced problems on tree diagrams. They would be an excellent challenge for a student who is advanced in his/her skills.

http://www.mathsisfun.com/combinatorics/combinations-permutations.html –This website has a tutorial and interactive lesson on permutations and combinations.

http://www.onlinemathlearning.com/permutations-combinations.html –This website has video, tutorials, and worksheets to help students with probability.

Probability of Independent Events

Goal

The goals of this lesson can be found in the following objectives.

- Find the probability of at least one of two independent events occurring.
- Find the probability of two independent events both occurring.
- Find the probability of three independent events all occurring.
- Use probability to predict the likelihood of real-life events.

Common Errors

A common error can occur if students think that they need to add independent events instead of multiply them. This can be corrected by going through each example in the text. Independent events are multiplied together and the product tells us the likelihood of the even occurring. This can happen with two independent events or with three independent events.

Additional Examples

This website will provide students with additional practice.

http://www.mathgoodies.com/lessons/vol6/independent_events.html –This website explains and provides students with additional practice on calculating independent events.

CHAPTER 3

Grade 6 - Differentiated Instruction

Chapter Outline

3.1 Number Sense and Variable Expressions

This first chapter *Number Sense and Variable Expressions* takes students from whole number operations into the world of beginning Algebra. This chapter covers the concept of whole numbers and defines the basic order of operations. Students are also introduced to variables and expressions; exponents; and estimation. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Operations with Whole Numbers
- Whole Number Estimation
- Powers and Exponents
- Order of Operations
- Variables and Expressions
- A Problem-Solving Plan
- Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Operations with Whole Numbers

Alternative Teaching Strategies

This lesson opens with a story problem. These story problems have connected themes for each chapter. The goal of each story problem is for the students to work with the content of the lesson yet see a connection to real life. Many students become overwhelmed with story problems. Therefore, in this initial chapter, it is recommended that you read the story problem out loud with the students. Then you can help them to break down the information in the problem. Once you read through the problem and the questions at the end, you can help the students to make a list of the information that they know and the information that they need. Helping them to organize information will assist them in becoming excellent problem solvers.

Information We Know

25 female seals

18 male seal

Total: 43 seals

11 pounds of food per day for each seal

25 pound buckets of seafood

Information We Need to Figure Out

How much for 1 week?

Break it down –how much for one day?

Seven days = 1 week, how much for one week?

How many buckets is this?

To differentiate the instruction in the lesson, you can use color coding to help the students to organize their thoughts and their work.

Look at this example from the text:

Millions Hundred-Thousands Ten-Thousands Thousands Hundreds Tens Ones

In the text, this is written in black. To help students to learn, you can take this place value list and change the colors for the students. This can be created on index cards or on large rectangles of poster board. Using colors can help students who learn visually or who have learning challenges such as auditory processing problems. By creating visual cues, you can help these students and others to organize their work. Look at this example.

Millions Hundred – Thousands Ten – Thousands Thousands Hundreds Tens Ones

Now continue to use these colors when organizing numbers too. For example, the digit in the hundreds place would always be in green, the tens in blue, etc. Colors can be changed for convenience, but the important thing is to organize by using color.

Alternative Assessment Options

The usual method of assessment in most classrooms comes in the form of tests and quizzes, however, there are ways to assess student learning on a daily basis. Keep a checklist with you in the front of the room of the key skills involved in each lesson. This will help you to be thinking of these skills when the students answer questions or participate in class discussions. By using observation and the targeted skills from each lesson, you can monitor student learning and understanding.

Classroom Modifications for Diverse Learners

Diverse learners can be challenging to any classroom environment. Students in this category could be students with learning challenges or gifted students as well as students for whom English is a second language (EEL).

For EEL students, the vocabulary will be the most challenging part of the lesson. Sometimes, words in one language mean something entirely different in another language. To work with this challenge, you can ask students to find a word that is close to the English translation of the vocabulary word in question.

For example, the word ***sum*** could have an alternative meaning in another language. Working in this way will help students to assimilate new vocabulary and information together.

Additional Resources

To challenge students or help students with learning challenges, here are a few interactive games or websites for student practice.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=207 –This website by NCTM has a great game where students have to add a sum quicker than their partner.

http://www.shodor.org/interactivate/activities/ArithmeticFour/ –This is a great website where students can play challenging games that use all four operations. The level of difficulty is adjusted as the student plays the game.

Whole Number Estimation

Alternative Teaching Strategies

One of the biggest challenges to teaching estimation is that students don't understand the concept of "reasonable". We say "it must make sense", but in that statement is the assumption that when a student selects an answer that he/she doesn't think that it makes sense. Most of the time, students think that their answer does make sense. Otherwise, they wouldn't have chosen it to begin with. Therefore, our biggest challenge is in helping students to select an answer that makes sense and is reasonable. We want to help students to choose estimates that are accurate.

To accomplish this task, provide students with a problem and with two answers. One of the answers should be much too big and one of the answers should be much too small.

For example: $498 + 120$

Answer a: 500

Answer b: 800

The next step is to ask the students if either one of these answers would be a "reasonable" estimate. Many will say neither. Then you can begin a discussion about why neither answer is an acceptable one. Here are some possible ways to explain it –the first answer is close to the first number in the problem. Therefore, it is way too small because 120 hasn't even been considered. Second, the 800 is too large because we can consider 100 and 500 as close to 600. Students will usually come to this realization on their own. When they do, you can use this as an opportunity to introduce rounding. Many students rounded naturally and so you can bring their attention to it and rounding will become a natural way of estimating.

Alternative Assessment Options

Listening and observing is an excellent way of assessing student understanding. Use class discussions to listen to what students have to say. Make a note of those students who don't respond during the discussions. These are students to check in with on a one–to–one basis to be sure that they understand the material.

Classroom Modifications for Diverse Learners

A modification that can be made is to offer students two choices and then have them circle the one that is the best estimate. This allows them an opportunity to see estimates that are reasonable before having to come up with them on their own. This will work for students with learning challenges or for students who are visual learners. See the next section for examples.

Additional Resources

These examples could be used in place of some of the review problems as the end of the lesson. This would provide an extra level of problems that are less challenging than the ones in the text.

Directions: Circle the answer that is the best estimate.

1. $479 + 102 = 500$ *or* 600
2. $345 + 299 = 650$ *or* 800
3. $201 + 987 = 1200$ *or* 1350
4. $23 + 87 = 100$ *or* 90
5. $165 + 389 = 400$ *or* 600

Here are few other places on the internet where you can find games or resources.

http://www.gamequarium.com/estimation.html –This website has many different games on estimation.

http://jc-schools.net/tutorials/interact-math.htm –This website links up to Mathtv which has many interactive games for middle school students.

Powers and Exponents

Alternative Teaching Strategies

When working with powers and exponents, it is useful for students to make some real –world connections. By connecting the mathematical concept in this way students are able to see value in learning the material. To differentiate the instruction, we want to work in ways that are visual and interactive.

One thing that you can do is to create a matching game with powers and evaluated answers. This will allow students to work in cooperative groups. Working in cooperative groups will help students to peer tutor each other. Students who are more advanced in their understanding are given the opportunity to help others. You can ask students to write their own powers and evaluate them. Ask each student to write 10 matching examples.

Here are a few examples.

1. $4^2 = 16$
2. $5^3 = 125$
3. $2^5 = 32$

After they have done this, then give them index cards to cut in half. On one half they write the power with the exponent and on the other half they write the answer. In groups of three, students combine their cards and then they have a matching game.

Alternative Assessment Options

Observation is a key method of alternative assessment. You can walk around while the students are working in their groups. Some of the students may be struggling. You can offer assistance while listening for student understanding. A student's ability to create successful pairs is another way of assessing student work.

Classroom Modifications for Diverse Learners

A modification for students who are advanced is to assign them an exponent. Sometimes, these students will only choose a power of 2 or 3 and create simple pairs even though they are capable of working at a higher level. You can assign these students a power of 4 or 5, so that they are forced to work at a challenging level. A student in this case could only create pairs where the exponent of choice was 4 or 5.

For example: $3^4 = 81$

You can provide this same modification for students with learning challenges by working with squaring or cubing only. These students would only be using a power of 2 or 3 to create their pairs.

For example: $2^2 = 4$

By working in this way, you are providing each student the opportunity to work at his/her own level. Groups should be mixed levels. That way, the students who are more advanced can assist students who have learning challenges.

Additional Resources

This websites provide additional examples for working with powers and exponents.

http://www.softschools.com/math/games/exponents_practice.jsp –This website has games using greater than and less than with exponents. The website allows students to work at their own level.

Here is a book that can assist students as they work with exponents.

G is for Google: A Math Alphabet Book by David M. Schwartz –This book is a math literature option for teaching exponents to students of all ages.

Order of Operations

Alternative Teaching Strategies

One of the ways to differentiate the learning of the order of operations is to use color coding. By putting each step in a different color, students can learn to organize the processes according to the order that they are to be performed. For example, if Parentheses is always in red, then students will associate that as being the first step to be performed. This helps students who learn in a visual way. It also assists students with language processing challenges. By creating

visual cues, these students, who have a difficult time making sense of verbal directions, are provided a pathway to success. Here is a recommended color code.

Parentheses

Exponents

Multiplication/Division

Addition/Subtraction

Using this type of color coding can help students to not only remember the acronym PEMDAS, but also the color associated with each operation.

Alternative Assessment Options

Creative writing is a way to assess student learning. You can provide the students with an equation and ask them to write a story about it using their creative skills.

For example: $4+1-2$

This is a simple example. Students write a story about four of something that joined one of something and then two of them went away. This can be imaginative and fictional. This is a very simple example, but the assignment could be differentiated based on the type of learners and their levels.

For example, you could ask students to include 2 operations, one set of grouping symbols and one exponent into their equation. Then they write a story. This is a more advanced option.

A simple option would be to provide the students with an equation and then they have to write the story. This would be an example of how you could use this exercise with students who face learning challenges.

It is also possible to include both options in the same class. This is differentiated instruction at its best. Each student participates at his/her own level.

Classroom Modifications for Diverse Learners

See the previous section on how to differentiate this lesson.

Additional Resources

The following are resources for working with the order of operations.

http://www.mathgoodies.com/lessons/vol7/order_operations.html –This website has a tutorial on the order of operations where students can work at their own pace.

http://regentsprep.org/Regents/math/ALGEBRA/AOP2/Torder.htm –This is a challenging grid puzzle. To solve it, students will need to know how to use the order of operations. A great challenge for an advanced student.

Variables and Expressions

Alternative Teaching Strategies

One way to differentiate this lesson is to have the students create a game. They are each given an expressions. You can take the expressions from the review problems at the end of the lesson. The students are placed in groups of three. Each of the three students is given the numbers 1 –9 on index cards. Then students have to evaluate their expression by substituting each value, 1 –9 into the given expression. When this happens, the students have solutions to each and every problem. Each students will have a different expression, therefore they will each have 9 different answers by the time they are finished. Let's look at an example.

Example: Here is the equation given to the student.

$$5x + 3$$

The students have numbers 1 –9 to work with, and here would be correct answers.

8, 13, 18, 23, 28, 33, 38, 43, 48

Then you can ask the students to detect the pattern between each of the values. There is +5 pattern. An advanced student will notice that because the variable was multiplied by 5, that each value in the answer was +5.

Alternative Assessment Options

An alternative assessment option could be to use this type of a format as a quiz. Each student is working on a different expression with the same values substituted into the expression. Then you can check each answer for accuracy.

Classroom Modifications for Diverse Learners

Because each student is working on a different expression, this exercise allows the teacher to differentiate the lesson based on level. Each student can be given an expression that matches his/her ability level. A student who has learning challenges might be given an expression with only one operation in it.

For example: $4x$

While another student might be given an expression with two or more operations in it.

For example: $3x - 7$

This allows each student to work at his/her own level.

Additional Resources

The following are resources for working with variables and expressions.

http://www.studystack.com/flashcard-7788 –This website has flashcards for practicing translating verbal statements into expressions. A nice review from an earlier lesson that will help students remember how to work with variables.

http://www.aaamath.com/g520_ex1.htm –This website has a tutorial about how to evaluate an expression with one variable.

A Problem-Solving Plan

Alternative Teaching Strategies

Encouraging students to learn the four parts of the problem solving plan can be a challenge. In Teaching Tips, it is recommended that you work with the students to create a bulletin board display of the four steps. When this has happened, students are able to work on learning each step so that they can be successful in solving problems.

Another way to do this is to have the students each take a different step of the four-part problem solving plan and work to explain each step to the class. This can be a way that students who learn well through interpersonal ways can excel.

Alternative Assessment Options

One way to assess student learning is to have them write out a description of how they solved the problem. Sometimes when problem solving, a student has an incorrect answer but the way that he/she tried to solve the problem was actually accurate. By only marking an answer incorrect, we, as teachers, will miss out on valuable insights into how our students think. By asking them to write an explanation of "how" they solved the problem, we can see how to help them in the future.

Classroom Modifications for Diverse Learners

A classroom modification for students who face learning challenges would be to allow them to solve a problem using a picture. Sometimes students can draw "how" they solved the problem but aren't able to articulate their thinking in words. While an alternative assessment is to write a description of how a student solved a particular problem, students can also be allowed to draw a diagram to explain their problem solving.

Additional Resources

The following are resources to help students with problem solving.

Math-terpieces: The Art of Problem Solving by Greg Tang –In this book, Greg Tang underscores the importance of four basic rules in problem-solving. Keeping an open mind, looking for unusual number combinations, using multiple skills (like subtracting to add) and looking for patterns, will guarantee any child success in math. In MATH-TERPIECES, Tang continues to challenge kids with his innovative approach to math, and uses art history to expand his vision for creative problem-solving.

One Minute Mysteries: 65 Short Mysteries You Solve With Math by Eric and Natalie Yoder –This book is geared for grades 5 and up and can provide students with many problem solving opportunities.

Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Alternative Teaching Strategies

Using the problem presented in the introduction, students can work on a problem of their own for orangutan adoption. Present the problem and the lesson first, this will allow students the opportunity to see how the problem from the beginning was solved using the strategies. Once the lesson has been taught and the solution to the problem presented, you can move on to the following activity which will differentiate the lesson for the students.

Provide each student with a different amount of money. You can write these amounts on an index cards. Keep in mind the learning levels of the students in the classroom. Here are some possible examples.

$50.00

$250.00

$123.75

Students who face learning challenges should be given a simple denomination. An advanced student will receive a money amount with a fraction of a dollar in it. The challenge of the money amount should match with the ability of the student.

Then students need to use their given amount to calculate their level of giving for the orangutan adoption.

Here are the levels of possible giving:

Bronze $35 - $100

Silver $100 - $500

Gold $500 - $1000

Then students should calculate their level if they were to earn their given amount of money for 8 weeks. This will parallel the work done in the original problem from the introduction.

Alternative Assessment Options

Allow students the opportunity to draw a picture and explain their problem solving. Students could also present their solutions orally to the whole class or to their peers in small groups. Discussion can be a way of assessing student understanding.

Classroom Modifications for Diverse Learners

See the section on alternative teaching strategies.

Additional Resources

Problem solving resources:

http://www.aaamath.com/men.htm –The following has resources on problem solving and on practicing mental math.

http://www.mathplayground.com/count_the_money.html –This is a game on mathplayground.com on how to calculate money using mental math.

3.2 Statistics and Measurement

This second chapter *Statistics and Measurement* students become familiar with measurements. Also covered are finding the perimeter and area of rectangles, frequency, creating and understanding graphs, and an introduction to the mean, median, mode, range, and central tendency. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Measuring Length
- Perimeter and Area
- Scale Drawings and Maps
- Frequency Tables and Line Plots
- Bar Graphs
- Coordinates and Line Graphs
- Circle Graphs and Choosing Displays
- Mean, Median and Mode

Measuring Length

Alternative Teaching Strategies

Many students are able to measure in inches, half inches and quarter inches without much difficulty. They have a naturally ability for discerning these measurements and since many have been measuring things for a long time, they are able to work with these quarter inch measurements quite accurately.

Many students have difficulty when it comes to eighths. They don't make the leap from dividing the inch into fourths to dividing it up into eighths. As a result, the measurements that require eighths are often inaccurate.

To help with this, you can explain to the students that the ruler actually has three different types of measurements on it. It has whole inches which are the numbers, it has quarters –that means that each inch is divided into four parts, and it has eighths which means that each inch is divided into eight sections. You can't measure in quarters and eighths at the same time. When you work with quarters, you measure in quarters. If you need smaller units, then you measure in eighths.

You can have the students create their own rulers to work with these concepts. Creating their own rulers will help you to see which students understand the different measurement units and which students don't understand them.

Alternative Assessment Options

Assign students many different things to measure. There should be a list that is consistent. For example: a notebook, a new pencil, the length of a desk etc. It is important that the list is the same for all students so that you can assess accuracy. After the students have measured each of the assigned items, then you can have them share their measurements. Discussing differences can help students to self-correct and learn from each other.

Classroom Modifications for Diverse Learners

If there are students who have learning challenges, assign students a partner to work with. There is safety in trying something new with a friend. This can also help students who are having difficulty to learn from a peer. Peer

coaching and teaching is one of the best strategies for working with a class of diverse learners.

Additional Resources

The following are children's books which can be used as a resource for measuring length for working with measuring length.

Famous Bridges Of The World: Measuring Length, Weight, And Volume (Powermath) by Yolonda Maxwell –This book branches off into other measurement concepts, but is an interesting book for students to explore.

The Great Pyramid of Giza: Measuring Length, Area, Volume, and Angles (Math for the Real World) by Janey Levy –Some of the things in this book can be too advanced for students, but it is still worth looking at for its real world connections.

Perimeter and Area

Alternative Teaching Strategies

Once you have covered the content in this lesson, the formulas and instructions on how to find the perimeter and area of a square and a rectangle, you can differentiate the actual assignment. Providing the students with a hands–on activity will help them to put the material in the lesson into action. Some students will need to stay with simple rectangles and squares, while others will need to work on irregular figures. The simple rectangles and squares can be given to students who have learning challenges and the irregular figures can be given to the more advanced students.

http://etc.usf.edu/clipart/galleries/math/geometry_general.php –This website has some detailed floor plans which can be given to students who are looking for a challenge. These figures will be challenging to work with and students will need to use problem solving skills to find the area and perimeter of some of the options. Teachers are able to select an image of choice.

There are also places on this site where you can find rectangles and squares with different dimensions. These can be printed out and given to the students for working with area and perimeter.

Alternative Assessment Options

By providing students with different figures to measure, you are providing them with alternative assessments. Assessments can be done by observing students while they are at work. It can also be done through discussion.

Classroom Modifications for Diverse Learners

A modification for students with learning challenges is to provide a key for them to use which contains the necessary formulas. Put the perimeter formulas in one color and the area formulas in another color. You can require students to write them in their notebooks, or put them on the board so that they are in view as the students work on their individual figures.

Additional Resources

The following are math literature resources for area and perimeter.

Perimeter and Area at the Amusement Park (Real World Math: Level 5) - by Dianne Irving

Teotihuacan: Designing an Ancient Mexican City : Calculating Perimeters and Areas of Squares and Rectangles (Powermath) - by Lynn George

Scale Drawings and Maps

Alternative Teaching Strategies

To provide the students with a real –world exercise that will require them to use the skills that they have just learned, select a room in the school that the students will need to measure to create a map. You can make this as simple or as complicated as you wish and this is how you can differentiate the lesson.

If the class has many challenges with mathematics, then you can use a simple method. Give the students a scale to work with. The size of the room and the size of the paper that you want them to work with will determine the scale that they are given. For example, if the room is 12×11, and you want it to fit on paper that is 11×14, then you could use $\frac{1}{2}'' = 1\ foot$. This will give students plenty of room to work on the determined size of paper.

If the class is doing well and seems to have an excellent grasp of the concepts, then you can give them more of a challenge. In this option, don't give them a scale. They will need to determine the scale that they are going to use on their own. You can give them the size paper you want them to use and tell them that their scale must work so that the design fits on the paper. This will be challenging and require some trial and error.

Alternative Assessment Options

You can assess student understanding by listening to their conversations. Group/individual observation is an excellent way to assess how students are working with the concepts. You can walk around and look and listen as they work. Asking questions is also a way to assess student understanding.

In the end, each drawing will provide you with a window into each students' understanding of the concepts.

Classroom Modifications for Diverse Learners

A modification for learners is to write the steps to the assignment on the board.

1. Measure the room
2. Select a scale or use given scale
3. Determine measurements for the drawing
4. Use a ruler and complete the drawing

Determining measurements for the drawing will be challenging to most students. Be sure to provide extra help during this section.

Additional Resources

http://www.basic-mathematics.com/scale-drawings.html –This website has a great tutorial on scale drawings.

http://www.ehow.com/list_6579859_scale-drawing-activities-middle-school.html –This website has a variety of scale drawing activities for middle school students.

Frequency Tables and Line Plots

Alternative Teaching Strategies

You can differentiate this lesson by creating a hands–on lesson out of the lesson in the text. Rather than having the students go through each example in the text, write the data on the board. Then have each student create a frequency table and a line plot to represent the data. In this way, the students have an interactive, hands–on way to learn the information. They create a display showing that they understand how to work with the two concepts in this lesson.

Alternative Assessment Options

The line plots that are created by the students can be used to assess student learning. You could even ask them to create a rough draft of one, check it for accuracy and then have them redesign it with color for a grade. This is a way to work with this lesson.

You could also use this chapter to create a portfolio of data displays. The different displays of data could be organized into one folder and then this portfolio would be graded as a test/quiz grade at the end of the chapter.

Classroom Modifications for Diverse Learners

The biggest confusion for students with learning difficulties is that a frequency table is written vertically and a line plot is created horizontally. To help them with this, be sure to go over these two directions. Pointing it out will help students to become conscious of the directions and will help students with spatial challenges to organize their work.

Additional Resources

http://www.shodor.org/interactivate/activities/PlopIt/ –This website has interactive games for working with line plots and other data displays.

Bar Graphs

Alternative Teaching Strategies

Differentiate this lesson by having students use the data in the lesson to create their own bar graphs. First, go through the information in the lesson. Once you have taught the content to the students, ie. how to create a bar graph, give them the data to work with on their own.

For example:

Here is data that gives us information on the number of visitors that went to the botanical garden in four days.

Day 1 = 310

Day 2 = 600

Day 3 = 550

Day 4 = 425

This is the data given at the end of the section in the problem section. This would be used by all of the students to create a bar graph.

The activity could be repeated when double bar graphs are taught. In this lesson, there are two graphs that can be created and added to a portfolio for evaluation at the end of the chapter.

After students have created a bar graph, then you can have them work in pairs to correct accuracy errors and recreate the graph for their portfolios.

Alternative Assessment Options

An alternative assessment option would be to include the graphs made during this lesson as part of a portfolio on data displays. Students collect their work throughout the chapter and then the portfolio of data displays is collected at the end of the chapter and graded as a test/quiz.

Classroom Modifications for Diverse Learners

To help students with learning difficulties, you can provide them a scale for the graph. Selecting an accurate scale will be the most challenging. By providing the students with a scale, you can help them to create accurate graphs.

Additional Resources

Bar Graphs (Making Graphs) by Bodach and Vijaya K –This is a great book for students on making bar graphs. It could be used as an additional classroom resource.

Great Graphs, Charts & Tables That Build Real-Life Math Skills: High-Interest Reproducible Activities That Give Kids Practice Interpreting and Creating Bar Graphs, Line Graphs, Pie Charts, and More by Denise Kiernan –Another great book on data displays. There are many, many different options for students to work with in this book.

Coordinates and Line Graphs

Alternative Teaching Strategies

First, notice that this lesson requires students to work in one quadrant and not in four quadrants as is taught in a later lesson. Some students will have already been introduced to working in four quadrants, therefore, you will need to reinstruct them to work in one quadrant.

An alternative teaching strategy is to use color coding to help students to keep track of the x and y values in the ordered pairs and on the coordinate plane.

For example: (x, y)

Here the x value is red and the y value is blue. If you use these colors consistently it will help students with learning challenges or who learn in a visual way to organize their work.

Alternative Assessment Options

An alternative assessment option would be to include the graphs made during this lesson as part of a portfolio on data displays. Students collect their work throughout the chapter and then the portfolio of data displays is collected at the end of the chapter and graded as a test/quiz.

Classroom Modifications for Diverse Learners

Color coding is a classroom modification. You can also write the steps to working on the graph on the board. Here are the steps from the lesson.

Now let's make a line graph.

The first thing that we need is two axes, one vertical and one horizontal.

Next, we plot the points on the graph and connect them with a line.

If you have students with severe learning challenges, you could also provide them with a template of two axes already drawn. This way, they would only have to fill in the data on the horizontal axis and then plot the points connected with the line graph.

Additional Resources

http://nces.ed.gov/nceskids/createagraph/ –This website provides students with the tools for making a variety of different graphs.

Great Graphs, Charts & Tables That Build Real-Life Math Skills: High-Interest Reproducible Activities That Give Kids Practice Interpreting and Creating Bar Graphs, Line Graphs, Pie Charts, and More by Denise Kiernan –Another great book on data displays. There are many, many different options for students to work with in this book.

Circle Graphs and Choosing Displays

Alternative Teaching Strategies

When working on this lesson, you can either require the students to convert the values into percentages on their own, or you can give them the percentages. If you have a class of students with learning challenges, you might want to start by giving them the percentages. Then the students take these percentages and create a circle graph of the data. This would be step one.

Step two would require the students to convert each value into a percent and then make a circle graph. Allowing students to work in two stages will help all to complete the assignments with accuracy.

To differentiate this lesson, be sure that the following indicators from the text are written in a visual display for students.

Here are some notes on each of the ways that we have learned about to display data.

1. Frequency Table-shows how often an event occurs.
2. Line plot-shows how often an event occurs-useful when there is a large range of numbers.
3. Bar graphs-useful when comparing one or more pieces of data
4. Line graph-shows how information changes over time
5. Circle graph-a visual way to show percentages of something out of a whole

Alternative Assessment Options

An alternative assessment option would be to include the graphs made during this lesson as part of a portfolio on data displays. Students collect their work throughout the chapter and then the portfolio of data displays is collected at the end of the chapter and graded as a test/quiz.

Classroom Modifications for Diverse Learners

A modification for students with learning challenges would be to assign different displays to different students for a presentation. In the last section of the lesson, students are asked to select an accurate way to represent the data. This can be challenging for some students. You can assign these students a specific type of graph. Ask them to do the following with their assigned graph.

1. Draw an example of the graph –this can be one from the text or one that is created on their own.
2. Write an explanation of when and why you would you use this graph to represent data.
3. Present the graph and information to the class in a short presentation.

This will also allow you an opportunity to assess student learning.

Additional Resources

http://nces.ed.gov/nceskids/createagraph/ –This website provides students with the tools for making a variety of different graphs.

Great Graphs, Charts & Tables That Build Real-Life Math Skills: High-Interest Reproducible Activities That Give Kids Practice Interpreting and Creating Bar Graphs, Line Graphs, Pie Charts, and More by Denise Kiernan –Another great book on data displays. There are many, many different options for students to work with in this book.

Mean, Median and Mode

Alternative Teaching Strategies

One of the biggest challenges of this lesson is helping the students to keep all of the different vocabulary words straight. They will need to remember the meaning of each word as well as the steps for finding each measure of central tendency. You can use the information in the text to create a chart of each term and the steps for finding each measure.

Mean –average –add up the scores and divide by the number of values in the set

Median –the middle score –arrange scores from least to greatest –then find the middle score

Mode –the value that occurs the most

Range –the difference between the smallest and greatest values in the set. The values must be ordered from least to greatest.

Alternative Assessment Options

Use the additional resources to have student work with real world data. Then they can provide an example of each term in their porfolio.

Classroom Modifications for Diverse Learners

See the above section for modifications. Providing students with a list of each term and the directions for finding each value is a classroom modification for this section.

Additional Resources

http://academics.uww.edu/cni/webquest/HallOfFame/Snacks/Index.htm –This is a data project using the measures of central tendency and MM's. You will need computers to complete this project.

http://www.teachervision.fen.com/statistics/lesson-plan/4912.html –This is a detailed data project using temperature.

3.3 Addition and Subtraction of Decimals

This third chapter *Addition and Subtraction of Decimals* introduces students to the Decimal System, covering addition and subtraction, rounding and estimation, and ordering decimals as well as introducing stem and leaf Plots. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Decimal Place Value
- Measuring Metric Length
- Ordering Decimals
- Rounding Decimals
- Decimal Estimation
- Adding and Subtracting Decimals
- Stem-and-Leaf Plots
- Use Estimation

Decimal Place Value

Alternative Teaching Strategies

In the text, there are many examples of how to create decimals using a hundreds grid. This can be made into a hands–on lesson to help students with learning challenges, visual and kinesthetic learners as well as learners with language processing disabilities. Here are a selection of decimals that students can work with.

.34

.67

.98

.01

.15

.60

.54

Write these on the board and then give the students a selection of hundreds grids to work with. Here is a website where you can print hundreds grids.

http://math.about.com/od/countin1/a/100chart.htm

Once students have successfully created these visual representations of the decimals, you can move on to the place value part of the lesson.

You can also expand this into a difficult assignment by providing students with hundreds charts and then asking them what a decimal in the thousandths would look like. This will require the students to build a new square or grid to represent a decimal in the thousandths.

For example: .324

You can ask them to create a visual representation of the decimal. This would be a challenge to the students.

Alternative Assessment Options

Assessment can be completed through observation and in class discussion. You could also collect the visual representations of each decimal. This is a hands–on way of assessing student understanding. You can look to see if the decimal is correctly represented or if there are accuracy errors.

Classroom Modifications for Diverse Learners

A classroom modification is to use color coding for students with learning challenges. Each of the different place value names can be written in a different color. Then each digit of the decimal can be written in a matching color. This will help any child on the Autism spectrum in his/her ability to organize work.

HundredTensOnesTenthsHundredthsThousandthsTen – Thousands

.483

You can adjust the colors as you wish, but by color coding the place value of each digit, students are better able to keep track of their digits and their work.

Additional Resources

Delightful Decimals and Perfect Percents: Games and Activities That Make Math Easy and Fun by Lynette Long –This book has many fun games and activities for students to do when learning to work with decimals. It does expand to percents as well.

http://www.factmonster.com/ipka/A0881929.html –This is a fun website where students can track the number 9 as it changes its value through the places.

Measuring Metric Length

Alternative Teaching Strategies

When teaching this lesson, be sure to have rulers on hand that show both millimeters and centimeters. Some students may make a connection between eighths and millimeters, but try to avoid their doing this. This will only confuse the students. Eighths and millimeters are not connected. While they are both small units, comparing them could encourage students to use the units interchangeably. This will confuse the students in the end.

Many students will not be familiar with a meter stick. Be sure to have one or two in the classroom. It is important that the students can see the meter stick and handle it. Using it to measure is also useful and brings this lesson from the page of the text to a real world experience.

The more you can encourage students to explore mathematical content in real life, the more connections the students will make and the easier it will be for them to find mathematics valuable. The text for this lesson should be a support for exploration in measurement.

Alternative Assessment Options

Observation is the best way to assess this lesson. Once the different units and tools have been introduced. Allow students the opportunity to experience measuring different items in the classroom. Ask them to keep track of what they measure and what the measurements are so that you can take time to discuss it at the end. Working in this way engages many different types of learners.

You can also take the students outside. Taking them out into nature includes children who learn best through the outdoors. There are many different things, both large and small for them to measure outdoors.

Classroom Modifications for Diverse Learners

Students with learning challenges or special needs will need a visual reminder of each of the metric units of measurement. If you choose to write them from the smallest to the largest unit, be consistent. If you write them that way once, then continue to teach them in this way. If you don't do this, then students can become very confused. Here is the list from the text.

Millimeter

Centimeter

Meter

Kilometer

Additional Resources

The Metric System (My Path to Math) by Paul C. Challen –This is a book for students on understanding the metric system.

http://www.mathleague.com/help/metric/metric.htm –This webpage has a straightforward list of metric measurements and conversions. While not interactive, it can provide students with a clear list of conversions and equivalents.

Ordering Decimals

Alternative Teaching Strategies

The most challenging part of this lesson is completing all of the conversions when comparing metric units of measure. Each measurement has to be converted to its equivalent so that we can compare like units. For example, if we are trying to compare centimeters and millimeters, we have to compare the same unit either both centimeters or both millimeters. Therefore, the students will need to work on converting one of the measurements to match the other.

To do this, it is recommended that you assign students a partner or a small group to work in. Then you can give them a list of values from the lesson to compare. You can use the examples in the text or the ones in the review problems. This will give the students practice working together. Discussion is a big part of small group work. Many students don't know how to work in a group and so you will have to teach them. Working in a group isn't just about getting the right answer, but it is about making sure that everyone in the group is valued, contributes and understands what is happening. If you begin talking about this from the very beginning, then students will understand that while getting the answer right is important it isn't the only goal working together.

Alternative Assessment Options

Observe the students working together in a group. Notice who is doing most of the talking and remind the students that everyone in the group needs to participate. You can see a lot about who understands what by watching as they work together in a group.

Classroom Modifications for Diverse Learners

A necessary modification is to give the students the operations involved in making conversions of metric equivalents. Here is a list that you can put on the board for them to copy in their notebooks. This will help students with organizational issues, learning challenges and attention issues to focus on completing the assigned work.

From larger units to smaller ones

Cm to mm divide $\times 10$

M to cm $\times 100$

M to mm $\times 1000$

Km to m $\times 1000$

From smaller units to larger ones

Mm to cm $\div 10$

M to cm $\div 100$

M to mm $\div 1000$

Km to m $\div 1000$

Additional Resources

http://www.aaamath.com/dec52_x2.htm –This website has an interactive lesson on comparing decimals.

http://themathgames.com/our-games/decimal-games/place-value/decimal-place-value-math-game –This is an interactive math game on comparing decimals with a space theme.

Rounding Decimals

Alternative Teaching Strategies

Students have challenges when rounding decimals without a number line because they forget which digit stands for which value. To help with this, keep a place value chart on the board. This way, students can identify the place value of the digit that they are working with and round accurately.

Anytime you can create a visual indicator for students, it will help them to remember the rules and the steps for solving problems. This helps students with learning challenges as well as students who learn in a visual way.

When working with leading digits, it is best to assign students small groups to work in. When students are in a small group they have a chance to discuss their work and use combined thinking to solve problems. This helps students to peer teach, the more advanced students can help struggling students, but also it helps students who learn best interpersonally.

For advanced students, it helps to have some challenge problems for them to work through. These problems are optional, but will provide the added challenge that is needed for some students.

For example: Round to the nearest hundred –thousandth place. .032456

Answer: .03246

You can have students create five problems for another student to solve too. This will provide the students the challenge of creating a problem and then another student solves the problem and it is checked by the originator.

Alternative Assessment Options

Assess student work as they discuss problems and solutions in their small groups. Observation is a key alternative assessment option.

Another assessment option is to collect all of the decimal problems that the students write themselves. Have the author of the problems write his/her name on the paper and have the student who solve them write his/her name too.

Classroom Modifications for Diverse Learners

To help students with learning challenges, it is best to write these rules on the board. Students can copy them in their notebooks, but being able to see them in front of them will help them to remember the steps as they complete the assigned problems.

To do this, we followed a few simple rules.

1. Look at the digit to the right of the place value you are rounding.
2. If the digit to the right is a five or greater, you round up.

3. If the digit to the right is less than 5, you round down.

Additional Resources

http://www.math-play.com/rounding-decimals-game-1/rounding-decimals-game.html –This website has a soccer game for students to practice rounding decimals.

http://www.sheppardsoftware.com/mathgames/decimals/scooterQuestDecRound.htm –This is a scooter game for students so that they can continue to practice rounding decimals.

Decimal Estimation

Alternative Teaching Strategies

You can differentiate this lesson by providing students with two different levels of problems. One level would be the level in the text which is for the average learner. The other level could be an advanced level for students who are looking for a challenge. Look at this website for a set of advanced problems.

http://www.numbernut.com/advanced/estimate_intro.shtml –This website has a tutorial for students and then connects students to a couple of games that become progressively more challenging as you play them. This would be excellent for a student looking for a challenge.

Alternative Assessment Options

Observation is an alternative assessment option. You could also use an interactive game on the computer to assess student learning.

Classroom Modifications for Diverse Learners

Here are a couple of things to make a note of for students who face learning challenges.

1. Under 1000 use rounding
2. Over 1000 use front–end estimation

It will be important to review both rounding and front–end estimation with the students.

Additional Resources

http://www.math6.org/decimals/3.2_quiz.htm –This is an interactive quiz on decimal estimation which uses real data as its content.

http://www.scribd.com/doc/21598754/Quiz-Rounding-Decimals-Estimating-Sums-and-Differences –This is a quiz on estimating decimal sums and differences using rounding.

Adding and Subtracting Decimals

Alternative Teaching Strategies

A fun way of differentiating this lesson is to allow the students to use play money to count different sums. You can write the following operations on index cards.

+ 12.33

+ 16.75

+ 22.20

+ 55.67

+ 18.76

+ 19.01

+ 11.20

- 13.23

- 12.25

- 12.98

- 14.11

- 10.01

- 34.75

Then you give each student an amount of money greater than $34.75. Each student receives a card that is a sum and a difference. They then have to figure out how much money they would have if the new number is added to their amount or if the new number is subtracted from their amount.

Working in this way provides the students with a hands–on, interactive group project that is fun and engaging. Not only will the students be working with sums and differences, but they will be having a great time doing it.

Alternative Assessment Options

As the students work on the interactive lesson, walk around and help when needed. You will be able to observe who is in need of help and who isn't. This is an excellent opportunity to assess student accuracy and understanding.

Classroom Modifications for Diverse Learners

Even though each students is working on his/her own assignment, encourage them to work in pairs. By working in pairs, students will be able to help each other and discuss their answers.

Additional Resources

http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=237

This is a great website. This lesson uses models to show students how to find decimal sums and differences. There is a PDF document for teachers which explains the lesson in detail. This would be an excellent resource for students with learning challenges or who are EEL students because of its organization and visual component.

Stem-and-Leaf Plots

Alternative Teaching Strategies

There are ten stem-and-leaf plots to be created in the *Time to Practice* section of this lesson. To differentiate the instruction, allow students class time to create each of the plots. Then they are to select 3 –5 of them to be graded.

Assign each student a partner. They are to check each other's work for accuracy. Once the stem-and-leaf plots have been checked for accuracy, the students can create a final product. This allows students discussion time and peer review time as well as a visual way to demonstrate what they have learned.

Alternative Assessment Options

An alternative assessment option for this lesson would be a portfolio. Students would be asked to create 3 –5 different stem-and-leaf plots and then put them into a folder. Each student would complete a rough draft and a final draft of their plots. Then these would be collected so that they could be graded as a test/quiz grade.

Classroom Modifications for Diverse Learners

There are no modifications in this lesson.

Additional Resources

http://www.mathgametime.com/videos/stem-leaf-plot-video –This is a video tutorial on how to create a stem-and-leaf plot.

Use Estimation

Alternative Teaching Strategies

This is a problem solving lesson. To begin, it would be best to review the four part problem solving plan. If the students have created a bulletin board or poster on these four parts, then this would be a great time to use the visual tool to review.

See classroom modification for help with this lesson.

Alternative Assessment Options

There aren't any alternative assessment options for this lesson. The assessment is in the students' ability to problem solve and find correct solutions.

Classroom Modifications for Diverse Learners

Reviewing is key for students who face learning challenges. When beginning the lesson, it is important to take the time to review the four part problem solving plan from the last chapter.

Write the key words for estimation on the board. These are the words that students will see when looking at or working on a word problem.

Here are some of the key words that we use when estimating:

- Close to
- Approximate
- Estimate
- An answer that makes sense
- About

Review the two different ways of estimating.

1. Rounding
2. Front–End Estimation

Students can choose either method, but be sure that they make a note of which one they did choose. In this way, you will be able to check their work for accuracy based on the method that they used to complete the estimation.

Additional Resources

http://www.iched.org/cms/scripts/page.php?site_id=iched&item_id=estimation –This is a great article on problem solving and estimating for teachers and parents. It is for research purposes.

3.4 Multiplication and Division of Decimals

This fourth chapter *Multiplication and Division of Decimals* furthers a student's understanding of decimals by introducing methods of multiplication and division. Also covered are the Distributive Property and the use and conversion of metric units. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Multiplying Decimals and Whole Numbers
- The Distributive Property
- Multiplying Decimals
- Dividing by Whole Numbers
- Multiplying and Dividing Decimals by Powers of Ten
- Dividing by Decimals
- Metric Units of Mass and Capacity
- Converting Metric Units

Multiplying Decimals and Whole Numbers

Alternative Teaching Strategies

The first skill to work on in this lesson is multiplying decimals by whole numbers. The second skill is estimating the products of decimals and whole numbers. Let's look at multiplying decimals by whole numbers first.

To differentiate this section, put any number with a decimal in one color and put the whole number in a different color. This will help the students to see where the decimal point is located.

Example: 1.34×6

The next step is to follow the directions as they are presented in the text. The student multiplies the two values as if there wasn't a decimal point. Then the decimal point is inserted in the product.

Answer -8.04

Remind students that we count decimal places from the right to the left. This will seem strange to some students because normally we work from left to right. This is one exception to the usual ways of doing things.

Alternative Assessment Options

As a teacher, you can assess student understanding by observing students as they work. When they have finished working, you can collect homework assignments. Sometimes a student can articulate the correct answer verbally but can't seem to get it down on the paper. When listening to students, listen for inaccuracies and for where these inaccuracies occur.

Here are some questions to think about when a student has an error:

1. Is it an error in multiplication that could have been corrected if the student had known his/her times tables better?

2. Is it an error because the student forgot to place the decimal point in the correct place?

Classroom Modifications for Diverse Learners

For students with special needs, be sure to write the steps on the board.

1. Multiply just like you would if there were whole numbers.
2. Put the decimal point in the product by counting decimal places from right to left.

Additional Resources

http://www.math-play.com/decimal-math-games.html –This website has many games for assessing student understanding. These games are based on decimal operations.

http://www.superteacherworksheets.com/decimals.html –This website has decimal worksheets of varying levels. There is a very challenging worksheet on decimal multiplication that would be excellent for an advanced student. The worksheet is noted as challenging on the page.

The Distributive Property

Alternative Teaching Strategies

To differentiate this lesson, begin by handing something out. You can make it fun, by handing out new erasers and new pencils. After this has been done, you can talk about how you distributed 2 things to the number of students that you have. Each student was given two items. Everyone received the same two items. This is the heart of the distributive property, and it has now been presented in hands–on way with a real world example. This is great for students with learning challenges, kinesthetic learners and visual learners.

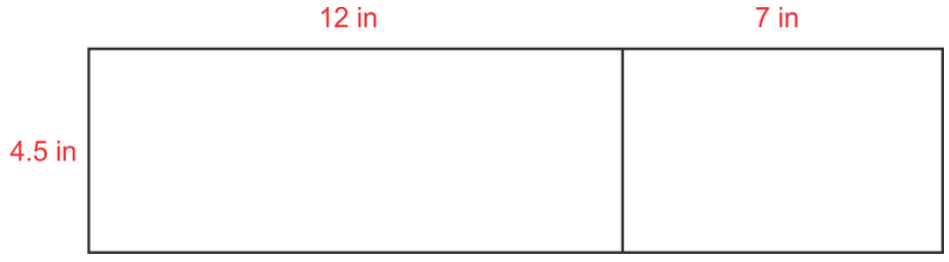
Then move to writing an example on the board.

$2(3+4)$

This means that we are going to distribute the 2 to both values in the parentheses. Just like we distributed the pencil and the eraser to each of the students. This should make sense to the students and help them to be able to make sense of the distributive property.

Some students may ask you if you can just add the two values inside the parentheses first and then multiply. The answer to this question is yes, but be sure that they understand that they are still distributing the value outside of the parentheses. You can go back to the pencil and eraser example to make this clearer.

The last section of this lesson focuses on area models and the distributive property. To work on this one, hand out rulers and ask the students to measure out two rectangles like the ones in the text.



This is the example. The students should use rulers and draw an accurate model of these two rectangles. Then you can use the model to teach them how to distribute the 4.5 inches in a hands–on visual way. This is excellent for all types of learners and makes the curriculum easier to understand.

Alternative Assessment Options

When students are working on the problem with the rectangles, you can walk around and observe them as they work. Here are some questions to be thinking about as you observe.

Did the students accurately recreate the model?

Are the measurements accurate?

Do they understand why 4.5 inches is distributed and not one of the other values?

Did the students use the distributive property to find the area or did they work on each rectangle individually and then add the areas together?

Both ways are accurate, but one uses the distributive property and one does not. If the majority of the students don't use the distributive property, you will need to show they HOW they could have done the problem using the distributive property.

Classroom Modifications for Diverse Learners

Allow students the opportunity to work in pairs and have students work cooperatively. Be sure that students who tend to struggle in mathematics are paired up with students who excel. This allows each to benefit. The student who struggles can be helped by a peer, and the other student can have the chance to teach what he/she knows.

Additional Resources

http://www.onlinemathlearning.com/distributive-property-1.html –This website has videos, worksheets, stories and songs for helping students learn the distributive property. It is geared for grade 6.

Multiplying Decimals

Alternative Teaching Strategies

If you think back to the common errors that students can make when multiplying decimals, these common errors can give us a starting point for differentiating the instruction of students. First, one of the most common errors is for students to forget or inaccurately count the decimal places in the product. To work with this, we can scaffold the number of decimal places and then increase the difficulty and the decimal places as the students become more facile with the work.

For example, starting with one decimal place is a good way to begin.

$$\begin{array}{r} 34.5 \\ \times\ 4 \\ \hline \end{array}$$

This is an example of a problem with one decimal place. Therefore, the students will multiply, as they would with whole numbers, but they only need to remember to count one decimal place in the product.

Then you can move to two decimal places.

$$\begin{array}{r} 45.6 \\ \times\ .3 \\ \hline \end{array}$$

This problem has a decimal point in the first value and a decimal point in the second value. Now the student must not only consider the first decimal point, but also the second. In working recently with middle school students, many remembered the decimal point in the first value, 45.6, but forgot to include the decimal point in the second value, .3.

Working in this way builds upon skills and helps the students to recall what they have done previously. In this way, you can tailor the difficulty of any assignments with the learning levels of the students in your classroom.

Alternative Assessment Options

One alternative assessment option for this lesson is to consider two things when correcting student work.

1. Did the student multiply the values accurately as whole numbers?

If the error is in the multiplication, then the product will never be accurate. By correcting this early, you can help students to strengthen their multiplication skills.

2. Did the student multiply correctly, but incorrectly place a decimal point?

This is another point where you can work with the students directly on remembering to count decimal places in the product.

Classroom Modifications for Diverse Learners

Color coding is a classroom modification for diverse learners. You can color code the first value and the second value to help the students to keep track of values and decimal points.

$$\begin{array}{r} 45.6 \\ \times\ .3 \\ \hline \end{array}$$

While this won't always work on a worksheet, by using this method when explaining or teaching the actual multiplication, students are given the chance to keep the values straight as they work.

Another modification for students with dyslexia or with severe learning challenges, is to use grid paper. Each digit, including the decimal point, is given a box on the grid. Then the students can keep track of the numbers that they are multiplying as well as the decimal point. This is also an excellent strategy for students with organizational issues.

Additional Resources

Here are some additional resources for multiplying decimals.

http://www.mathsisfun.com/multiplying-decimals.html –This website has a great visual tutorial for students on multiplying decimals.

Dividing by Whole Numbers

Alternative Teaching Strategies

Many students struggle with division. For some reason, the idea of splitting up values or items equally is challenging. Through the years, middle school students have shared with me that this is one of their most challenging content areas. To help with this, this chapter scaffolds division of decimals into three sections. The first is dividing a decimal by a whole number, then we move to dividing by decimal powers of ten and finally dividing by decimals. Let's begin with dividing by a whole number.

To help students to make this real, you can begin with a real life example for them to think about.

A group of 8 students went to see a concert. They each spent $30.10 on a ticket. Sarah was in charge, she collected all of the money and paid the person at the counter $300.00. She was given back $59.20. This was the change that needed to be divided evenly between all of the concert attendees. How much change did each student receive?

A problem like this one helps the students to see the value of division. Now that you have the problem, to differentiate it for visual learners, after reading the problem aloud, hand out a copy of it to each of the students.

Then ask them to underline the important values in different colors. Finally ask the students to write a problem that best explains HOW they are going to figure out how much change to give back.

Once students have a problem written ask them to check their thinking with a neighbor. This is how you can help them to help each other to learn.

Next, take suggestions and write a problem on the board.

Example: $59.20 \div 8$

Finally, you can have the students figure out the solution.

Answer: $7.40

Alternative Assessment Options

Assess each step of the problem solving.

1. Did the student underline the correct information in the problem?

Looking at this can help you figure out if a student is able to identify the most valuable information. Many times, students will underline information that is not useful to them.

2. Did the student figure out that this problem required division?

3. Did the student write an equation that could be used to solve the problem?

4. Did the student arrive at the correct answer?

If yes, then the student has each of the steps. If no, then go back and look at where the student had difficulty. Was it in the procedure of dividing or was it in the concept of division? Asking these questions can help you to correct student errors and increase accuracy in learning.

Classroom Modifications for Diverse Learners

A modification for diverse learners is to use grid paper. Another could be to demonstrate a division problem on the board so that students can be reminded of the steps needed in division. This would be in a division box complete with all of the steps.

Example:

$74.70 \div 6 = 12.45$

Show all of the work for the students and go step by step on the board.

Additional Resources

http://guidedmath.wordpress.com/2010/08/26/decimal-squares-in-guided-math-groups/ –This website has some interesting ideas on how to work with the conceptual and procedural skills needed in working with decimals. This is an excellent resource for teachers.

Multiplying and Dividing Decimals by Powers of Ten

Alternative Teaching Strategies

Multiplying and dividing by decimal powers of 10 depends on number sense and a student's ability to see patterns. As a result, this is a valuable lesson for students and for a teacher it provides an opportunity to observe how a student is thinking and whether or not he/she is understanding place value.

When multiplying and dividing by decimal powers of 10, you can create a kinesthetic activity for the students to practice. First, teach the lesson as it is presented in the text, then you can work on this activity called "Movable Decimals."

To prepare, write the following digits on index cards. The values should be large and in marker so that students are able to see them no matter where he/she is sitting.

45.678

Each digit is written an index card and the decimal gets its own card. Include a zero, just in case a place holder is needed. This is a kinesthetic, visual, fun way to teach this concept.

Choose 7 students to come to the front of the room. Give each a card and have them stand in order so that the decimal written above is represented. One student holds the zero card. Then ask one students to come to the front of the room and write each number on the board as it is created.

Finally, you ask the students to call out the following commands-you can let them choose. Write these choices on the board. The decimal point with be moving based on the command.

$\times 10$

$\times 100$

$\times 1000$

$\div 10$

$\div 100$

$\div 1000$

Then each student still sitting has a chance to command the decimal point. For example, say the first students says $\times 10$. Then the decimal point moves and creates a new number.

$45.678 \times 10 = 456.78$

This would be the new value represented. As each new number is created, the student writing, keeps track of them on the board. In this way, the students are working with multiplying and dividing decimals.

Alternative Assessment Options

When finished, you can assess student understanding by asking the students to explain the patterns that they see on the board. There should be a good long list on the board. Students now have to articulate the different patterns that they see. In this way, see if the students can decipher any rules to multiplying and dividing by decimal powers of 10.

Classroom Modifications for Diverse Learners

A modification for special needs students is to write the following rules on the board:

$\times$ = move the decimal to the right

$\div$ = move the decimal to the left

Number of places = number of zeros

Example: $\times 100$ = move decimal to the right two places

This can help special needs students to remember the rules. Ask all students to write this information down in their notebooks.

Additional Resources

http://www.coolmath.com/prealgebra/02-decimals/index.html –This website has wonderful instructional tools for teaching decimals. It also has many puzzles and games that require working with decimals.

Dividing by Decimals

Alternative Teaching Strategies

Begin this lesson by reviewing dividing by whole numbers and by reviewing multiplying and dividing by decimal powers of ten. This is important background information for students to be successful in this lesson. You can point out to students that it was definitely easier to divide by whole numbers than it is to divide by decimals. Then show the students this example.

Example: $.5678 \div .2$

Now you can point out that we are dividing by a decimal. In order to make our work easier, we want to divide by whole numbers always.

Question: How can we do this if we are dividing by .2?

This is a great challenge question for advanced students. They will have to think about all that they have learned. You want the students to conclude that if they multiply $.2 \times 10$ that it will become a whole number. Allow students time to grapple with this question. It provides a needed challenge for advanced students. Don't worry if the students with learning challenges aren't grasping the question. This is for the advanced students to think about.

Once the students have arrived at this conclusion, you can move on to more challenging problems.

$.4272 \div .12$

Here is another problem where students will need to multiply by 100 to move the decimal point so that .12 becomes a whole number. Then you can move on to working through the division.

The answer is 3.56.

Alternative Assessment Options

Here are some points to look for or questions to ask yourself about each student as you assess his/her understanding.

1. Does the student understand how to multiply by a decimal power of 10 to change the decimal to a whole number?
2. Is the student able to complete this procedural step successfully?
3. Is the student able to successfully complete the division?
4. Is the student's answer accurate?

Based on your answers, you can determine whether or not the student is challenged with a concept or a procedure. Procedures require review of steps while concepts require deeper one on one conversation and instruction.

Classroom Modifications for Diverse Learners

A modification for special needs students is to write the following rules on the board:

$\times$ = move the decimal to the right

$\div$ = move the decimal to the left

Number of places = number of zeros

Example: $\times 100$ = move decimal to the right two places

This can help special needs students to remember the rules. Ask all students to write this information down in their notebooks.

Review these rules before beginning student work on dividing decimals.

Additional Resources

http://alex.state.al.us/lesson_view.php?id=26251 –This is a hands–on project from Alabama that requires students

to use all operations with fractions and decimals. This is an aquarium project where the class is given money to purchase fish for a class aquarium. Students must use the size and lifestyle requirements for the fish to decide which type to buy. Students must present their proposal to the principal in a letter.

Metric Units of Mass and Capacity

Alternative Teaching Strategies

Differentiating this lesson would involve students taking on a project that would involve Metric units of mass and capacity. The lesson is very straightforward in its approach with equivalents. Teach the procedural work in the lesson and then use the additional resources listed here to help students with conceptual understanding.

Alternative Assessment Options

See additional resources for projects with assessment components.

Classroom Modifications for Diverse Learners

Write the units for mass and capacity on the board from the smallest unit to the greatest unit. Be sure that this is visual for the students to see. Then write the following rules on the board.

To convert from a large unit to a small unit, we multiply.

To convert from a small unit to a large unit, we divide.

Having these items in view will help the students to remember them as they work through the problems in this lesson.

Additional Resources

http://lamar.colostate.edu/~hillger/ideas.html –This website has tons of ideas for educators on how to help students to use the metric system. There are project ideas here as well as teaching tips and instructional assistance.

Converting Metric Units

Alternative Teaching Strategies

Begin this lesson by reviewing multiplying and dividing by decimal powers of 10. The students will need these procedural skills to be effective in the lesson.

A fun game that you can create with the students is a concentration game using equivalent units of measurement. The best way to do this is to have the students complete the review problems at the end of the chapter. Then they take each measure and its equivalent and write them on index cards. You can even cut the index cards in half to conserve resources. Then the measure is written on one index card and the equivalent measure is written on the other index card. Finally, students pair up and play the game.

Alternative Assessment Options

Assessment can be completed through observation. When students are playing the game, encourage them to work together to help each other. Some students will be able to figure out if one value is an equivalent to the other in their heads. Other students will need time to work through this.

Classroom Modifications for Diverse Learners

For special needs students, it is recommended that the equivalent units for length, mass and capacity be written on the board. Many students will have them written in their notebooks, but students with organizational challenges may have difficulty flipping through pages to find the information that they need. In this way, the students will need to

have this information in front of them so that they can refer to it as they work.

http://www.metric-conversion.net/unit-dictionary.htm –This website is an excellent resource for students to look up and research all kinds of metric conversion units.

Additional Resources

http://lamar.colostate.edu/~hillger/ideas.html –This is a website from the U.S. Metric Association with many ideas, guidelines and teaching tips for students to help with the learning of the metric system.

3.5 Number Patterns and Fractions

This fifth chapter *Number Patterns and Fractions* covers prime and composite numbers and factorization. Students will also explore equivalent fractions, how to order fractions, and the relationship between fractions and decimals through methods of conversion. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Prime Factorization
- Greatest Common Factors
- Equivalent Fractions
- Least Common Multiple
- Ordering Fractions
- Mixed Numbers and Improper Fractions
- Changing Decimals to Fractions
- Changing Fractions to Decimals

Prime Factorization

Alternative Teaching Strategies

As students begin working with factors and multiples they often become confused about these and begin to use them interchangeably. To help with this, the more that you can do to show how a number breaks down into its factors, the greater the chance that the students will remember what they have been taught and remember factors.

Factoring is the process of breaking down a number into its smaller parts. In fact, we want to break down a number into the smallest parts that we can. This can be difficult for students to remember. Often, a student will break down a number once and think that they are finished. Prime factorization is about breaking a number down into its smallest, prime parts.

Here is an activity that you can do to help the students. First, draw a chart of a factor tree on the board. You can use the one in the text so that the students can see one already written down. This is also a modification for special needs students.

Next give each student the numbers 1 –10 on index cards. They are going to build factor trees on their desks. Then give them a number to work with such as 48. Next, the students must use the cards that they have been given to build a factor tree.

Here is one option.

$$48$$
$$6 \times 8$$
$$2 \times 3 \times 2 \times 4$$
$$2 \times 2$$
$$2 \times 2 \times 2 \times 2 \times 3$$
$$Or\ 2^4 \times 3$$

Many students will notice that they need more than one 2 or more than one 3. Have extra copies of these numbers on your desk and you can hand them out to the students as they are needed. You will need a lot of 2's and 3's –you might want to make at least 15 extras of each.

Alternative Assessment Options

You can assess student learning through their hands–on work. Here are some questions to think about as you observe.

1. Do the students understand how to break down a number?
2. Can they use the term "factor" appropriately? For example, does the students say "two factors for 48 are 6 and 8?" This would demonstrate an understanding of the term. If not, then you will probably need to review the vocabulary with the student.
3. Does the students break the number down to its primes?
4. Can the students represent the repeated multiplication with exponents?

Classroom Modifications for Diverse Learners

Special needs students will need a list of the smallest prime numbers. This list can serve as a reference for them. In this way, when they think that they have factored a number completely, they can check their answer with this list of smallest primes and see if their work is accurate.

Small Primes

2

3

5

7

Often if a student has not factored all the way to these primes or to some combination of these primes then their work is incomplete.

If you have students with special needs, another modification is to allow them to write out the primes in expanded form and not include exponents. Other students should be writing their primes with exponents to show the repeated multiplication. With special needs students you can begin without this step and progress to it.

For advanced students, include values like 144 or 256 to challenge them into thinking of larger factors. Some students will need greater challenge and in this way you can differentiate the class by providing numbers that are more challenging.

Additional Resources

http://www.mathsisfun.com/prime-factorization-tool.php –This website has a tool for helping students to understand prime factorization. It uses a student's birthday in the puzzle.

http://amby.com/educate/math/2-1_fact.html –This website has a great tutorial for helping students to use division to find prime factors. It walks the student through the prime factorization by going step-by-step.

Greatest Common Factors

Alternative Teaching Strategies

If students have done well with prime factorization and have a good grasp of the concept of factors, then moving on to greatest common factors becomes one of the simpler transitions for them. The problems in the book with greatest common factors on a level that is appropriate for most students.

To differentiate this lesson, provide students who are advanced or who are needing a challenge with numbers that are larger and more difficult to work with or with more than two numbers. Here are some possible choices.

1. 54, 63 and 180

Answer: 9

2. 84 and 96

Answer: 6

You can add to this list and even ask students to challenge each other.

Alternative Assessment Options

Accuracy is the best way to assess student understanding. If students are not working accurately, then be sure to review each step involved in factoring.

Classroom Modifications for Diverse Learners

See teaching notes above.

Additional Resources

http://www.basic-mathematics.com/greatest-common-factor-game.html –This website has a wonderful game for helping students to identify greatest common factors.

http://www.funtrivia.com/playquiz/quiz2715661f17598.html –This website has challenging word problems for students to solve. It would be an excellent site for a student who is advanced or looking for an additional challenge.

Equivalent Fractions

Alternative Teaching Strategies

To work with equivalent fractions, you can given students a strip of paper a length of your choosing and tell them that this piece of paper represents one whole. Then you can ask them to create the following fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

Next, taking each fraction at a time, you ask them to create three equivalent fraction models for $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$.

Students will need paper of different colors and scissors to complete this task. In addition, they can work on measuring their strips to be sure that they are accurate. This hands–on task brings the idea of equivalents into a real context for the students.

Example:

The student has a strip of paper that represents $\frac{1}{2}$. $\frac{3}{6}$ is equivalent to $\frac{1}{2}$. Therefore, the students cuts another strip of paper the same length as the $\frac{1}{2}$ and divides it into three equal sections. Each of these is $\frac{1}{6}$. There are $\frac{6}{6}$ in one whole, so to equal $\frac{1}{2}$, the student has created $\frac{3}{6}$. This is an example of one part of the activity.

Alternative Assessment Options

Here are some questions to ask yourself as you assess student learning.

1. Does the student understand how to create a strip that is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ of the whole?
2. Is the student's work accurate and measured?
3. Is the students able to identify an equivalent fraction for each of these?
4. Can the student accurately create these equivalent fractions?
5. Can the student explain how and why he or she has created the equivalents that he/she selected?

Classroom Modifications for Diverse Learners

Success in this lesson relies on a student's understanding of the vocabulary. Be sure to review the term "equivalent" with the students before beginning the lesson. You might even write the word on the board with an equals sign = to help students to remember the definition of the world.

Notes: To create an equal fraction, we multiply the numerator and the denominator by the same number.

Students should write this on the board and memorize it as a rule.

The activity is kinesthetic, visual, auditory and requires students to use interpersonal and intrapersonal skills.

Additional Resources

http://www.aaamath.com/fra42ax2.htm –This website has an interactive game about working with equivalent fractions.

http://www.shodor.org/interactivate/activities/EquivFractionFinder/ –This is a visual game where students have to match equivalent fractions.

Least Common Multiple

Alternative Teaching Strategies

Multiples in general are challenging for students because they mix them up with factors. You can make this interactive and hands–on by having students work in small groups. Give them two or three numbers to work with. Then ask the students to list out all of the multiples for each of the numbers they were given. They can list these out on paper or on small chalkboards. Once this has been done, ask the students to look at their lists and circle all of the common multiples. Finally, ask the students to find the least common multiple of all of their numbers.

You can differentiate this further by giving advanced students four numbers to work with in their groups. Some students will be better served by working with two numbers at a time. This is a way that you can differentiate the difficulty of the activity.

Alternative Assessment Options

Here are some questions to think about as you observe students at work.

1. Are the students listing multiples or are they looking for factors? If this is the case, then the students are confused and will need your help in correcting their errors.
2. Are the students able to find the common multiples?
3. Are the students able to find the least common multiple?
4. Are all students actively participating in their groups? Do some students need more time or a simpler set of numbers to work with?

You can use the answers to these questions to alter the lesson as needed.

Classroom Modifications for Diverse Learners

Build this lesson with a progression. You can begin by working with multiples alone. Ask the students to list out multiples, even write them on the board. Then move on to common multiples of two different numbers. Practice

finding all of the common multiples. Finally, move on to finding the least common multiple. This is a perfect way for students to progress through this assignment. It scaffolds the content for students with special needs and will help to avoid reteaching throughout the lesson.

Additional Resources

http://www.aaamath.com/fra66ix2.htm –This website has a tutorial and interactive lesson on least common multiples for students to explore.

http://www.coolmath.com/prealgebra/01-gcfs-lcms/04-least-common-multiples-01.htm –This website has a fun interactive game for students on least common multiples.

Ordering Fractions

Alternative Teaching Strategies

In this lesson, students will learn to first compare fractions and then order them from least to greatest or from greatest to least. To help those students who are visual learners, you can begin by reviewing how to find a common denominator. Comparing fractions becomes easy when the fractions being compared have common denominators. Most students will easily make this connection and pay attention to the numerators before choosing a symbol to compare the given fractions.

You can differentiate by having students work in small groups. This helps students who learn in auditory or interpersonal way to be working at their best. The students can work through the examples in the text in this way.

Students who are looking for a challenge might be given fractions that have different denominators. These denominators will require more work to find a common denominator and so the students are given an additional challenge to work through. Here are some possible examples.

1. $\frac{4}{78}$ *and* $\frac{2}{96}$
2. $\frac{6}{34}$ *and* $\frac{8}{40}$
3. $\frac{9}{80}$ *and* $\frac{6}{240}$
4. $\frac{4}{124}$ *and* $\frac{6}{186}$
5. $\frac{5}{335}$ *and* $\frac{15}{315}$

Students may figure out that they can simplify these fractions to make them easier to work with, and this is an added step. If they figure this out on their own, then that is excellent. This also allows students the opportunity to work with the divisibility rules that were presented in the last lesson.

Alternative Assessment Options

Assessment can be done through observation, or through the accuracy of student's answers. If students write down an incorrect answer, then it might make the best sense to try to figure out if the students made the error because they don't understand the size of fractions or if they forgot how to find a common denominator. If the students made an error because they don't understand the size of fractions, then use visual cues to help them –see the resource section.

Classroom Modifications for Diverse Learners

For special needs students, you can write the following hint on the board.

When comparing fractions, first see if they have the same denominator.

If so, then compare the numerators.

If not, then find a common denominator by using equal fractions.

Additional Resources

http://www.visualfractions.com/ –This website has many examples of visual fractions for students. This would be an excellent resource for students who need extra practice or who are still struggling with fraction concepts.

http://www.mathplayground.com/fractions_compare.html –This is a fun game on comparing fractions where students need to choose the correct symbol to make a true statement.

http://abcteach.com/directory/basics/math/fractions/ –This website has many fractions for students to cut out and use in hands–on visual way. Excellent for different learning styles.

Mixed Numbers and Improper Fractions

Alternative Teaching Strategies

To work with the first part of the lesson, make it as hands–on as possible by including rulers. These rulers can be wooden or plastic as long as they have all of the lines for the different measurements on them. You can also ask the students to make their own rulers. They can do this by using a wooden or plastic ruler as a model.

For students who need a challenge, you can assign or offer them the chance to make an accurate ruler without a model. This means that they will have to remember each of the fractions of an inch and build a cardboard ruler without looking at a wooden or plastic ruler. This can be an excellent challenge for some students.

Alternative Assessment Options

To assess student learning, begin by looking at the rulers that the students have made. Here are some questions to think about.

1. Was the student able to use a model?
2. Is student work neat and legible?
3. Are the inches measured accurately?
4. Did the student use a model or draw it on his/her own?
5. Can the student measure accurately to the nearest fraction of an inch?

Classroom Modifications for Diverse Learners

When working with rulers, there are several websites that offer models for students to work with. Some students may need a model of a paper ruler to work with. This is a modification that can work for many students.

http://www.vendian.org/mncharity/dir3/paper_rulers/

Additional Resources

http://www.amazon.com/Measurement-Mania-Games-Activities-that/dp/0471369802/ref=sr_1_5?s=books&ie=UTF8&qid=1322250174&sr=1-5 –The following link will connect you to amazon.com where there are many excellent books on measurement. Some of the books will be in story form and others will be actual math activity books.

Changing Decimals to Fractions

Alternative Teaching Strategies

This lesson is about changing decimals to fractions. If the students have a good grasp of place value, then this lesson is often a smooth transition from decimals to fractions. Students can sense the place value of the decimals and this easily translates to the denominator of the fraction.

A fun game can be to take decimals and their equivalent fractions or mixed numbers and make a matching game. You can write one decimal on an index card and the equivalent on another index card. Continue in this way until you have a bunch of matches. Each student in the class should get a card. Mix them up and then hand them out. When you say "go", students must hunt around the room for their match. This is a kinesthetic matching game. It is excellent for students who learn through movement as well as students who learn through auditory and interpersonal ways.

You can also vary the difficulty of the matches. For example, a student with learning challenges might receive a simple fraction or decimal. While a student who is more advanced would receive a mixed number or a fraction that needed to be reduced to find its equivalent. See the resource section for examples.

Alternative Assessment Options

As the students play the game, you can use the following questions.

1. Are the students able to find their matches?
2. Are students simplifying if necessary?
3. Are the students able to identify the place value of decimal digits?
4. Is there anyone who needed more/less of a challenge?

Classroom Modifications for Diverse Learners

To help students with learning challenges, begin by reviewing place value with the students. Once the students understand how to identify the places of tenths, hundredths, thousandths, etc, it will be much simpler for them to write the decimal as a fraction. By reviewing place value, you can help them to see that the place of the last digit is also the denominator of the fraction. Here are some notes that you can give students to help them with this lesson.

Place value = denominator of the fraction

.3 = tenths so the denominator is $10 = \frac{3}{10}$

.15 = hundredths so the denominator is $100 = \frac{15}{100}$

Additional Resources

Here are possible matches for the game. There are fractions and decimals of varying degrees of difficulty. Some have two possible equivalents based on the degree of difficulty desired for the student.

$.34 = \frac{34}{100}$

$.7 = \frac{7}{10}$

$.25 = \frac{25}{100}$ *or* $\frac{1}{4}$

$.16 = \frac{16}{100}$ *or* $\frac{4}{25}$

$.2 = \frac{2}{10}$ *or* $\frac{1}{5}$

$3.25 = 3\frac{25}{100}$ *or* $3\frac{1}{4}$

$7.20 = 7\frac{20}{100}$ *or* $7\frac{1}{5}$

$3.33 = 3\frac{33}{100}$

$16.22 = 16\frac{22}{100}$ *or* $16\frac{11}{50}$

$.325 = \frac{325}{1000}$ *or* $\frac{65}{200}$ *or* $\frac{13}{40}$

Changing Fractions to Decimals

Alternative Teaching Strategies

Now we are going to be working the other way around. If students were successful with the activity in the last lesson, then this lesson should also be a smooth transition. If students had difficulty with the last lesson, then you can take some time to differentiate the lesson so that students who are able to work at a challenging level are and other students are working on simpler problems.

For example, students with learning challenges would be working with fractions and decimals in base ten.

$$\frac{45}{100} = .45$$
$$\frac{3}{10} = .3$$

This is the beginning step. Some students may remain at this step for the entire lesson and only move on to more complicated problems in another class period.

Other students will be able to move on right away. These students will either by dividing numerators by denominators or will be working with equal fractions as they are instructed to do so in the text. These students will progress from one section to the other based on ability and level of understanding.

Alternative Assessment Options

Alternative assessment is done through the examples in the text. You can use these examples to listen to each student's response. You can also ask the students if they understand or use the question, "If you are ready to move on please raise your hand." Use this information to notice who will need more help and who is ready to move on.

Classroom Modifications for Diverse Learners

See alternative teaching strategies for suggestions.

Additional Resources

http://www.mathplayground.com/fractions_convert.html –This is a great interactive game for students to help them to connect fractions to decimals and decimals to fractions.

http://math2.org/math/general/arithmetic/fradec.htm –This is a conversion table for students to complete. It is an advanced assignment and should be given to students who are working on an advanced level or who are in need of an additional challenge.

3.6 Addition and Subtraction of Fractions

This sixth chapter *Addition and Subtraction of Fractions* further develops a student's capability with fractions by introducing methods of addition and subtraction. Also covered are box-and-whisker plots, solving problems using diagrams, and basic problems involving time. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Fraction Estimation
- Adding and Subtracting Fractions with Like Denominators
- Adding and Subtracting Fractions with Different Denominators
- Adding and Subtracting Mixed Numbers
- Subtracting Mixed Numbers by Renaming
- Elapsed Time
- Box-and-Whisker Plots
- Problem-Solving Strategy-Draw a Diagram

Fraction Estimation

Alternative Teaching Strategies

You can practice rounding the nearest half by creating a fun game for the students to play. First, create a card with a 0 on it, then one with a $\frac{1}{2}$ and one with the number 1. Put these three numbers in different corners of the room. Then write out the following fractions on index cards. You can ask the students to move to the corner of the room where their fraction would be rounded to if we were rounding by halves. Then you can assess the accuracy of each student's choice by conducting a whole class discussion. If you have made enough fractions, switch them out and play again.

Here is a list of possible fractions to play with.

All of these would round to 0.

$\frac{1}{6}$

$\frac{1}{7}$

$\frac{2}{9}$

$\frac{2}{11}$

$\frac{1}{12}$

$\frac{1}{19}$

$\frac{3}{33}$

$\frac{2}{27}$

$\frac{1}{18}$

$\frac{4}{90}$

$\frac{5}{212}$

All of these would round to $\frac{1}{2}$.

$\frac{4}{9}$

$\frac{3}{7}$

$\frac{5}{11}$

$\frac{6}{13}$

$\frac{6}{14}$

$\frac{7}{15}$

$\frac{9}{20}$

$\frac{11}{25}$

$\frac{16}{30}$

$\frac{16}{29}$

$\frac{17}{33}$

$\frac{25}{53}$

$\frac{33}{67}$

$\frac{32}{65}$

$\frac{70}{135}$

All of these would round to 1.

$\frac{6}{7}$

$\frac{7}{8}$

$\frac{8}{9}$

$\frac{9}{11}$

$\frac{10}{13}$

$\frac{13}{16}$

$\frac{15}{17}$

$\frac{22}{24}$

$\frac{22}{27}$

$\frac{30}{32}$

$\frac{33}{35}$

$\frac{67}{70}$

$\frac{78}{81}$

$\frac{89}{92}$

$\frac{100}{111}$

The rest of this lesson is very straightforward and can be taught by using the examples in the text.

Alternative Assessment Options

When you have played the game, you will be able to see the each student's choice of corner, then you can assess each student's understanding based on the accuracy of his/her choice. If you play it more than once, you will be able to see if a student who made an error the first time was able to self-correct or not. If a student was incorrect both times the game was played, spend some time reviewing this lesson with him/her while the other students are completing their independent work.

Classroom Modifications for Diverse Learners

In the list of fractions above, there are fractions of varying degrees of difficulty. This will allow you to differentiate the lesson. A student with special needs can be given a simpler fraction, such as $\frac{6}{11}$ while another students might be given a fraction that is very challenging to decipher such as $\frac{16}{29}$. Because all of the students play the game at the same time, the students will be able to learn from one another and more advanced students can "peer teach" other students. Always allow time for sharing during these games because most of the teaching happens during this time.

Additional Resources

http://www.echalk.co.uk/tasters/taster2/taster.html –This is a very cute chicken game about rounding and estimating fractions. Correct answers earn an egg. There is a visual picture of the fractions as well as a number line.

Adding and Subtracting Fractions with Like Denominators

Alternative Teaching Strategies

A fun way to differentiate this lesson is to have the students make different fraction strips with given denominators. In this way, adding and subtracting fractions becomes a hands–on kinesthetic experience. While it may seem like a very basic lesson, a student's ability to understand this lesson conceptually will help to ensure a student's success in later lessons. Therefore, it is a great one to make interactive with the fraction strips.

To do this, first decide which denominators you want the students to work with. Some possible options are sixths, eighths and tenths. Some students may want to work with more challenging denominators. For example, you may have a student who says that he/she wants to work with eighteenths. That is fine, in fact is a great way for students to choose their own challenge, however you will need to remind the student that the fractional pieces still need to be exactly the same measurement whether they are working with sixths or sixteenths.

Then students can create fraction strips. Each student should make two parts of a whole that use the same denominator. You can either let them choose their own two fractions or assign them.

Then the students write a number sentence showing how to add the fractions and a solution. Ask each student to work with a partner and have them check each other's work.

This activity can work with subtraction too, only the student would "cut off" with scissors the fraction part of the whole that he/she is subtracting.

Alternative Assessment Options

You can assess two things with this activity. The first is the students understanding of fraction concepts. Think about these questions to assess the understanding of fraction concepts.

1. Did the students understand how to create a fraction strip using the numerator and the denominator?

2. Did the student accurately create the two fractions?

Next, think about procedural knowledge.

3. Did the student accurately write a number sentence to represent the addition problem?

4. Is the sum that the student wrote accurate?

Classroom Modifications for Diverse Learners

For students with learning challenges, assign them the two fractions they will be creating in the fraction strips.

For example: $\frac{2}{6}$ and $\frac{3}{6}$

The student would be assigned these two fractions. Then he/she would use the fraction strips to create the fractions and write a number sentence with an answer to show the sum. Once this was done, the students would be able to "see" the sum of the two fractions.

Additional Resources

http://www.mathcats.com/grownupcats/ideabankfractions.html –This website has a fraction idea bank for working with fractions. There are activities listed here as well as books. This website would be a great resource for working with all different types of learning styles.

Adding and Subtracting Fractions with Different Denominators

Alternative Teaching Strategies

This lesson builds on the previous one by moving from adding and subtracting fractions with like denominators to adding and subtracting fractions with unlike denominators. To differentiate this lesson, we can work with this concept in a way that includes visual, auditory, kinesthetic as well as students who learn through interpersonal and intrapersonal methods. We are going to use the basics of the fraction strip activity from the last lesson, but we are going to build on it so that the students can "see" creating common denominators.

To work on this lesson, take out the fraction strips from the last class. Now have the students create a new strip. It should be the same length as one of the ones that they already created, but be divided up into a different number of sections. For example, if yesterday they created $\frac{2}{6}$, then today the strip would be the same but would represent $\frac{1}{4}$.

Once students have these two strips of different denominators created, you can ask the students how they can add them if the denominators are different. We have to find a common or same denominator for them both.

Now students must determine a common denominator. The student decides that 12 is the common denominator for 4 and 6. Next, use tracing paper to show the 12. To do this, simply cut a piece of tracing paper the same length as the fraction strip. Next divide it into 12^{ths}. Then put the tracing paper over the previously created fraction strip and color in the same number of bars as was drawn when the denominator was 4^{ths} or 6^{ths}. Now the students can see what part of the 12 is $\frac{2}{6}$ *or* $\frac{1}{4}$.

Finally, he/she can write a problem to represent the new problem and add the fractions. This is a visual way of working through adding fractions with unlike denominators.

To help students to work through this lesson, allow them the opportunity to work in groups or pairs.

Alternative Assessment Options

Assessment is completed through observation. Here are some questions to keep in mind as you observe.

1. Can the student create a new fraction with a different denominator on his/her own?
2. Did I have to give them the new fraction to create as a fraction strip?
3. Was the student able to determine a common denominator?
4. Was the student able to create a fraction strip to represent the new denominator?
5. Was the student able to write a problem to represent the new addition problem with the renamed fractions?
6. Is the student's answer accurate?

By pondering and answering each of these questions, you will understand which students need help and which skills must be reviewed or worked on.

Classroom Modifications for Diverse Learners

A modification for this activity would be to provide the student with the second fraction with the different denominator.

Another modification would be to provide the student with three possible common denominators for he/she to choose from.

For example: 4 and 6

12, 20, 24

Remind the student that we are looking for the least common denominator. Then allow the student to choose. He/she should notice that one doesn't work at all. 12 is the correct common denominator because it is the lowest common denominator.

Additional Resources

http://www.youtube.com/watch?v=Fg9fOUXkrf4 –This is a youtube video on how to use fraction towers. These fraction towers are created by etacuisaire.com, but the same concept could be applied to the fraction strips that are shown in this lesson.

Adding and Subtracting Mixed Numbers

Alternative Teaching Strategies

When adding and subtracting mixed numbers, students are working with wholes and parts. To work successfully with wholes and parts, students will need to add the parts first and the wholes afterwards.

To differentiate the instruction, be sure to provide steps for students to follow on the board. This will help visual learners as well as students with special needs to remember each step. Here are some guiding steps for all students.

1. Look to see if the fraction part has a common denominator.
2. If so, then add.
3. If not, then find a common denominator, rename the fractions and then add.
4. Finally simplify your answer if needed.

Be sure to do each example on the board for the students. By doing all of the examples with the students, they will have had the opportunity to practice each type of problem that will be assigned to them when they complete their independent work. This will help to prevent unnecessary reteaching.

Alternative Assessment Options

You can assess student understanding by looking at how students have worked their way through the steps. If student answers are incorrect, you can look at one of two things. Either the student did not follow the steps, in this case a simple reminder is all that is necessary. The other option is that the student forgot what he/she needed to do when completing the steps. If this is the case, then you will need to reteach or use peer teaching to help this student.

Classroom Modifications for Diverse Learners

The most challenging part of adding mixed numbers for special needs students is when the parts equal one whole or greater than one whole all on their own. Look at this example.

$$2\frac{3}{4}+1\frac{3}{4}=3\frac{6}{4}$$

Many students will add in this way and leave this as an answer. The answer here is not complete because we have an improper fraction remaining. This is not a common occurrence but there are times when students may see an

example like this one. Review how to figure out the mixed number from the improper fraction and then add the wholes together. Finally the fraction remaining will need to be simplified.

Answer $= 4\frac{1}{2}$

Write these questions on the board to help students to remember the steps.

1. Does the fraction part have a common denominator?
2. If so, then add.
3. If not, find a common denominator, then add.
4. Is my answer in simplest form?
5. Remember: improper fractions are not in simplest form.

Additional Resources

http://www.mathsisfun.com/mixed-fractions.html –This website has a tutorial on identifying and understanding mixed numbers as well as how to add and subtract mixed numbers.

http://www.analyzemath.com/middle_school_math/grade_6/fractions.html –This website has a great challenging problem for students looking for an additional challenge. It has a visual geometric square divided up into different colors and sections. Students are given a series of questions that they need to answer. This requires problem solving skills and would be great for some students.

Subtracting Mixed Numbers by Renaming

Alternative Teaching Strategies

Renaming mixed numbers when subtracting is one of the most challenging things for students. To help with this, begin by working only with whole numbers that subtract mixed numbers. Look at this example.

$$8 - 2\frac{1}{2}$$

Working with whole numbers first will help students to practice simply renaming a whole number into a whole and a fraction part. This will be tricky for some. Be sure to walk through each step in a visual way. As you perform each step talk through exactly what you are doing, this will help auditory learners to work through the problem.

Alternative Assessment Options

Observe students as they work and make yourself available for student questions. Here are some questions to think about if students have an incorrect answer.

1. Did the student rename the whole number correctly?
2. Did the student add the renamed fraction to the original fraction?
3. Is the answer in simplest form?

One of these places is probably where the student "got stuck" if the answer is incorrect. Helping the student to trace back through his/her work will help the student to self-correct in the future.

Classroom Modifications for Diverse Learners

If you have a class with many students with IEP's or with special needs, using color coding will help these students. Here is what color coding would look like on the example above.

$$8 - 2\frac{1}{2}$$

Begin with the fraction in red and the whole numbers in black. Then as you rename the whole number 8, you can change it to black and red. Now the students can see the whole number part and the renamed fraction part.

$$7\frac{2}{2} - 2\frac{1}{2}$$

Point out to students that the whole has been renamed in terms of halves because the denominator of 2 is what is being subtracted. Now you can subtract as you normally would.

Answer: $5\frac{1}{2}$

This class could be taught in two lessons. Spend the first lesson focused on subtracting mixed numbers from whole numbers. Then in the second lesson teach the students how to rename mixed numbers and add the new fraction to the previous one. By breaking this down, you will help all students to be included in the lesson.

Additional Resources

http://www.onlinemathlearning.com/6th-grade-word-problems.html –This website has two sixth grade word problems that require students to use fractions and ratios. The solutions are worked out and have visual aids with them. These could be used for whole class instruction or for students who are looking for an additional challenge.

http://math-and-reading-help-for-kids.org/word_problems_involving_fractions.html –This website has some wonderful resources for both teachers and parents. There are some fraction word problems, but also some different helpful hints/strategies for student success.

Elapsed Time

Alternative Teaching Strategies

To differentiate this lesson, you can use an actual clock with moving hands to help the students to calculate the time. This can be a fun, visual hands–on way to work with the content. It is also kinesthetic in its approach. The students can work in pairs or small groups too, so students who learn best through interpersonal and intrapersonal ways can be included as well. Working with the clock faces and hands would be a first step to working through the problems. Eventually, you want the students to move to the mathematical way of working as presented in the lesson.

http://www.enchantedlearning.com/crafts/clocks/clock/ –you can use the instructions and design on this website as a guide for making clocks.

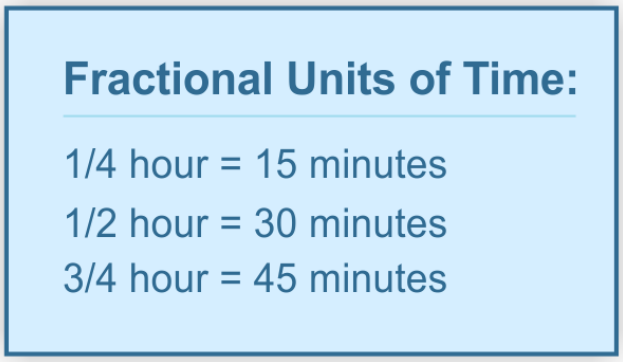
Some students may want to work in a strictly mathematical way as is presented in the text. This is fine as well. Have the clocks available for use, but students should have the choice as to whether or not they wish to use them. This is true differentiation.

Alternative Assessment Options

You can assess student understanding through observation. Walk around as the students are working on the problems in the text. Notice which students are using manipulatives and which ones are already working mathematically. Make a note so that you can help the students to transition from manipulatives to the mathematical way of working through the problems. The guidelines in the text provide a step-by-step way of accomplishing this task.

Classroom Modifications for Diverse Learners

Be sure to have students copy down the fractional units of time in their notebooks. Many will know them intuitively, but not know the actual numbers. This is especially common with special needs students or with students who are used to looking at a digital clock. In our day and age, digital clocks are so common that we have to keep this in mind when working with problems having to do with time.



Additional Resources

http://marg.mhost.com/MathGr5/elapsedtime.htm –This is a fifth grade elapsed time quiz. It could be used as a pretest prior to teaching the lesson or as a homework assignment.

http://teacher.scholastic.com/maven/timefor/index.htm –This is a fun, interactive story from scholastic which uses elapsed time. It is very engaging and comes in a pdf format as well.

Box-and-Whisker Plots

Alternative Teaching Strategies

To differentiate this instruction, use the story problem at the beginning of the lesson as a group work activity. Working in groups requires students to be cooperative and engages visual, auditory, kinesthetic, interpersonal and intrapersonal learning styles. Provide students with a printed out copy of the story problem. Then teach the content in the lesson on how to build a box-and-whisker plot. Do not show students the solution at the end of the lesson.

After teaching the content, allow students the chance to go to work. Students should present a mathematical explanation of their work as well as a drawing of a box-and-whisker plot.

Once each group is finished, allow time for the students to "teach" their solution to the other students in the class. This way of working helps them to share what they know.

Alternative Assessment Options

When students are showing their work, here are some questions to think about.

1. Do the students understand the vocabulary words in the lesson?
2. Are the different measures included in their presentation?
3. Is the box-and-whisker plot accurate?
4. Are all students taking part in the explanation?
5. Is there anything that is unclear or needs further explanation?

Classroom Modifications for Diverse Learners

Write the steps for creating a box-and-whisker plot on the board.

Here are the steps to drawing a box-and-whisker plot.

1. Draw a number line labeled to show the range of data from least to greatest.
2. Mark the median, the upper quartile, the lower quartile, the lower extreme and the upper extreme on the number line.
3. Draw in a box around the quartiles. The median is the middle line of the two boxes.
4. Then draw in the whiskers. These are lines that extend from each quartile to the upper and lower extremes.

Additional Resources

http://www.shodor.org/interactivate/activities/BoxPlot/ –This is an interactive website where students work with box-and-whisker plots. This could be used as an enrichment activity or as an additional activity for students who are looking for an additional challenge.

http://www.learnnc.org/lp/pages/3767 –This is an interactive activity to create a human box-and-whisker plot. It is very engaging and includes multiple learning styles in its approach.

Problem-Solving Strategy-Draw a Diagram

Alternative Teaching Strategies

To work through this problem-solving lesson, review the four parts of the problem solving plan. The previous lessons on elapsed time and box-and-whisker plots provide students with excellent examples for drawing diagrams to find solutions. Begin by reviewing these lessons with the students.

To make this lesson fun and interactive have the students create a comic strip that tells the story of the word problem. They can choose one of the word problems from the practice section. Then they draw out the problem and the solution in a comic strip style. Finally, the students share their work with each other.

Alternative Assessment Options

Assessement can be completed through observation.

Classroom Modifications for Diverse Learners

You can assign students the problem that you want them to draw as a comic strip. Sometimes students with special needs have a difficult time choosing a problem and assigning it helps them to select one.

Encourage or assign students a partner to work with. By working with a partner, the students can work together to find a solution.

Additional Resources

http://www.fi.edu/school/math2/ –This website has many open –ended math problems for students in middle school. These problems could be worked on in small groups or used as a challenge problem for students who are in need of additional challenge.

http://theteacherscafe.com/Math/Story_Problems.php –This website is a resource site for teachers. It has many different story problems and games for students.

3.7 Multiplication and Division of Fractions

This seventh chapter *Multiplication and Division of Fractions* covers the methods needed to multiply and divide fractions and mixed numbers. Students are also introduced to using and converting customary units. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Multiplying Fractions and Whole Numbers
- Multiplying Fractions
- Multiplying Mixed Numbers
- Dividing Fractions
- Dividing Mixed Numbers
- Customary Units of Weight and Capacity
- Converting Customary Units
- Problem-Solving Strategy: Choose an Operation

Multiplying Fractions and Whole Numbers

Alternative Teaching Strategies

In this lesson, students learn to multiply fractions and whole numbers. Here are two different ways to differentiate this lesson to engage students with a variety of learning styles. The first way is to hand out index cards with whole numbers on them. You can vary the whole numbers that are given out to differentiate the level of challenge. For example, a student with learning challenges might receive a 2, while another student could receive a 12 or 13. Then you write a fraction on the board.

Example: $\frac{2}{3}$

Then the students write the product of their whole number times the fraction on the board at their desk. Each student is working with the same fraction, but each product is different. Then you can allow time for students to share their answers and how they arrived at their answers.

Another way to differentiate the lesson is to use the word problem with Jessica. She gave $\frac{2}{9}$ of the cake to 3 friends. Ask the students to draw a picture of the problem and a picture of the solution. They can divide their paper in half vertically. The left side of the paper shows the problem and the right side the solution. You can engage the students' creativity while helping them learn to use pictures to solve word problems.

Alternative Assessment Options

Assessment for this lesson can be done through observation and class discussion. During the first differentiated activity, you can walk around and watch students as they work to solve problems. Then in the second activity, the drawings themselves will demonstrate student understanding. You can also see who understands the concept of multiplication and who doesn't. There may be some students who understand the concept yet didn't get a correct answer. These students may be weak at multiplication skills and need practice.

Classroom Modifications for Diverse Learners

A modification for special needs students would be to provide a multiplication chart. This would be used for students who have severe learning challenges and haven't learned their times tables yet. The goal should be for all students to learn their tables if they haven't yet done so.

Additional Resources

http://www.ixl.com/math/grade-4/multiply-fractions-by-whole-numbers-word-problems –This website has an interactive section where students practice solving word problems by multiplying fractions and whole numbers.

http://astore.amazon.com/k5matteares-20?_encoding=UTF8&node=17 –This math website has a list of math read alouds that include fraction concepts.

Multiplying Fractions

Alternative Teaching Strategies

To differentiate the instruction, you can take the first example for multiplication in the text and have the students create a visual version of this themselves.

Example: $\frac{3}{4} \times \frac{1}{2}$

In the text, a rectangle is used to create a model of three –fourths. Then one –half of the three fourths is shaded showing a model of three –eighths. You can either have the students color in the squares to make the model or fold paper. Sometimes folding paper and then coloring will help the students to see it clearly. Some students may say that the answer is four –eighths because the fold shows four squares. However, this isn't accurate because only three out of four squares were shaded in the beginning because the first fraction was three –fourths. This is a creative, hands–on way to not only reinforce fraction concepts, but also to show students how to multiply the fractions.

Alternative Assessment Options

As students work on this lesson, think about these questions.

1. Can the student create the two fractions by folding the paper or by coloring?
2. Is the product easy to identify?
3. Can the student identify the product by looking at the section that overlaps both fractions?
4. Can the student transfer this concept to a mathematical problem?
5. Is the student struggling with fraction concepts or with times tables?
6. Did the student simplify on the diagonals but forget to multiply horizontally?
7. Is the student's answer in simplest terms?

Classroom Modifications for Diverse Learner

A modification for special needs students is to write the following steps on the board when it comes to cross simplifying.

1. Simplify fractions that can be simplified first.

For example: $\frac{6}{8} = \frac{3}{4}$

2. Cross simplify means to simplify the numbers on the diagonal.

$$\frac{\overrightarrow{3} \quad 6}{4 \times \underrightarrow{9}}$$

You can use color coding to show which numbers you are simplifying and arrows to remind students of the direction that they are multiplying.

Additional Resources

http://www.basic-mathematics.com/multiplying-fractions-word-problems.html –This website has multiplying fractions word problems with a tutorial to remind students how to work with them.

Multiplying Mixed Numbers

Alternative Teaching Strategies

One of the ways to differentiate this lesson is to create a "conversion" game to practice converting mixed numbers to improper fractions and improper fractions back to mixed numbers. This is a key skill for this lesson because the students will be combining the work of multiplying fractions with mixed numbers. To be successful in this lesson students will need to convert mixed numbers to improper fractions before multiplying them.

Here is how to create the game. Use the pairs below and put the mixed number problem on one index card and the matching improper fraction problem on a different index card. Once you are finished, then you shuffle the cards and hand –out one card to each student. Some students will receive a problem written in mixed number form and some students will receive a problem written in improper fraction form. Then the students try to find the matches. You can work on this by timing students for an additional challenge or not. Once the students think they have found a match, then the two students have to come up with a way to show or prove that the match is indeed true. This will show how well they know how to convert mixed numbers and improper fractions. You can play this game multiple times too.

Alternative Assessment Options

Assessment can be done through observation as the students play the game. If a student is not able to convert either an improper fraction to a mixed number or a mixed number to an improper fraction, then it will be obvious as they play the game. This is an opportunity to make note of the students who are struggling, use your time to help them or have them work with a partner so that peer teaching is incorporated in the lesson.

Classroom Modifications for Diverse Learners

A modification for special needs students would be to write the steps to converting a mixed number to an improper fraction on the board. Then the notes on the board will remind the students of the process as they play the game.

If the class has many special needs students in it, then you can assign them partners from the beginning. This means that two students would have one card to work with and so on.

Additional Resources

Here is a list of matches that you could use for the game. Notice that the matches have varying degrees of difficulty so you can scaffold the lesson.

$$3\frac{1}{2} \times 2\frac{1}{3} = \frac{7}{2} \times \frac{7}{2}$$
$$4\frac{1}{5} \times 6\frac{1}{2} = \frac{21}{5} \times \frac{13}{2}$$
$$8\frac{2}{3} \times 4\frac{1}{6} = \frac{26}{3} \times \frac{25}{6}$$
$$6\frac{1}{2} \times 4\frac{1}{9} = \frac{13}{2} \times \frac{37}{9}$$
$$1\frac{1}{2} \times 2\frac{1}{3} = \frac{3}{2} \times \frac{7}{3}$$
$$5\frac{1}{4} \times 2\frac{1}{2} = \frac{21}{4} \times \frac{5}{2}$$
$$6\frac{1}{9} \times 4\frac{2}{7} = \frac{55}{9} \times \frac{30}{7}$$
$$5\frac{4}{5} \times 6\frac{2}{3} = \frac{29}{5} \times \frac{20}{3}$$
$$1\frac{7}{9} \times 2\frac{1}{2} = \frac{16}{9} \times \frac{5}{2}$$
$$2\frac{3}{4} \times 5\frac{2}{5} = \frac{11}{4} \times \frac{27}{5}$$

Dividing Fractions

Alternative Teaching Strategies

To differentiate this lesson, use the problem from the introduction as a way to explore the division of fractions. Use this problem before teaching any of the content in the lesson. The content of the lesson contains a lot of procedures. The procedures are necessary for students to be successful in actually dividing fractions. Using the problem from the introduction will provide students with an experience of the concept behind the procedures.

You will need rulers, pencils and chart paper. A large roll of paper is ideal for this kind of problem. Students will be working in groups.

Begin by reading the problem aloud. Then write the following important information on the board. This can be generated through class discussion and you can have the students generate the important information.

Paper $= 20''$ wide

Each strip $\frac{3}{4}''$

25 students each need a strip

Is the paper large enough? Will this size help Julie? Why or why not? Prove your answer.

Next, divide the students up into groups. Provide them with materials and let them go to work. This activity allows students the opportunity to work collectively on problem solving. They also have to prove that they are correct with a demonstration. It is an activity that includes all learning styles and where students can work together to find a solution.

Allow time for the students to share their work when finished.

Alternative Assessment Options

While the students are working, you can walk around and offer help as needed. The real assessment will be during the presentations or sharing at the end of the class. When students are sharing, listen for the process of how they

arrived at an answer. Here are some questions to think about.

Which strategies were used?

Did all students take part in the process?

Is the work accurate?

Can the students explain their answer in words?

Is the explanation physical by using paper?

Is the concept of dividing fractions clear in the explanations?

Classroom Modifications for Diverse Learners

Rather than having the students generate the important information, you can provide them with it. Write it on the board for them and then ask them to find it in the story problem and underline each part. Give the students the information first so that you can be certain that they identify the correct information. Then ask them to find it themselves and underline it. This reinforces the important content from the problem.

Additional Resources

http://www.internet4classrooms.com/skill_builders/divide_fractions_math_sixth_6th_grade.htm –This website has a list of resources for teachers to use when teaching their students to divide fractions.

Dividing Mixed Numbers

Alternative Teaching Strategies

Write the steps on the board and have them visual throughout the lesson. Teach the students to use this as a check list. When all of the steps are done, then they are done too. Writing them on the board includes visual learners and will help students with special needs to remember the procedural steps involved in dividing mixed numbers. Be sure to talk through each of the steps orally. As you complete the practice problems, refer back to the step number that you are working on. This will help students to connect the procedural steps to actually dividing the mixed numbers.

1. Change the mixed number to an improper fraction so that you are working in parts. If you think about the example we were just working on this makes perfect sense. We need to work in parts.
2. Change the division to its inverse multiplication and multiply by the reciprocal of the fraction.
3. Multiply and simplify to find the quotient.

Alternative Assessment Options

There isn't an alternative assessment for this lesson. Students need to follow steps and find accurate answers.

Classroom Modifications for Diverse Learners

For students with special needs, it is best to divide this lesson into two class periods. To successfully divide mixed numbers, there are many steps involved. Providing the students with two class periods will give them more time to incorporate each of the steps.

On the first day, teach dividing a mixed number by a fraction. On the second day, teach dividing a mixed number by a mixed number.

You can also divide the problems in the practice section up according to the two day division. Students practice dividing mixed numbers by fractions on the first day and dividing mixed numbers by mixed numbers on the second day of the lesson.

Additional Resources

http://www.math.com/school/subject1/lessons/S1U4L8DP.html –This website has a word problem that involved dividing whole numbers and mixed numbers. It provides a step by step solution and could be used as a tutorial.

http://www.mathtv.com/videos_by_topic –This website has excellent math videos designed by topic. The students can select mixed numbers, multiplying and dividing and see videos that teach yet are fun and engaging.

Customary Units of Weight and Capacity

Alternative Teaching Strategies

Working with weight and capacity can be fun for everyone and can be taught in a hands–on interactive way. Here are some suggestions for differentiating this lesson. There are many more activities possible, but here is an option for weight. These activities require students to work cooperatively and in pairs. They also teach to different learning styles including auditory, kinesthetic, visual, musical, naturalist, interpersonal and intrapersonal learning styles.

For weight, you will need a couple of different scales. You can use traditional scales that you would find in homes, or you can use older scales too. Students can compare and contrast the weights that they get on each scale. When weighing items, the weight of each will be in pounds. Then the students will use mathematics to translate the weight into ounces and tons. For ounces, they will multiply. For tons, they will divide and have a decimal answer –a part of a ton.

Allow students the chance to choose their own items. The students record the weight of each item, then make two other columns on a piece of paper so that they have the weight of each item in pounds, tons and ounces. This will require all types of mathematics and practice with division and multiplication.

Alternative Assessment Options

Observe students as they work on this lesson. Here are some questions to think about.

Do the students understand how to weight each item accurately?

Can they write the weight in ounces?

Can they find an accurate decimal for the weight in tons?

Collect each paper and check student work for accuracy. Provide feedback and an opportunity for the students to correct their work.

Classroom Modifications for Diverse Learners

Modify this activity by writing the following information on the board.

16 ounces = 1 pound

To convert from pounds to ounces, multiply by 16.

1 ton = 2000 pounds

To convert from pounds to tons, divide the number of pounds by 2000. This means that the pounds will be written inside the division box. You will have a decimal answer.

Additional Resources

http://www.proteacher.org/org/c/322_Capacity__Volume.html –This is a project for middle school students on capacity and volume. It could be used as an additional assignment. This website is a resource for teachers.

Converting Customary Units

Alternative Teaching Strategies

To differentiate this lesson, you can have some fun. You will need some different items. If you plan it ahead of time, parents might even be willing to donate the necessary items. Here are some suggestions for making this a hands–on lesson.

Two or three long ropes –it doesn't matter the length, students will be figuring this out.

Two jars of pennies –they can be different sizes and have different amounts in them.

2 five gallon jugs –they can be empty

For this activity, students will be working in groups to exploring converting different measurement units.

Each group is assigned a different item.

Students first need to figure out the length, money amount or capacity of what they have. Then they will need to convert it a different measurement unit.

Length is measured in feet and changed to inches.

Money is measured in cents and changed to dollars or fractions of dollars.

Capacity is measured in gallons and converted to ounces.

Allow time for the students to share their work when finished.

Alternative Assessment Options

Some of these tasks are more challenging than others. You can differentiate the lesson by planning which students are going to work on which task. For example, length is very straightforward whereas capacity will require more problem solving skills. Be sure to help the students as they work through this activity.

Classroom Modifications for Diverse Learners

For special needs students, this activity could take several days. You could treat it as a project and work with length one day, money the next and capacity on the third day. Breaking down the project in this way will help the students to work through the content in a procedural way. You can also reinforce skills.

Additional Resources

http://mathforum.org/library/drmath/sets/mid_word_problems.html –This website has many different challenge problems for middle school students. The problems are on many different topics, but could be used as an additional resource for students looking for a challenge.

Problem-Solving Strategy: Choose an Operation

Alternative Teaching Strategies

When working on problem solving, there are a few key strategies that can help students in their work. These strategies can help a student to "break down" a word problem into smaller sections. First, have the students read the problem out loud. Then ask them to share what the story is about. This verbal retelling of the problem will help you to know what information the students were able to glean from the story. Then help the students to underline the key words in the story. This would be any words that indicate numbers or operations. In this lesson, we are focused on choosing an operation as a solution for solving a word problem. To help students with this, it is recommended that you put a copy of the list of key words in a place that is visual to all students.

Alternative Assessment Options

You can assess student understanding by listening to what they gleaned from the reading of the word problem. If word problems are a regular part of your lesson, you can also keep track of which students are participating in the discussion. Is it always the same ones? If so, see if you can find a way to engage these other students in the discussion. Asking specific questions would be helpful.

For example, "Do you see a key word that would show us which operation we will need to solve the problem?"

Then you can write the key word shared on the board and then move on to which operation is needed given the key word.

Classroom Modifications for Diverse Learners

Students with special needs will need additional help with word problems. Writing them in double spaced type can help. If the students have more space between the lines of text, it will be easier for them to follow the story of the problem.

Additional Resources

http://msteacher.org/epubs/math/math2/math.aspx –This is a great website for teachers. It offers thinking from the NCTM on problem solving and how to work with students. It has great articles and links that are all connected to problem solving in the middle school.

3.8 Ratios, Proportions and Percents

This eighth chapter *Ratios, Proportions and Percents* introduces students to ratios and rates, basic uses of proportions including understanding scale drawings, and percents. Also explored is the relationship among percents, decimals, and fractions. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Ratios
- Rates
- Solving Proportions
- Proportions and Scale Drawings
- Understanding Percent
- Percents, Decimals and Fractions
- Finding a Percent of a Number
- Problem-Solving Strategy: Use a Proportion

Ratios

Alternative Teaching Strategies

One of the first ways that you can differentiate this lesson is to look at the three different ways of identifying ratios. We can write them as fractions, with a colon or by using the word "to". The three different forms can be used together and students can practice interchanging the forms. It shouldn't matter if a fraction is equivalent to a ratio in the form with a colon or not. This is an excellent way for students to be challenged during this lesson. For students with special needs, see the notes below.

To challenge students, look at these examples.

Directions: Ask students to write an equivalent fraction by simplifying. Use a different form of ratios for each answer.

1. 15 to 45
2. $\frac{90}{270}$
3. $\frac{77}{49}$
4. 69 *to* 13

Alternative Assessment Options

Assessment can be done through observation. Here are some questions to think about.

1. Is the student able to understand how to simplify?
2. Can the student simplify the ratio in all three forms?
3. Can the students use the three forms interchangeably?
4. Does the student need more practice with simplifying?

Classroom Modifications for Diverse Learners

A modification for students with special needs is to only work with one form of the ratio at a time. In other words, students would be simplifying ratios only in fraction form first. Fraction form is the simplest way for students to first work with simplifying ratios because it is the form that they are already familiar with. This is an excellent place to start. Begin by having the students simplify ratios in fraction form. Here are a few examples.

1. $\frac{6}{10}$
2. $\frac{4}{12}$
3. $\frac{6}{9}$

Then have them move on to simplifying ratios written using a colon. You can begin by using the same three examples above so that the students can make the connection of them being the same.

1. 6:10
2. 4:12
3. 6:9

Finally, move to using the word "to". Again, you can use the same three examples. By now students will see how to work with them and be able to interchange the forms of the ratios when simplifying or finding equivalent ratios.

Additional Resources

http://msteacher.org/epubs/math/ratios/math.aspx –This is a resource page for teachers on how to engage middle school students when teaching ratios.

Pythagoras and the Ratios: A Math Adventure by Julie Ellis is a book that could be used to differentiate ratios through a geometry integration.

Rates

Alternative Teaching Strategies

A fun way to differentiate working with rates is to use actual advertisements from a newspaper or from a supermarket advertisement. Many times, these advertisements will have notes about how much per pound, etc. For example, in thinking about Thanksgiving, you can say that turkeys are .89 a pound. This is the rate. It is being compared to 1.

.89 to 1 pound of turkey

A possible problem would be to figure out the cost of a 10 or 12 pound turkey.

To work with the students, you can begin with an example like this one. Then have them work in small groups to write a problem that uses the same rate as in the example, but a different number of pounds. Working in this way helps to engage many different learning styles including auditory, visual, kinesthetic, interpersonal and intrapersonal. Allow time for students to share their work at the end of the lesson.

This will also be a good warm –up for the enrichment activity presented in another flexbook.

Alternative Assessment Options

When assessing student work, you can use the following questions as a guide.

1. Did the students understand that they needed to multiply?
2. What did the students multiply to figure out the price?
3. Did they do this correctly?
4. Is the price accurate?

5. Are the students able to explain their thinking to their classmates?
6. Did everyone in the group participate in the work?

Classroom Modifications for Diverse Learners

Provide special needs students with a format for working on the problem. Here is a suggested set of ratios.

$$\frac{.89}{1} = \frac{x}{12}$$

Then students can see that they need to figure out how much a 12 pound turkey costs if it is .89 per pound. Many will know that they need to multiply. If the proportion is confusing, then allow students the chance to work on the problem intuitively.

Additional Resources

http://msteacher.org/epubs/math/ratios/ratiosinfrac.aspx –This is a resource page for teachers about teaching ratios and fractions as rates.

http://karen.mcnabbs.org/worksheets/rates_ratios/index.html –This website has worksheets designed for 7^{th} graders on rates and ratios. These could be used as a challenge for students needing some additional skill work.

http://www.helpingwithmath.com/by_subject/ratio/rat_unit_rate_6rp2.htm –This website has a guided tutorial and several worksheets on rates and unit rates. One is a warm –up and then another can be used for additional practice for the students.

Solving Proportions

Alternative Teaching Strategies

There are two different methods taught in this lesson for identifying and solving proportions. The first is to help students to simplify the ratios to see if they are equivalent. The second is to use cross–products. To help the students to remember these two ways, make a list of them on the board. You want to keep the list visual while the students are working through the lesson. When things are left in front of the students, you can help them to use their reasoning skills to figure out that if they can't use simplifying, then they should use a cross–product.

Word problems are a great way to challenge advanced students. You can provide them with a word problem like the one in the text and ask them to solve it using a proportion. Be very specific about the proportion as the method of solving it. Then you can ask them to illustrate how they solved the problem and their solution. Illustrating word problems is excellent for all students. The students really enjoy drawing it out and many will be very creative in their methods.

Alternative Assessment Options

When assessing how a student has solved a word problem using an illustration, you can look to see if they could *show* which values were being compared in the proportion. Look at this example from the text.

$$\frac{25\ campers}{1\ tent} = \frac{75\ campers}{?\ tents}$$

This is a proportion from the text. The students would be illustrating this proportion and it's solution. An acceptable drawing would be one where one tent is drawn with an arrow or equals sign with 25 campers drawn. Students could also draw one big tent with 25 people drawn inside it. Wow! A big tent! Then the student would draw the correct number of tents for 75 people (3 tents) and show how this ratio is equal to the other ratio.

Classroom Modifications for Diverse Learners

To modify this lesson, begin by having special needs students work on simplifying problems first. Once they have mastered determining proportions through simplifying, you can move on to cross–products. For students with where English is a second language or who have a language based learning disability, keep the word problems for last. Then you can allow them the chance to work with a partner or in a small group with you as the leader.

Additional Resources

http://www.ixl.com/math/grade-7/solve-proportions-word-problems –This website has interactive word problems that require students to solve proportions.

http://www.internet4classrooms.com/grade_level_help/solve_with_ratios_math_seventh_7th_grade.htm –This website has many internet links for parents and teachers to work with students on ratios and proportions.

Proportions and Scale Drawings

Alternative Teaching Strategies

When teaching this lesson, you can differentiate the instruction by first teaching the content and then using the problem from the introduction as the differentiated activity. After teaching the students how to identify proportions and work with scale measurements, ask the students to create the actual poster. They have been given a scale and the problem. Students can use their own creativity to make the poster. This is an excellent opportunity to connect the content to a real world activity. Students with creative ability can be leaders in this activity, and students who excel in mathematics can help their groups with the measurement. You can use the problem solution at the end of the lesson so that students can check their work when finished.

Alternative Assessment Options

Assessment can be measured a couple of different ways. Here are some questions to think about as you assess student work.

First, did the students write the proportion accurately?

Is the measurements accurate?

Is the art work creative and shows a best effort?

Can the students explain how they arrived at the answer that they did?

Do the measurements match the solution in the text?

Classroom Modifications for Diverse Learners

One way to modify this lesson for special needs students is to show a set –up proportion that students will need to solve. Remind the students that they can solve a proportion through simplifying or through cross products. Here is an example of a proportion for the problem in the introduction.

$$\frac{8'' \; long}{5'' \; wide} = \frac{x}{2.5 \; feet}$$

This example gives the students the dimension for width. This is one way to modify the assignment. The final step is for the students to complete. They will need to figure out the measurement of the length.

The correct answer is 4 feet because $1'' = 6''$ which means the measure is 48 inches or 4 feet.

Additional Resources

http://academic.brooklyn.cuny.edu/geology/leveson/core/linksa/scale.html –This website has a wonderful tutorial to scale and map measurements.

http://www.ilovethatteachingidea.com/ideas/011206_proportions_scales_ratios.htm –This website has an article for teachers about an educator's experience teaching ratios and proportions to students.

Understanding Percent

Alternative Teaching Strategies

In this lesson, students are going to make the connection between a fraction, a decimal and a percent. Fractions, decimals and percents are all part of a whole. Because of this, we can use them interchangeably. There are two goals in working with this content. First, we need the students to understand these parts conceptually and then procedurally understand how to work with them.

To create an interactive game with this content, you can use the following list to create index cards. Each card has either a fraction, decimal or percent or written words describing one of the equal ratios. Then the students work in pairs to identify all of the other forms of the equivalent ratios.

Alternative Assessment Options

Student work can be collected an evaluated for accuracy. Students should have four columns to show the words, the fraction, the decimal and the percent that they have been given.

Classroom Modifications for Diverse Learners

A modification for diverse learners is to actually receive a paper with four columns created on it. Each column is labeled with the following headings: words, fraction, decimal, percent. Then the students only have to fill in the missing values on the chart.

Additional Resources

Here are possible choices for the cards:

14 out of 100

12%

34%

13 out of 100

50 out of 100

60 out of 100

$\frac{3}{4}$

$\frac{2}{5}$

$\frac{1}{10}$

.40

.55

.60

.80

90%

$$\frac{9}{10}$$

Students use these given values and then work together to write the missing ratios.

Percents, Decimals and Fractions

Alternative Teaching Strategies

If you did the activity in the last lesson, then many of your students have already had practice working with converting basic fractions, decimals and percentages. The tricky part is working with ones that aren't basic such as one –third. This is where students are introduced to terminating and repeating decimals.

To help students to understand these concepts, you can make this visual by writing examples on the board. You can also challenge the students to come up with as many repeating decimals as they can. This is a fun way to engage them in working together to figure it out. The students pair off or work in small groups of three. The students work together and create a list of repeating decimals.

Here are some possible options:

$\frac{1}{3}$

$\frac{1}{6}$

$\frac{1}{9}$

$\frac{1}{11}$ has repeating parts

$\frac{1}{15}$

Alternative Assessment Options

You can assess student understanding through observations. Here are some questions to think about.

Do the students understand how to convert a fraction to a decimal through division?

Are the decimal points in the correct place?

Do the students understand the difference between terminating and repeating decimals?

Classroom Modifications for Diverse Learners

A modification for diverse learners is to have them simply work on converting fractions to decimals through division. Many special needs students won't understand how to test fractions to see if they turn into repeating or terminating decimals. Therefore, you can give them a list of fractions and have them divide them to turn them into decimals. Only when a repeating decimal comes up do you introduce the term. Then it has a real connection for the students. Let them discover first and then explain the different terminology.

Additional Resources

http://www.eduplace.com/math/mathsteps/7/a/7.rationals.develop1.html –This is a great tutorial on converting fractions to different types of decimals. It also has some fun suggestions for teachers on how to make this an engaging activity for students.

Finding a Percent of a Number

Alternative Teaching Strategies

Alternative Assessment Options

When assessing student understanding, you will be able to determine whether or not students understand percents as a concept by checking for accuracy in this lesson. Up until now, there have been specific procedures for working with percentages. This lesson approaches percentages from a conceptual point of view. If students understand the concepts involved in this lesson, they will be able to complete the work accurately. If their understanding is strictly procedural, as in knowing "what to do" they will struggle with the lesson.

Classroom Modifications for Diverse Learners

When working with special needs students, you can provide them with the following steps.

1. Change the percent to a decimal over 100.

$50\% = \frac{50}{100}$

2. Then use the number as the second part of the proportion.

$\frac{50}{100} = \frac{x}{26}$

3. Simplify or use cross–products.

$\frac{50}{100} = \frac{1}{2}$

4. Now solve = 13

This is a number not a percent. We found 50% of the number 26 which is 13. It is a common error for students to want to label everything with a % sign. Be on watch for this it shows that a student does not completely understand the concept being taught.

Additional Resources

http://www.learningwave.com/lwonline/percent/percent_num.html –This website has examples that are color coded and use illustrations. A nice website for a visual learner or for someone who needs an additional tutorial.

http://www.teachingideas.co.uk/maths/contents_fractions.htm –Here is a website of resources for teachers for teaching fractions, decimals and percents. Many have links to PDF downloads.

Problem-Solving Strategy: Use a Proportion

Alternative Teaching Strategies

The best way to differentiate this lesson is to have the students work with the information in the problem along with their own height to use a proportion to solve the problem. In the introduction, Tim's height is being used, but this problem would be more relevant to the students if they were using their own heights. You want the student's to figure out how to use the proportion to solve the problem. Therefore, don't give them too much information. Read the problem. Tell them to use their own height and then let them work. Allow students the chance to work with a partner so that they have someone to talk things over with. Ask students to illustrate their solution.

Alternative Assessment Options

Student work can be assessed by thinking through the following questions.

1. Did the students accurate write a proportion that compares height to distance and height to distance?
2. Does the illustration accurately show this proportion as well?

3. Is the work creative and does it show best effort?
4. Is the answer accurate given the students height?

Classroom Modifications for Diverse Learners

A modification for special needs students would be to show them how to set up the proportion.

$$\frac{frog\ height}{frog\ \ jump\ length} = \frac{Student\ height}{Student\ \ jump\ length}$$

Remind students to keep in mind that this jump length is 20 times the height.

Additional Resources

http://www.yale.edu/ynhti/curriculum/units/2004/5/04.05.08.x.html –This is a great resource site for teachers and is all about problem solving and proportions with middle school students. It is a wonderful site for teaching strategies and contains some wonderful problems to challenge those students who are in need of an additional challenge.

3.9 Geometric Figures

This ninth chapter *Geometric Figures* covers basic principles of geometry. Students are introduced to angles, triangles, quadrilaterals, and polygons. Also explored are congruent and similar figures, line symmetry, and strategies for using Venn Diagrams to solve problems. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Introduction to Geometry
- Classifying Angles
- Classifying Triangles
- Classifying Quadrilaterals
- Classifying Polygons
- Congruent and Similar Figures
- Line Symmetry
- Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Introduction to Geometry

Alternative Teaching Strategies

One way to differentiate this lesson is to assign the students posters on the different terms. This can be carried out throughout the entire chapter. On each poster, students would make different figures and write a short student-friendly definition. The students can create these on smaller pieces of paper, then cut them out and glue them on the posters. Posters can be displayed around the room.

Students might also be required to create a portfolio. This could be done throughout the chapter. Students might have to draw an example of each term, but then they would need to find a real world example of each term. This activity would engage all of the learning styles including the naturalist student who bonds best with the outdoors. There are many geometric examples in the outdoors.

Alternative Assessment Options

Because the vocabulary is so essential to understanding Geometry, it is recommended that students take periodic quizzes. These quizzes will help them to identify the terms that they know and the terms that they don't know.

Another assessment strategy could be a portfolio where students create their own examples of each term. This is described in the section above.

Classroom Modifications for Diverse Learners

On quizzes for vocabulary provide special needs students with a word bank. This word bank can help them to remember each term and find the appropriate definition for each term. You can also provide visual definitions as well as written ones.

Additional Resources

http://jc-schools.net/tutorials/tools/math-ms.html –This website has many links and resources for teaching geome-

try. It also has links to games that use geometry and some interactive websites.

Classifying Angles

Alternative Teaching Strategies

A way to differentiate this lesson is to provide students with many opportunities to see angles in real life. Many times, students don't understand the different angles because they haven't seen them in real contexts only in abstract drawings. Provide the students with tracing paper and many architecture books. Ask them to find angles in the architecture. They will each then need to select a building or piece of architecture to work with. The students must trace their building, etc and then label the angles that are found in the architecture. This engages special needs students, visual, mathematic, kinesthetic as well as students who work well interpersonally and intrapersonally.

Be sure to ask students to include the book title and page number of their original choice.

Alternative Assessment Options

Assess student work based on the following criteria.

Is the drawing accurate to the piece selected?

Did the student trace it with precision and does the work show the student's best work?

Are the angles labeled accurately?

Classroom Modifications for Diverse Learners

A classroom modification for special needs students would be to have the different terms on the board with their definitions. This can serve as a visual reminder for the students.

Acute

Right

Obtuse

Straight

Complementary

Supplementary

Additional Resources

http://www.edutopia.org/mountlake-terrace-geometry-design –This is a great website that shows how a group of students used angles and geometry to understand architecture.

Classifying Triangles

Alternative Teaching Strategies

The best way to differentiate this lesson is to have the students work with the ruler and protractor to create the angles of the triangle. Provide students with one angle measurement for their triangle. Then using a protractor, the students are assigned the task of creating the triangle. You can let students work in groups or by themselves. Working alone could be an added challenge to students who are in need of some advanced work. Another variation would be to require students to draw one of each of the triangles not just one.

Students can also be required to measure the other two angle measures of their triangle and to label each angle

measure on their drawing.

Here are some possible options:

110°

60°

90°

25°

55°

140°

115°

45°

Alternative Assessment Options

Assess student work by examining the triangles created by each student. Here are some questions to think about.

Does one angle of the triangle match the measurement that the student was given?

Is the angle measurement accurate?

Did the student accurately measure the other angles of the triangle?

Did the student label each angle of the triangle?

Is the student's work neat?

Classroom Modifications for Diverse Learners

Special needs students can be assigned one triangle to create. Be sure to review how to use a protractor with the students. The students can complete the same activity listed above, but you may want to allow them time to work with a partner to help them with understanding.

Additional Resources

There are many pictures of classified triangles. These could be used to create a classroom display.

http://www.geom.uiuc.edu/~ekeel/Project/tri1.html –This website has a review of how to classify triangles and then a Sketchpad activity for creating triangles accurately.

Classifying Quadrilaterals

Alternative Teaching Strategies

In the last lesson, the students practiced creating triangles using a ruler and a protractor. Now students are going to create quadrilaterals using a ruler and a protractor. Focusing on constructions is a way to differentiate this lesson for all kinds of learning styles. Students create a quadrilateral and then they can share their work with a peer. After the construction is completed, then you can ask the students to switch papers with a friend and check the other student's work for accuracy. Then students can peer –teach with corrections.

Students are given an angle measure for their quadrilaterals.

With rulers and protractors, students create a quadrilateral. Any quadrilateral can be made as long as one of the angles matches their given measure.

Students then exchange papers and correct another peer's work.

Finally, students share their work with the class.

Alternative Assessment Options

Assessment is completed through peer review and sharing. Allow time for the students to correct each other's work and then to share their quadrilaterals with the class.

Classroom Modifications for Diverse Learners

Review how to use a protractor with the students. Then allow special needs students the opportunity to work with a partner. The students can complete the same activity as listed above.

Additional Resources

http://education.ti.com/calculators/downloads/US/Activities/Detail?id=4058 –This website has instructions for creating quadrilaterals on calculators. Students will need the following equipment and software.

□ TI-83 Plus Family, TI-84 Plus Family

□ **Software**

TI Connect^{TM}

□ **Accessories**

TI Connectivity Cable, Unit-to-Unit link cables

Classifying Polygons

Alternative Teaching Strategies

There are two ways to scaffold this lesson for students. First, the students need to be able to identify the different polygons by sight. Second, the students will be required to work with a formula to figure out the sum of the angle measures of each polygon.

One of the things that you can do to help the students with the identification piece is to have the student make a diagram of each of the figures. The student can work in small groups for this and create pictures of each polygon with its description.

For the second piece, first be sure that the students can identify each polygon. Then move on to the angle measures. This is mathematics and following a formula.

Keep the information in this lesson visual. Once students have learned how to find the sum of the angle measures for each polygon, they can add this information to the chart that was previously created.

Alternative Assessment Options

Here are a few questions to think about when assessing student work.

Are the polygons drawn accurately?

Are the angles accurate?

Can the students distinguish between a regular and an irregular polygon?

Are the sum of the angle measures for each polygon correct?

Did the students add these measures to the chart?

Classroom Modifications for Diverse Learners

Special needs students will need the steps for working with the formula. You can tell them that the 180 never changes. Then they can change the number of sides for each polygon. Finally, multiplication will give them the

correct answer.

Additional Resources

http://www.math-play.com/Polygon-Game.html –This website has a game where students have to identify different polygons based on their properties. If students make a mistake, then the game begins again.

Congruent and Similar Figures

Alternative Teaching Strategies

There are many things taught in this lesson. The first concept is for students to be able to identify similar figures and congruent figures and to be able to distinguish between them. To do this, students must know the definitions. Be sure that the students are adept at figuring out which figures are similar and which are congruent.

In the next part of the lesson, students work on identifying corresponding angles and sides. This is key to their success in figuring out missing side lengths when we get to the proportional reasoning piece of the lesson. A goal in teaching this section is to have the students be very engaged in the lesson. To do this, cut out triangles of different sizes. Then in small groups, you can have the students work on making a triangle that is congruent to the one that they were given and then have them make one that is similar.

You will have more errors with the similar ones. That is okay. This is a chance for students to experiment in a hands–on visual way. Students can work in small groups so that they can discuss their thinking and hopefully do some self –correcting.

Provide rulers and pencils for students to work.

Alternative Assessment Options

You can assess student work by observing their group work and also by looking at the triangles that are created. Here are some questions to think about.

Did the student create a triangle that is exactly congruent?

Did the student create a triangle that is similar?

Classroom Modifications for Diverse Learners

For special needs students to be successful in this lesson, you will need to provide some scaffolding. The first thing to be conscious of is the triangles that you give out. Be sure that the side lengths are even measurements. Second, hand out rulers and remind the students that a congruent triangle is exactly the same.

Next, students will need to create a triangle that is similar. If you have given out triangles with even side lengths, then you will be able to create a similar triangle by dividing or multiplying by 2. This is the simplest way for special needs students to work on this activity. Many will be willing, but not know how to create a similar triangle. They won't understand where the dimensions for the other triangle come from. By teaching how to multiply or divide by two, you add steps to proportional reasoning which will help students to be successful.

Additional Resources

http://www.educationworld.com/a_tsl/archives/math.shtml –This website is a treasure trove of lesson plans and activities for teachers. There are games and lessons on all kinds of mathematical content across the grades. It is a wonderful resource.

Line Symmetry

Alternative Teaching Strategies

www.mathwire.com/problemsolving/patternblocksymmetry.pdf

This is a pdf about how to create symmetrical patterns with your students by using pattern blocks. Working with pattern blocks engages every learning style from hands–on learners to kinesthetic learners. Students can work independently or in pairs on creating different patterns with their blocks. To do this work, you will need a set of pattern blocks.

http://www.etacuisenaire.com/catalog/product?deptId=&prodId=740&q=pattern+blocks –Cuisinaire has a great set that can be used with students of all ages.

To work on the activity, ask students to draw a line down the middle of their page. Then they must create a pattern on one half of the line. Afterwards, ask them to trace the blocks and draw in the pattern. Then the students must recreate a mirror image of the pattern on the other side of the line.

To expand on this first step, students can also draw a line vertically and horizontally on their paper. This divides it into four quadrants. After doing this, students create a pattern in one corner and then must figure out how to mirror the pattern in the other three quadrants. This will be very challenging for some students.

Alternative Assessment Options

The key to this lesson is observation. Observe the students as they work and think about these questions.

Are the students able to create the symmetrical patterns accurately?

Were there students who found this difficult?

Why do you think he/she was challenged?

How could you have helped to make this simpler in the beginning?

Classroom Modifications for Diverse Learners

There aren't any modifications for this lesson.

Additional Resources

http://www.mathplayground.com/patternblocks.html –This is an interactive game where students use two –dimensional pattern blocks on the computer to explore symmetry and other geometric concepts.

Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Alternative Teaching Strategies

Begin by reminding students of the four problem solving steps. It is helpful to have these written on the board. To help differentiate this lesson, you can have some students work on the problem from the introduction. These would be your advanced students who are looking for a challenge.

Other students could be working on designing three or four examples of patterns. The patterns can be number patterns or patterns with shapes, but they should repeat in some way. At the end of the class, students will share these patterns with their peers. Challenge the students to come up with patterns that would be challenging for students to figure out.

Alternative Assessment Options

Assessment can be completed through discussion. Allow time for the students to share their work at the end of the

class. The students working on the problem from the introduction should be able to explain how they solved it. Students working on the patterns should share their number patterns.

Classroom Modifications for Diverse Learners

Special needs students should be encouraged to work on creating patterns.

Additional Resources

http://www.purplemath.com/modules/venndiag4.htm –This website has a great tutorial on how to use Venn Diagrams for solving word problems.

3.10 Geometry and Measurement

This tenth chapter *Geometry and Measurement* further covers geometric principles through methods for finding the area of parallelograms, triangles, and circles. The student is also introduced to the concepts of solid figures, surface areas, and volumes of prisms and cylinders. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Area of Parallelograms
- Area of Triangles
- Circumference of Circles
- Area of Circles
- Classifying Solid Figures
- Surface Area and Volume of Prisms
- Surface Area and Volume of Cylinders
- Problem-Solving Strategy: Solve a Simpler Problem

Area of Parallelograms

Alternative Teaching Strategies

One way to differentiate this lesson is to provide the students with graph paper to draw rectangles and then parallelograms. The connection between the area of a rectangle and the area of a parallelogram is a key concept of this lesson so you can begin by asking the students to draw 4 or 5 different rectangles using the graph paper. Then remind them of the formula for finding the area of a rectangle by writing it on the board. This will help to prompt visual learners. Be sure to use rulers so that the work is neat. Once students have drawn their rectangles, they should go ahead and find the area of each rectangle.

Then move on to parallelograms. The students will need to draw a line on the diagonal to complete the parallelogram. Have them work with the same rectangles as they just drew. Then the connection will be very clear as they transform the rectangles into parallelograms. Once this has been completed, write the formula on the board. Be sure that the students understand where the height of the parallelogram is compared to the slanted side. Then ask the students to find the area of each parallelogram that has been drawn.

The use of graph paper will also help you to remind the students that we write area in square units. Since they are creating figures using squares, this becomes very obvious in a visual way.

Alternative Assessment Options

When the students work on this activity, you can think about the following questions.

Did the students draw rectangles that were accurate and had straight lines?

Are the areas of those rectangles correct? *You can have the students ask a partner to check their work on each of the areas. Then students can correct any errors.*

Were the students able to accurately transform the rectangles into parallelograms?

Are the areas of those parallelograms accurate?

Classroom Modifications for Diverse Learners

Be sure that special needs students have the formulas written out for them in a key. Each formula can have a small picture of the figure that it is used with next to the formula.

$A = lw$ 

This is an example of a key that students would have in their notebooks. Working in this way will help to remind the students that they are looking for a rectangle shape when working with area.

An added challenge for advanced students would be to skip using the graph paper and use only a ruler. This will add an extra challenge of working accurately with a ruler to be sure that the measurements are exact.

Additional Resources

Area of Triangles

Alternative Teaching Strategies

Use the rectangles and parallelograms from the last lesson to work on this lesson on triangles and their areas. You can ask the students to draw them newly, but it is sometimes easier just to work on the ones that have been already created. Students will need colored pencils and rulers. Then ask the students to create two triangles by drawing a line down the diagonal of each figure. Now each parallelogram and rectangle will be in two triangles.

You can tell the students that $\frac{1}{2}$ will need to be part of the formula because one –half of each parallelogram and rectangle has become a triangle.

Then you can ask them to draw a square and show how a square can also be divided into two triangles.

Once you have given the students the formula, write it on the board. Then ask the students to find the areas of the triangles that have been created when the quadrilaterals were divided. Once this is finished, ask them to draw a few new triangles. You can either have them figure out these areas themselves or ask them to exchange papers with a partner.

Alternative Assessment Options

When assessing student work, here are a few questions to think about.

Do the students see how these figures can be changed into two triangles?

Do they understand where the $\frac{1}{2}$ in the formula for the area of a triangle comes from?

Are the diagonals straight and therefore the triangles accurately created?

Are the areas of each triangle accurate?

Were the students able to draw new triangles on their own with straight lines and accuracy?

Classroom Modifications for Diverse Learners

Continue to add to the key created in the last lesson. Each formula should have a picture next to it so that the students can visually connect the formula with the figure that it is used for.

When it comes to the drawing part of the assignment, you can continue to have students work on graph paper, or you can give them plain white paper.

If they draw on graph paper, ask them to draw the missing triangle that would complete the figure. For example, the student draws one right triangle on their paper. Well, if they were going to complete a square, for example, the students would need to draw the missing triangle too. This can be an added challenge for some students. They can

draw one triangle in one color and the missing triangle in another color.

Additional Resources

Circumference of Circles

Alternative Teaching Strategies

Circumference of circles is fun and many students really enjoy the concept and the exploration that can take place around it. A great way to do this is to bring some string and a piece of colored chalk outside. You can have on for each pair of students too. You are going to need some kind of pavement, so a parking lot that is not being used is perfect.

Once outside, tell the students that they are going to draw a circle. How can they do this with only a piece of string and a piece of chalk. The circle must be accurately drawn, so freehand won't work.

Someone in the group will figure out that the length of string will be the diameter, although he/she probably won't use that word. The circle is created by attaching the string to the chalk. One person stands in what will the center of the circle and holds an end of string. The other person uses the chalk to draw the circle.

When all circles are complete, examine them. Allow the students time to share about how they did what they did. Then go inside the classroom.

Now it is time to introduce the formula for finding the circumference of a circle. Brainstorm with the students HOW they could use their string to figure out the circumference of their circle. This exploration could take more than one day and also could require that students write about their work at home.

Alternative Assessment Options

Assess student work by looking at each circle. You will be able to see the accuracy of each by looking at it. Assessment for this lesson is completed through observation and discussion.

Classroom Modifications for Diverse Learners

If your class has many special needs students in it, be sure to explain what you are going to do prior to going outside. Then allow time for the students to ask questions. This will help the students to think through the process before they arrive outdoors and are given the chalk and the string.

Also, be sure to define a circle so that all students are clear.

Additional Resources

Circumference and the Dragon of Pi –a wonderful book you can use to teach circumference and circles to the students. Fantastic, imaginative and not to be missed!

Area of Circles

Alternative Teaching Strategies

To work on differentiating this lesson, you can work with the area of circles by first drawing circles on graph paper using a short string and a pencil. This is similar to the activity that the students completed when they were working on the circumference of a circle except this will be on graph paper. When the circles have been drawn, the students will see that there are partial squares and whole squares in the circle. The difficulty calculating the actual area will be obvious. This is the perfect place to introduce the formula to the students.

First, you can talk about the partial squares and brainstorm ways to measure the area. Then introduce the formula.

Alternative Assessment Options

When assessing student work, be sure to notice which students want to simply count the pieces and guess on the actual size of the partial squares. These are students who have difficulty with formulas and will probably need additional help. Other students will be looking for the formula.

Classroom Modifications for Diverse Learners

For special education students, the activity listed above is a great way to make the connection between the task of figuring out the area of a circle and the actual formula.

Be sure to review pi and what pi actually measures.

Additional Resources

http://www.youtube.com/watch?v=l0ddichTNLE –This is a great video on youtube on how to find the area and perimeter of a heart shaped cake. Since a heart isn't a circle, it teaches students how to combine figures to find the area of a irregular object. This also has some problem solving components and could be expanded into a supplemental class by itself.

Classifying Solid Figures

Alternative Teaching Strategies

When working advanced students who are in need of an additional challenge, you can differentiate this lesson by using books on sculpture and architecture. This activity will differentiate the lesson for all types of learners including those who are advanced in the area of mathematics.

After presenting all of the solid figures so that students can identify them, you can send the students on a "shape hunt" through architecture books. Go to the public library or the school library and take out all of the books that you can. Students will need to find examples of each of the solid figures in real world architecture.

Prisms

Cones

Pyramids

Cylinders

Spheres

The students should make a note of where they found each including a book title and a page number. Allow time for students to share their findings in small groups.

Alternative Assessment Options

Assessment can be completed through observation of student work. After the observation, be sure to look at each student's selection. Here are some questions to think about.

Did the student find each figure?

Are the choices accurate? For example is the prism really a prism?

Did the student follow directions and has he/she listed a book title and a page number for each figure?

Can the student readily identify each figure?

Classroom Modifications for Diverse Learners

For special needs students, build a key with a figure and some of its characteristics. Here are some suggestions.

Prism –rectangle sides always –base names type of prism

Pyramid –one point always –base names type of pyramid

Cylinder –circle top and bottom

Cone –rounded bottom and pointed top

Additional Resources

http://www.amazon.com/Exploring-Solid-Figures-Math-World/dp/0836893867 –Here is a book, "Exploring Solid Figures on the Web" to help students to identify and work with solid figures.

Surface Area and Volume of Prisms

Alternative Teaching Strategies

Giving the students accurate clear notes will help visual learners and special needs students. Clear notes that are "talked through" will also help those students who are auditory learners. Here are some suggestions for notes.

Surface area –total areas of all surfaces

–Draw each surface

–Be sure you have them all

–Check them off when you have each area

–when you have all of the areas, add them up

Volume –multiply to find the filling up

Units are cubed^3 because we don't fill with flat squares by cubes

Large B means find the area of the base

Alternative Assessment Options

Assessment for this lesson is done by checking the accuracy of student answers. When students have made an error, you can check and see if the error is the procedure. A common error of students is to forget to find the area of one side.

Classroom Modifications for Diverse Learners

When teaching surface area and volume to special needs students, you can help them by identifying key words.

Surface area means the covering of a surface

Volume means the filling up of a figure

Special needs students will need their notes on formulas for plane figures.

Additional Resources

http://www.teacherschoice.com.au/maths_library/area%20and%20sa/area_9.htm –This website has wonderful visuals and a good tutorial for students needing a review. It could be offered to students as a site to refer to for extra review.

http://www.learner.org/interactives/geometry/area_surface.html –This website has great hands–on ways to interact with prisms to find the surface area and volume of them.

Surface Area and Volume of Cylinders

Alternative Teaching Strategies

To differentiate this lesson, you can use nets. Then looking at the net, you can ask "What areas do we need to find?" The students will answer that they need to find the area of two circles and a curved or rounded surface. Once students have made this connection, you can introduce the formula for finding the surface area of a cylinder. This will make a real–world connection for them.

For a hands–on activity on volume, you will need a clear plastic spaghetti container and some unifix cubes. Ask the students to fill the container with unifix cubes. They will see that the volume can be measured in this way, but not accurately. Now you have the chance to show the students how to use the formula for finding the volume of a cylinder.

Alternative Assessment Options

Assessment is done through the discussion with the students. Be sure that the students understand where the formula for surface area and volume comes from. You will be able to tell by listening to how they explain WHY each part of a formula is used. Be sure to take the time to break down each formula and talk about this with the students.

Classroom Modifications for Diverse Learners

For special needs students, continue creating a key. You will need a picture of a cylinder with each formula for surface area and volume written next to the cylinder. This will provide the students with a visual prompt for each figure.

Additional Resources

http://www.basic-mathematics.com/surface-area-of-a-cylinder.html –This website has a great tutorial on surface area and volume of cylinders. It also goes over the same content as the lesson to help students to understand why each formula is the way that it is.

Problem-Solving Strategy: Solve a Simpler Problem

Alternative Teaching Strategies

To teach this lesson, you want the students to understand that they will need to take a problem and break it down into smaller steps. You can teach this as a recipe for solving a problem.

Here is a set of guidelines to help students to break down a problem.

- How many steps are there?
- Steps = # of numbers
- The number of steps = Column 1 on the table
- Values or numbers on each step = column 2 on the table
- Look for a Pattern –How does the number change on each step?
- Write the rule of the pattern?
- Solve the problem for the answer

Alternative Assessment Options

Assessment is done through observation of students in small groups. Check for accuracy of answers and the student's ability to explain HOW he/she found that answer. Did the students break down and look for a pattern or did he/she solve the problem in another way?

Classroom Modifications for Diverse Learners

For special needs students, set up the table for them.

TABLE 3.1:

Steps	Values

Then the students can use this table to fill in the steps and the values that correspond to each step.

Additional Resources

http://www.studyzone.org/testprep/math4/d/simplerproblem4l.cfm –A great series of problems to solve. They are from a fourth grade and could be used for special needs students or as a pretest for the lesson.

http://www.thefutureschannel.com/problem_solving/solve_simpler_problem.php –A great page of several different activities that require students to use the problem solving strategy: solve a simpler problem.

3.11 Integers

In this eleventh chapter *Integers* students will learn about positive and negative numbers, adding numbers, sums and differences of integers, multiplication, division, coordinates to name locations, transformations, and all about data displays. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Comparing Integers
- Adding Integers
- Subtracting Integers
- Multiplying Integers
- Dividing Integers
- The Coordinate Plane
- Transformations
- Surveys and Data Displays

Comparing Integers

Alternative Teaching Strategies

To differentiate this lesson on comparing integers, students can create and play a game like the card game "war". To do this, you will need to create a set of "playing cards". See the additional resources section for a list of possible values to compare. The students write each of these problems on index cards to make the game. Then they shuffle the card and put the cards face down. Each student playing (groups of 2 or 3) has a card with a >on one side and a <on the other side. Then the students flip a card from the top of the deck and the first student to put the correct sign into the blank spot wins the card. At the end you can count up the number of cards each person has and the one with the most cards is the winner.

Alternative Assessment Options

Assessment can be completed through observation. By walking around while the students are working on comparing the integers, you will be able to help the students and to observe which students are easily able to identify the correct sign for each pair of integers.

Classroom Modifications for Diverse Learners

Special needs students are able to play the game described above. A modification would be to put the >sign in red and the <sign in blue. This will help through color coding to remind the students which sign means which measure.

Additional Resources

Here is a possible list of problems for students to use to create game cards.

6 ____ -3

-5 ____ -15

4 ____ -9

9 ____ 11

16 ____ -18

-11 ____ -5

-9 ____ -19

-3 ____ 5

-10 ____ -11

11 ____ 12

-19 ____ 10

-5 ____ 16

-11 ____ 20

-20 ____ 15

-15 ____ -25

-45 ____ -47

-6 ____ -9

-8 ____ -12

-15 ____ 7

You can also write more problems given the size of the class or have the students design their own too. This could be a possible homework assignment for the night before the game is played.

Adding Integers

Alternative Teaching Strategies

To differentiate this lesson, you can take the number line content from the text and make it movable. To make it movable, students will be moving along the number line as they add each of the integers together. Then they will be able to see the actual sum as they move. See the additional resources section for a list of possible problems and solutions.

The number line can be made out of colored electrical tape and should contain values from -20 through +20. Then the students will be able to move the addition problems. This is great for every learning style and is a lot of fun for the students.

Alternative Assessment Options

Students can be observed as they add on the number line. Through observation you will be able to see which students are able to arrive at an accurate sum and which students are not. The students who struggle with this task will be obvious because they will need help finding each sum.

Classroom Modifications for Diverse Learners

If your class has many special needs students, then it might be best to have you make two number lines. Then the students can be paired up and work together to find each sum. One student will be walking the sum on the number line and other student can be coaching. Then the students can switch places.

Additional Resources

Here are some possible problems for the number line. You can decide which students receive which problems based

on ability. This allows you to tailor the problems given to the needs of the students in the class.

$6+-3=3$

$4+-9=-5$

$16+-18=-2$

$11+-5=6$

$-9+-9=-18$

$-10+-8=-18$

$6+-9=-3$

$10+-11=-1$

$-19+10=-9$

$-5+16=11$

$-11+20=9$

$-20+15=-5$

$-6+-9=-15$

$-8+12=4$

$-15+7=-8$

Subtracting Integers

Alternative Teaching Strategies

Working with the same number line or number lines, you can work with the students to help them to figure out differences. Figuring out differences is always tricky because the students have to "take away" a value, sometimes that value can be negative and sometimes positive. Students will need to think on their feet to work through each problem and come up with accurate sums.

There are four different types of problems students can work to figure out. The first type is positive taking away a positive, then negative taking away a negative and finally positive taking a way a negative or negative taking away a positive.

Alternative Assessment Options

Students can be observed as they subtract on the number line. Through observation you will be able to see which students are able to arrive at an accurate difference and which students are not. The students who struggle with this task will be obvious because they will need help finding each difference. You will also see if the students are able to add the opposite in their heads or whether or not they will need paper.

Classroom Modifications for Diverse Learners

If your class has many special needs students, then it might be best to have you make two number lines. Then the students can be paired up and work together to find each difference. One student will be walking the problem on the number line and other student can be coaching. Then the students can switch places.

You can also provide small white boards or chalkboards for the students to add/subtract on.

Additional Resources

Here are some possible problems for students to solve on the number line.

$-2-4=-6$

$-8-2=-10$

$-5-7=-12$

$-9-1=-10$

$-2-9=-11$

$-3-12=-15$

$-6--4=-2$

$-8--4=-4$

$-9--10=1$

$-4--12=8$

$-6--6=0$

$-5--10=5$

$-6--13=7$

$4--8=12$

$3--4=7$

$9--5=14$

$6-20=-14$

$5-10=-5$

$6-8=-2$

$3-12=-9$

$10-11=-1$

$9-11=-2$

Multiplying Integers

Alternative Teaching Strategies

Once students understand the rules of multiplying integers, the rest is very straightforward. However, students may struggle when it comes to evaluating numerical expressions where more than one operation is being used. Let's look at this example from the text.

Example

$(-3)(-5)+(-2)(7)$

This operation requires students to have two operations, first they have to multiply and then they need to add.

Remind the students of the order of operations. You can review this and write it on the board. Then help the students to work through the multiplication and then the addition.

$15+-14=1$

Here is the final answer.

By creating visual reminders and by going through each step verbally, you will help students to remember how to

solve the problems because you will be addressing different learning styles.

Alternative Assessment Options

Assessment is done by correcting student work. Only through accuracy can you determine which students understand the material and which students still need additional help.

Some students may make errors because they don't know all of their times tables. Continue to help all students to reinforce and learn these tables.

Classroom Modifications for Diverse Learners

Be sure that the students have the rules for multiplying integers written down in their notebooks in a place where it can be easily located. This would be a great place for students to put a tab where they could easily find and turn to the page where the rules were written. Color coding these rules can also be helpful for students.

You will need the rules for multiplying integers to evaluate expressions.

$$\textbf{Positive} \times \textbf{positive} = \textbf{positive}$$
$$\textbf{Negative} \times \textbf{positive} = \textbf{negative}$$
$$\textbf{Negative} \times \textbf{negative} = \textbf{positive}$$

Notice that the rules are written here with positive in red and negative in blue.

Additional Resources

http://www.aaamath.com/mul65_x2.htm –This page has a great tutorial for students on multiplying two and three integers together.

http://www.xpmath.com/forums/arcade.php?do=play&gameid=47 –This is a math game where students practice multiplying integers.

Dividing Integers

Alternative Teaching Strategies

Differentiate this lesson by asking the students to write division problems in different ways. You want to help the students to identify the fraction bar as meaning division and the division sign as meaning division too. Ask the students to each write five different problems. Each problem will need to written in two different ways.

Alternative Assessment Options

When students write their five problems, you will be able to tell who is understanding the use of the fraction bar because they may put the wrong value as the numerator or the denominator. Watch out for this.

For example: $12 \div 3 = 4$ This is a first example created by a student.

$\frac{3}{12} = 4$ This is the second example. But this problem is written incorrectly. The student is confused about which is the divisor and which is the dividend. You can watch for these errors and help students to correct their work.

$\frac{12}{3} = 4$ This is the correct answer.

Classroom Modifications for Diverse Learners

Be sure that the students have the rules for dividing integers written down in their notebooks in a place where it can be easily located. This would be a great place for students to put a tab where they could easily find and turn to the page where the rules were written. Color coding these rules can also be helpful for students.

You will need the rules for dividing integers to evaluate expressions.

$$\text{Positive} \div \text{positive} = \text{positive}$$
$$\text{Negative} \div \text{positive} = \text{negative}$$
$$\text{Negative} \div \text{negative} = \text{positive}$$

Notice that the rules are written here with positive in red and negative in blue.

Additional Resources

http://worksheetplace.com/index.php?function=DisplayCategory&showCategory=Y&links=4&id=132&link1=40&link2=46&link3=25&link4=132 –This website has many worksheets for students to use to practice dividing integers.

http://www.ixl.com/math/grade-6/divide-integers –This website has an interactive game for students on dividing integers. Fun and easy to follow!

The Coordinate Plane

Alternative Teaching Strategies

A way to differentiate this lesson is to create a coordinate grid transparency that can be used by the students for practice. To do this, you create a transparency of a coordinate grid for each student in the class. Then the students are given two different colors of wipe –off markers and a rag to work with. As the problems are presented in the lesson, you allow time for the students to work out each solution the coordinate grid transparency. In this way, the students are given a hands–on interactive way to work with the content of the lesson. Students can use these coordinate grids with the examples in the text. Additional problems aren't needed.

Alternative Assessment Options

Student understanding can be assessed by listening to student answers. If a problem is posed to the students, for example, graph the point (-9, 2), the student is given the chance to graph it one their transparency, then you can ask, "Which quadrant is the answer in?" The students will answer the question. By listening to their answers, you will be able to determine if the student understands how to plot a point and if the student is able to identify each quadrant accurately."

Classroom Modifications for Diverse Learners

To modify this lesson for students, provide them with wipe –off markers in at least two different colors. Students can graph points in different quadrants with the different colors.

Additional Resources

http://www.mathwire.com/geometry/coordgeom.html –This is a fun game where the students have to "capture the penguins" by using coordinate geometry. There are pdf downloads that can be used. This is fun interactive way to learn about coordinates and graphing.

Transformations

Alternative Teaching Strategies

To work with the content of this lesson, you can once again use the transparencies of the coordinate grids that were created in the last lesson. The students are given transparencies of a coordinate grid, wipe off markers and a rag to work with. Then the content of the lesson is taught. As you teach each part of the lesson, you can make the

examples interactive by having the students work at their seats to learn how to graph each transformation. Students can correct their work easily and you can work along side them on a transparency on an overhead projector or through a computer shown large on the wall. A smart board is another way to show each example.

Each example from the lesson is taught in this way. Then the students work with the example and finally an answer is completed. The students should work hand in hand with the teacher and not be looking at their textbooks for this lesson. This will keep the students engaged in the process of exploring each type of transformation.

Alternative Assessment Options

Assessment is done through observation. You can observe students as they work on each example from the lesson. By walking around as the students work, you will have the chance to see how each student is doing and make corrections or suggestions.

Classroom Modifications for Diverse Learners

To modify this lesson for students, provide them with wipe –off markers in at least two different colors. Students can transformations in different quadrants with the different colors.

Additional Resources

http://www.saskschools.ca/~math68im/strands/algebra/coordinate_plane.html –This is a coordinate plane activity sheet with a transparency connected to it.

Surveys and Data Displays

Alternative Teaching Strategies

Talk through the problem in the introduction. Ask the students to think about how they could conduct a survey. You can put the students in small groups and ask them to come up with a survey question that they would like to ask.

Surveys are fun, and students can learn a lot by conducting an actual survey. You may choose this as a hands–on way to teach the content.

Then if you do conduct a survey, assign small groups of students different data displays. Ask the groups to use the survey data to create each data display. Allow time for students to share their work with the class.

Alternative Assessment Options

You can think about the following questions as students work.

Is the question relevant to the students?

Is it written in a way that isn't biased?

Did the students conduct the survey accurately?

Do the students understand the data collected?

Does the data display accurately display the data?

Are the students able to share about their data display and how it is connected to the data from the survey?

Classroom Modifications for Diverse Learners

Use the notes from the lesson and help the students to make a list of each type of graph and when it is best used. Then you can have the students create a small drawing of each graph next to the description. This will help the students to make a visual connection with each type of graph.

Additional Resources

http://www.survey.jokai.net/ –This website has an example of a survey conducted by a sixth grade math class in

Indiana. A good resource for working on surveys.

3.12 Equations and Functions; Probability

In this twelfth chapter *Equations and Functions; Probability* students will begin learning all about expressions and equations. They will also learn about solving equations, single-variable equations, graphing functions, probability, and how to calculate the probability of independent events. In this flexbook, titled Differentiated Instruction, you will be given teaching strategies to help reach a diverse community of learners. There will be suggestions for students with special needs as well as those with different learning styles. Alternative assessment options will also be suggested.

Lessons

The following lessons are part of this chapter.

- Writing Expressions and Equations
- Solving Equations Using Addition and Subtraction
- Solving Equations Using Multiplication and Division
- Functions
- Graphing Functions
- Introduction to Probability
- Finding Outcomes
- Probability of Independent Events

Writing Expressions and Equations

Alternative Teaching Strategies

To differentiate this lesson, you can take strips of paper or index cards for the students to use. This is a kinesthetic matching game. On one card, you write a verbal sentence and on another card you write the expression that matches. Then you shuffle the cards. The cards are handed out randomly, and each student must find the other student who has the matching card. Then the cards can be reshuffled and the students can play again.

As an advanced step, students can create the cards themselves. To do this, each student writes a verbal sentence and a matching expression. You collect the cards and correct an inaccuracies before the game is played.

Alternative Assessment Options

Assessment is completed through observation. You will be able to watch which students can easily find their matching partners and which students are in need of more assistance.

Classroom Modifications for Diverse Learners

Help special needs students to remember key words that indicate the different operations by creating a special place in their notebooks for the key words.

Addition

Sum, plus, altogether, plus, and

Subtraction

Difference, less than, subtract, take away

Multiplication

Product, times, groups

Division

Split up, quotient, divided

Then ask the students to include the steps needed for working through a number sentence.

When deciphering phrases, you will be looking for the same things as the last section.

1. **Identify any numbers**
2. **Identify the operation involved**
3. **Identify the variable**

Additional Resources

http://www.algebralab.org/studyaids/studyaid.aspx?file=Algebra1_1-6.xml –This website has many examples of verbal expressions that can be written as numerical expressions.

Solving Equations Using Addition and Subtraction

Alternative Teaching Strategies

Combining like terms is the first part of the lesson where you can differentiate student work. Begin by writing a bunch of terms on different index cards or small cards. Be sure to use three different variables $x, y,$ *and* z. Then hand out the cards to the students. They need to find the other like terms and figure out the sum of the terms. You can also have them figure out the difference of the terms. This may result in a negative coefficient, but it will definitely have the students thinking about like terms and unlike terms.

Alternative Assessment Options

Assess student learning through the accuracy of their answers on the *Time to Practice* problems. Solving equations is simply that –students must learn to solve equations. The accuracy of their answers is the best way to assess student understanding.

Classroom Modifications for Diverse Learners

For special needs students, you can also teach them to check their work. If their answer does not work in the original equation, then they have made a mistake. Teaching students to check their work with equations is a wonderful way to ensure accuracy in higher grades.

Additional Resources

Here are some possible terms for the activity described above.

$5x$
$15x$
$7x$
$-9x$
$-6xy$
$25xy$
$18xy$
$9y$
$13y$
$-11y$
$7z$
$-8xyz$
$-12z$

You can add a variety of more terms and include exponents as well to really challenge some of the students.

Solving Equations Using Multiplication and Division

Alternative Teaching Strategies

When differentiating this lesson, keep in mind that the students will need a strong grasp of their multiplication tables to be successful. If you have students who have special needs or are weak in their tables, take the time to work with them on these. The students will need to know them well so that they can work forwards and backwards on solving equations.

When working on this lesson, you can have students work in small groups on solving multiplication and division problems. Allowing students the chance to work together will help them to solve problems successfully.

Remind students that they need to complete the check of their answers as is taught in the lesson in the text.

Alternative Assessment Options

Assess student learning through the accuracy of their answers on the *Time to Practice* problems. Solving equations is simply that –students must learn to solve equations. The accuracy of their answers is the best way to assess student understanding.

Classroom Modifications for Diverse Learners

When completing multiplication problems, review times tables with the students. For those students who are still struggling with their tables, you may want to provide them with a multiplication chart while you continue to work with them on learning these tables. Remind the students that in multiplication that terms do not have to be alike to be multiplied together.

When working on simplifying division problems, encourage students to cross out terms that they are cancelling in different colors. This is one way to be sure that students don't become mixed up.

Additional Resources

http://www.studygs.net/mathproblems.htm –This website has a great page of notes for students on how to solve math problems involving equations.

Functions

Alternative Teaching Strategies

One of the ways to differentiate this lesson is to write a bunch of different operations that could be in an input/output machine. Write these on index cards (see examples). Then you give the students two dice and put them in groups of four. Each group has two teams that play together.

First, the teams roll the dice. Then they flip a card and each team has to take the dice roll, the input and come up with an accurate output. The team that does it first wins a point. The game can continue until you decide it is done.

A variation on the game is that the students roll a die five or eight times and have to use the same operation card to build a table. Then they pass their table of values to a neighboring group. The group has to figure out the rule.

For example: rule $\times 3$

Here are possible dice rolls and the table created by the team.

TABLE 3.2:

Input	Output
4	12
12	36
3	9
5	15
9	27

Alternative Assessment Options

Observation is an excellent way to assess student learning. You will see them work with the input/output game and be able to assess student understanding.

Classroom Modifications for Diverse Learners

For special needs students, allow extra time for calculating each output. You may want to have special needs students focus on accuracy and not speed. You can alter the activity to be one about accuracy rather than speed of completion.

Additional Resources

http://www.shodor.org/interactivate/activities/NumberCruncher/ –This is an interactive game where students have an input and then an output is created through the operation on the game.

Graphing Functions

Alternative Teaching Strategies

In the last lesson on Functions, the students rolled dice and created tables based on a given operation. These tables are going to be used in this lesson. We want the students to see the connection between a table of values and a function graph. These first graphs will not necessarily be linear graphs and that is alright. If the students graph the function and do the vertical line test, then they can determine if the graph is a function or not.

Students will need a transparency of a coordinate grid, a ruler and a wipe off marker along with the tables that they created in the last lesson. You can allow the students the chance to choose which table they want to graph. Then you can introduce the input as the x value and the output as the y value. Now the students will be able to graph the functions.

After the graphs have been created, you can ask the students to determine whether each graph is a function or not. They can do this work in small groups and then bring the conversation to the whole class through a large class discussion.

Alternative Assessment Options

You can assess student learning through observation. Walk around as the students are graphing these functions. Be sure that the graphs are accurate and that the students know how to connect each point on the graph.

Classroom Modifications for Diverse Learners

A modification for special needs students would be to assign them a specific table to graph. This will ensure that the vertical line test will work and that the students will be able to distinguish the function.

TABLE 3.3:

Input	Output
1	3
2	6
3	9
4	12

For advanced students, you can ask them to create table for a set of values that they think would not be a function. Then ask them to graph these values and prove that it is not a function.

Additional Resources

Key Curriculum Press has a great book called "The Pattern and Function Connection" for middle school students. This is a great hands–on program that works systematically with functions. An excellent supplement to any textbook.

Introduction to Probability

Alternative Teaching Strategies

To differentiate this lesson, create a bunch of games with the students using dice and spinners. You will need to divide the students up into groups and then give each group two or three dice and a spinner. Then ask the students to generate a list of probabilities that they can calculate. You can do this as a large class discussion and make the list of probabilities to investigate on the board.

Then let the students go to work. Be sure that each question from the board is written down on a piece of paper and that the students write down their probabilities as well. Finally you can discuss the findings of the group in a large class discussion.

Alternative Assessment Options

Observation is the best way to assess student learning. You can walk around as the students explore each question and listen to each conversation. Be sure to point out accuracies and inaccuracies in the class discussion at the end of the lesson.

Classroom Modifications for Diverse Learners

Make each group a multi–leveled group. In this way, students can help each others and through discussion and experimentation special needs students will have a better grasp of the material.

Additional Resources

http://www.keypress.com/x5736.xml –This is a great Probability unit from Key Curriculum Press geared for middle school and high school students. It is engaging and could be a great supplement of activities for any classroom.

Finding Outcomes

Alternative Teaching Strategies

As you work on this lesson, tree diagrams are a wonderful way for students to express their creativity and artistic skills. After teaching the content in the lesson, you can differentiate it for different learners styles by engaging the students with this hands–on interactive activity. First, you will need to put the students into groups.

Each group will need to write a problem that would require a tree diagram to solve it. On one sheet of paper, they will write an illustrated story problem. On a second sheet of paper, they will need to write the solution and illustrate it through a tree diagram.

Then collect the problems and pass out the problems to different groups. Each group works on solving the problem and then goes back to the original authors for the solution. Students should be encouraged to draw, talk and be able to explain their tree diagram and how it solves the problem.

Alternative Assessment Options

You can assess student learning by looking at their problems and tree diagrams. Here are some questions to think about.

Did the students write a problem that can be solved with a tree diagram?

Is the problem written clearly?

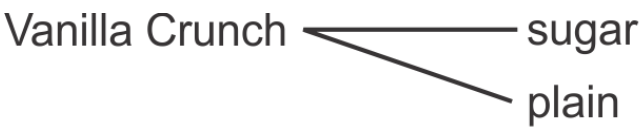
Is it illustrated?

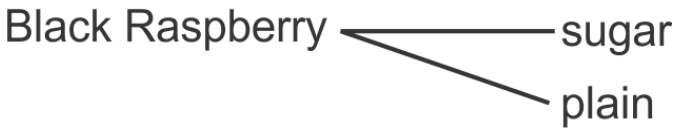
Is the solution accurate?

Does the solution clearly explain the answer through a tree diagram?

Classroom Modifications for Diverse Learners

For special needs students, you can highlight each level of the tree diagram in different colors. This will help students to keep each of the categories and options straight. Look at this example.





In this tree diagram, Each flavor would be in one color and each of the two options in another color. Students with organizational challenges will be helped a lot by color coding.

Additional Resources

http://www.keypress.com/x5736.xml –This is a great Probability unit from Key Curriculum Press geared for middle school and high school students. It is engaging and could be a great supplement of activities for any classroom.

Probability of Independent Events

Alternative Teaching Strategies

To differentiate this lesson, you are going to use this example from the lesson to turn a written example into a hands–on project.

There are 9 marbles in a bag. There are three blue, three green and three orange. What is the probability of selecting one blue marble, then putting that one back and selecting one green marble?

You will need to divide the students into groups. Each group will need a bag with 9 colored marbles in it. Be sure to follow the colors from the example. Then the students will actually explore calculating probabilities by actually doing each experiment. Here are some possible explorations.

Probability of selecting one blue and then two orange?

Probability of selecting two orange and one red?

Probability of selecting two green and one blue?

Probability of selecting two orange alone?

Probability of selecting one orange, then one blue and then one green?

Alternative Assessment Options

Observation is the best way to assess student learning. Walk around and be sure that the students are accurately calculating the probabilities. Allow time for the students to share their answers at the end of the lesson.

Classroom Modifications for Diverse Learners

Special needs students may need some help with the actual calculations. Be sure to arrange groups so that different levels of students are working together.

Additional Resources

www.teachinteract.com/pdf/INT872EX_ProbabilityandStatistics.pdf –This is a supplemental unit from Interact which could be used to help students to understand the content in a new way.

CHAPTER 4

Grade 6 - Enrichment

Chapter Outline

4.1 Number Sense and Variable Expressions

This first chapter *Number Sense and Variable Expressions* takes students from whole number operations into the world of beginning Algebra. In this FlexBook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Operations with Whole Numbers
- Whole Number Estimation
- Powers and Exponents
- Order of Operations
- Variables and Expressions
- A Problem-Solving Plan
- Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Operations with Whole Numbers

Lesson Project

This lesson focuses on the four operations and whole numbers. This project will combine the skills that the students are learning on the four operations with the theme of the lesson concerning animal eating habits at the zoo.

The purpose of the activity is for students to use the four operations to figure out the eating habits of an animal at a zoo. You can have the students choose this animal themselves and research using the internet or books. See the Lesson Extension for another option as well. The following could be used to create a worksheet or project sheet for students.

Project Outline

Volunteering at the Zoo

Imagine that you are going to be volunteering at a local zoo. Like Jonah, you will be investigating and using mathematics to figure out the eating habits of an animal at the zoo. You can choose an animal that you would like to investigate. Using the internet and books, figure out the eating habits of your animal. Be sure to keep track of your sources and list them in the place provided.

Part One: Data Collection

Animal Name:

Resource Investigated:

Type of Food Consumed:

Amount of Food Eaten Per Day:

Amount of Food Eaten Per Week:

Amount of Food Eaten Per Month:

Part Two: Writing It Up

Now that you have gathered all of this information, write a paragraph that describes your animal and its eating habits. Be sure to check your spelling and use complete sentences. Include a drawing of your animal in a habitat at the zoo.

Portfolio Assessment

When assessing student work, there are two different parts of this project to assess. The first one involves the information gathered. The second part is the writing piece. Be sure that student work is accurate and clear. When assessing student work, here are some questions to keep in mind.

Did the student complete the questionnaire?

Is the arithmetic accurate?

Did the student understand and use the correct operation?

Is the writing neat and clear?

Did the student include all of his/her data?

Multimedia Links/Additional Activities

- http://www.khanacademy.org/video/adding-whole-numbers-and-applications-1?playlist=Developmental+Math –This is a video on adding two digit whole numbers.
- http://www.khanacademy.org/video/subtracting-whole-numbers?playlist=Developmental+Math –Subtracting whole numbers
- http://www.khanacademy.org/video/level-4-division?playlist=Arithmetic –Dividing Two Digit numbers into a larger number.

Whole Number Estimation

Lesson Project

The project for this lesson will require that students use the estimation skills taught in the lesson. Students will be using the theme from the lesson. The student in the story problem at the beginning of the lesson has to estimate the amount of food needed for the penguins. In this project, students will select an animal and estimate the amount of food that the animal will eat. After completing this step, the students will figure out the actual food amounts and compare the data with the original estimation.

Project Outline

Part One: Data Collection

Animal Name:

Resource Investigated:

Type of Food Consumed:

Amount of Food Eaten Per Day:

Estimate the Amount of Food Eaten Per Week:

Estimate the Amount of Food Eaten Per Month:

Part Two: Writing It Up

Now that you have gathered all of this information, write a paragraph that describes your animal and its eating habits. Be sure to check your spelling and use complete sentences. Include a drawing of your animal in a habitat at the zoo.

Part Three: Actual Data

Now you will need to check on your estimate by completing the actual food amounts. To do this, go back to the

actual amount of food eaten per day by your animal. Then figure out the actual amount of food eaten per week and the actual amount of food eaten per month. Is your estimate reasonable? Why or why not?

Portfolio Assessment

When assessing student work, there are two different parts of this project to assess. The first one involves the information gathered. The second part is the writing piece. Be sure that student work is accurate and clear. When assessing student work, here are some questions to keep in mind.

Did the student complete the questionnaire?

Did the student estimate during Part One?

Are the estimates reasonable?

Does this student understand the value of estimating?

Is the work in Part Three accurate?

Did the student understand and use the correct operation?

In Part Two, is the writing neat and clear?

Did the student include all of his/her data?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=lNfZQNWZklI –This is a Khan Academy video on using rounding to estimate sums.

http://www.youtube.com/watch?v=bSemNdW9_wE –This is a video on estimating sums and differences.

http://www.youtube.com/watch?v=w_4VDQtESxs –This is a video on estimating products and quotients using multiplication and division.

Powers and Exponents

Lesson Project

This project uses the theme from the story problem in the lesson. Students will be assigned the task of using the information in the story problem to draw and label the different tiger cages. Since scale has not been introduced yet, you can use large sheets of paper (chart paper) and give the students a scale of $1'' = 1\ foot$. This way the students can use a simple corresponding measure to draw their cages.

Advanced students could be given a scale of $\frac{1}{2}'' = 1\ foot$.

Project Outline

Cage Design

In this project, you will be using the information in the lesson to draw different tiger cages. Use the questions provided below to guide you.

Scale $= 1'' = 1\ foot$

Part One: Information

What are the dimensions of the first cage?

What are the dimensions of the second cage?

Part Two: Drawing

Now use what you have learned to draw two different cages. The length, width and height of each cage should be

labeled in the drawing. Be sure to use a ruler and to label neatly. You can even include a drawing of a tiger in or next to one of the cages. Be sure that your work is accurate and neat. Use the scale provided or choose one of your own. If you choose your own scale, be sure to check it with the teacher before drawing.

Portfolio Assessment

Here are some questions to keep in mind as you assess student work.

Does the student have the accurate measurements for each cage?

Did the student follow directions?

Are there two cages drawn neatly with a ruler?

Are the measurements accurate and labeled?

Did the student use the scale correctly?

Did the student create his/her own scale and implement it accurately?

Does the design show careful work that represents the student's best efforts?

Multimedia Links/Additional Activities

- G is for Googal by David M. Schwartz is a great book that explains many different mathematical terms. It does explain exponents and powers within its pages and presents the material in a fun engaging way.
- http://www.youtube.com/watch?v=0GAMbuPJGOY –This is a James Sousa video on properties of exponents.
- http://www.youtube.com/watch?v=KOnQpKSpVRo –This is a James Sousa video on evaluating exponents and writing exponents in exponential notation.

Order of Operations

Lesson Project

This project will use the same number sentence as the one presented in the story problem. In the problem, Keisha is trying to figure out bird values by using the order of operations. She comes up with a number sentence to describe the bird counts.

$256+3\times5-2+3$

In this problem, she tries to figure out the total count of birds and comes up with 1296 because she doesn't use the order of operations.

In this project, students will need to insert parentheses so that the number sentence actually does equal 1296. Then they will write a story about how this became the total number of birds in the aviary.

Here is the solution:

$(256+3)\times5-2+3$

Here is a possible story line:

Keisha arrived at the zoo to find that the new count was 256 birds. That very morning three more birds were born. Over the next year, the number multiplied by five times the original total that Keisha counted on the first day. But two birds had died and three more were born after that.

This is only one possible variation.

Project Outline

Bird Counts at the Aviary

This project will use the same number sentence as the one presented in the story problem. In the problem, Keisha is trying to figure out bird values by using the order of operations. She comes up with a number sentence to describe the bird counts.

$256+3\times5-2+3$

In this problem, she tries to figure out the total count of birds and comes up with 1296 because she doesn't use the order of operations.

Imagine that you are Keisha, except this time, rather than coming up with a number much smaller than 1296, your bird count actually IS 1296. Where will the parentheses need to go for this to be true.

Part One

In this first part, add the parentheses to the equation so that it is a true statement.

$256+3\times5-2+3=1296$

Part Two: Story Time

Now write a story that describes the number sentence that you created above. Be sure to group the values in the correct way so that the total number of birds in the story ends up being 1296. You can illustrate your work as well.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student put the parentheses in the correct place?

Does the story match the operations in the number sentence?

Does the student understand the purpose of parentheses?

Does the student understand the order of operations?

Are there corrections that need to be made to the student's story?

Would peer correction make a difference to this student?

Are there revisions needed?

Is the work neat, accurate and shows the student's best work?

Multimedia Links/Additional Activities

- http://illuminations.nctm.org/Lessons/EveryBalance/Balance-OVH-OoOPSolutions.pdf

This is from an NCTM activity on the order of operations. It provides students with a list of questions about the order of operations.

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L730 –Here is an illuminations extension activity from NCTM for order of operations bingo. Fun for all!
- http://www.youtube.com/watch?v=_tHQgBt1KQ4 –This is an introductory video on the order of operations by James Sousa.

Variables and Expressions

Lesson Project

A possible project for the students to work on is a project where they need to write different expressions with variables and create a number game using a set of die.

Students will be required to write ten expressions. The first three will be given to them. After writing each number sentence, the students will write them on index cards, then they will create the game using the directions. Students can play in groups of three or four.

Project Outline

Variables and Expressions Game

To create the game, begin by writing seven additional expressions to go with these first three. Use a combination of the variables x and y. Then write your expressions on index cards.

1. $12x - y$
2. $\frac{12}{a} + 4$
3. $5x + 3y$

Now write each expression on an index card.

To play the game, you will need a single die and a small group of three or four people. The object of the game is to evaluate the expression first and correctly.

Here's how to play.

The index cards are placed face down. Then one student rolls a die, the first card is flipped over and the challenge begins.

Work as quickly as you can to solve the equation. Accurate work earns a point.

Portfolio Assessment

Assess student work through observation.

Are the expressions accurate?

Is the student work accurate?

Ask students to keep track of their answers and then check student work with the students.

Multimedia Links/Additional Activities

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L291 –This lesson provides an introduction to the use of variables, and develops students' algebraic thinking. Students create variable machines to discover the value of words.
- This lesson was adapted from a lesson by Gilbert Cuevas and Karol Yeatts, as found in *Navigating Through A lgebra in Grades 3 -5* , from the *Navigations* Series, NCTM 2001.
- http://www.youtube.com/watch?v=NI95DpVZX3Q –This is a great video by James Sousa on how to write variable expressions.

A Problem-Solving Plan

Lesson Project

A possible extension for this lesson and the next one is to investigate animal adoption. There are many possible ways for children to adopt an animal. This could also be adopted as part of a class project. To do this, students will need to first choose an animal to adopt and one could be adopted by the entire class. Then they will need to figure out a way to raise money such as a bake sale or car wash.

http://www.worldwildlife.org/gift-center/Default.aspx –This website could be used as a research site for students.

Project Outline

Directions: Follow each step of this project. Some of it will involve sharing your ideas with your classmates. Then come up with a plan to support an animal.

1. Research three different endangered animals that you are interested in having the class support financially. Write down the three animals below and write down any websites where you found information on adopting them.
2. How much would it cost to support each of these animals? Research some information about each and make notes below.
3. Share your findings with the class.
4. At this point, the class should vote and select one animal to support. As a class, decide on a goal for an amount of money that you would like to send to support the animal. Write that amount here.
5. Brainstorm three different ways to raise money.

Now it is time to put the plan into action. As a final step, write about how much money was raised and how it was raised. Make notes on successes or struggles. Share about how you felt during this project.

Portfolio Assessment

To assess this project, think about the following questions.

Did the students work together to solve problems?

What mathematical skills were used?

What were the struggles?

What were the triumphs?

Were the students able to articulate their process in words?

Is the arithmetic clear and accurate?

Does each write up represent the student's best work?

Multimedia Links/Additional Activities

As an extension, you could assign the students an early form of numbers to investigate. For example, the video mentions the Babylonians. You could have the students research or learn about the Babylonians from a book. Then they could write their ages in Babylonian writing. This writing of early numbers could be included in their portfolio.

Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Lesson Project

Expanding on the animal adoption, the students would now need to use mental math to problem solve figuring out how much money could be donated. This can work in a couple of different ways. First, before raising any money, students would estimate how much they think they can earn. Then they have the event and then count the money raised. Finally, they can compare the original estimate with the actual sum. All of this is a real –world application of the content of this chapter. Finally, the money is donated to help an animal in need.

Project Outline

Directions: Follow each step of this project. Some of it will involve sharing your ideas with your classmates. Then come up with a plan to support an animal.

1. Research three different endangered animals that you are interested in having the class support financially. Write down the three animals below and write down any websites where you found information on adopting them.

2. How much would it cost to support each of these animals? Research some information about each and make notes below.
3. Share your findings with the class.
4. Write down a goal that you would like to see your class achieve with regard to raising money.
5. After fundraising, report on your findings. Did your class reach the desired goal? Why or why not?
6. If you were going to do this again, what would you do differently next time?

Portfolio Assessment

To assess this project, think about the following questions.

Did the students work together to solve problems?

What mathematical skills were used?

What were the struggles?

What were the triumphs?

Were the students able to articulate their process in words?

Is the arithmetic clear and accurate?

Does each write up represent the student's best work?

Multimedia Links/Additional Activities

- http://www.mathstories.com/strategies_guesscheck.htm –This website has math stories that use the problem solving method –guess and check
- http://www.teachingideas.co.uk/maths/contents_mentalmaths.htm –This is a website that has a lot of additional ideas for teaching mental math to students.

4.2 Statistics and Measurement

In this second chapter, *Statistics and Measurement*, students become familiar with measurements. Also covered are finding the perimeter and area of rectangles, frequency, creating and understanding graphs, and an introduction to the mean, median, mode, range, and central tendency. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Measuring Length
- Perimeter and Area
- Scale Drawings and Maps
- Frequency Tables and Line Plots
- Bar Graphs
- Coordinates and Line Graphs
- Circle Graphs and Choosing Displays
- Mean, Median and Mode

Measuring Length

Lesson Project

This project is a hands–on measurement project designed to assist students as they learn to apply different measurements to in school items. It begins with the students identifying ten different things that they wish to measure. They list the items and then choose a tool which would best measure that item. Then they estimate the given length. Finally, they actually measure the item.

Allow time for students to share their work.

Project Outline

Measuring Out and About

Directions: To complete this project, you will be measuring different items that you can find in your classroom or around your school. List ten items that you would like to measure. Then make a note of the best tool to measure each item. Next, estimate what you think the length of the item will be and finally, measure the item and record its length.

TABLE 4.1:

Item	Tool	Estimate	Actual Length
1.			
2.			
3.			
4.			
5.			
6.			

TABLE 4.1: (continued)

Item	Tool	Estimate	Actual Length
7.			
8.			
9.			
10.			

Portfolio Assessment

Here are some questions to think about as you assess student work.

Did the student follow the directions?

Is each column of the worksheet complete?

Did the student choose the appropriate measurement tool?

Did the student make a reasonable estimate?

Did the student accurately measure each selected item?

Was the student able to self –correct if errors were made?

Multimedia Links/Additional Activities

http://math4children.com/Videos/measurement%20tools/index.html –This is a video on how to identify the different tools that are needed for different measurements.

Perimeter and Area

Lesson Project

This is a project where students will use the concepts of length and width to investigate perimeter and area. Students will be divided into groups of four. They will each be given four stakes, a ball of yarn and a measuring tape. The students will be working outside. They will need to create a rectangle that is exact. Once they do this, they measure and record the length and width of the rectangle to find the perimeter and area of the rectangle that they have designed. You can tell them that the group should be able to fit inside the rectangle.

This is a tricky project because students will need to think about the characteristics of a rectangle. The opposite sides are congruent and parallel. Students will eventually need to problem solve using the diagonals to be sure that their work is accurate. This is an excellent problem solving project. Allow plenty of time for the students to complete it.

Project Outline

Directions: Here is your data sheet for the rectangle project. Make notes below by using the questions as a guide. Then complete the final write –up.

1. What is the length of the rectangle?
2. What is the width?
3. Are you sure that your rectangle is a rectangle?
4. How can you be certain?
5. Does your rectangle have opposite sides that are congruent and parallel?
6. What is the perimeter of the rectangle?
7. What is the area of the rectangle?

Now that you have answered these questions, write up how your team worked together to complete this assignment.

Use complete sentences and include all of the answers to the questions in your write up. It should be at least five sentences in length. Include a labeled drawing of the rectangle that you created outside.

Portfolio Assessment

Here are some questions to think about as you assess student learning.

Did the team accomplish the task of designing a rectangle?

Are you sure that it is a rectangle?

Did each student complete the questionnaire?

If the students had difficulty, how did you help them to work through the problem?

Is the write up complete and does it show the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=ZASBmoylCPc –This is a James Sousa video on geometric formulas for area and perimeter.

http://www.youtube.com/watch?v=TN4tm_rONNc –This is a James Sousa video on figuring out the area of a rectangle.

http://www.nctm.org/standards/content.aspx?id=26770 –In this activity from NCTM, students are able to manipulate different side lengths so that they can see how the area of a figure changes.

Scale Drawings and Maps

Lesson Project

In this project, students will be designing a map of their classroom. You could even expand it to include a map of the school. But the basic outline will be for a map of the classroom. To complete this project, students will need to accurately measure their classroom and the things in it. Then they will need to create a scale to draw the map of the classroom. The map is designed to be an aerial view.

You may want to have students work in pairs. Each student should complete his/her own map, but can help each other with the pieces of the map.

Project Outline

A Bird's Eye View

Directions of Part One: This project will require you to create a map of your classroom. You will complete each part of the assignment and then draw a map that accurately represents the classroom. Follow the directions carefully and complete each part of the project.

1. Measure the perimeter of the classroom. Make a sketch of the shape of the classroom and label the actual measures of the room.
2. Measure the different things in the classroom. For example, you will need measures of desks, chairs, tables, bookcases, etc. Draw them in the sketch with their corresponding measurements.
3. Label additional things in the sketch including blackboards, computers, windows, doors, plants, etc.

Directions of Part Two: Now that you have an sketch and accurate measurements, you will need to draw the map of the classroom to scale. First, you will need to decide on a scale.

For example: $1'' = 1\ foot$

Keep the size of your paper in mind. You will need to find a scale that will work. Then complete the map with detail

and creativity.

Portfolio Assessment

To assess student work on this project, you will need to complete the assignment yourself. Be sure to measure and create a sketch of the classroom. Be sure that you know the real measurements of the room so that you can assess whether or not the student has accurately measured the things in the room.

Multimedia Links/Additional Activities

http://illuminations.nctm.org/LessonDetail.aspx?ID=L516 –This is an activity where students use real-world examples to solve problems involving scale as they examine maps of their home states and calculate distances between cities.

http://www.youtube.com/watch?v=93xYDoEA7CQ –This is a youtube video on scale reading and applying this information to a map.

http://www.youtube.com/watch?v=GC4aTrXNFJQ –This is a James Sousa video on scale factor and on how to understand scale factor.

Frequency Tables and Line Plots

Lesson Project

This project involves data collected over time. The students can use the provided data to complete the project or they can collect their own data. The best data is data that has meaning to the students. This is a project about white milk or chocolate milk. The outline provided is one that the students would use to keep track of the number of students who choose white milk or chocolate milk for two weeks.

To conduct this data collection, put the students in pairs. In each pair, one is assigned a white milk count and one is assigned a chocolate milk count. Each day the pairs change until everyone in the class has had a chance to collect the data.

After the data is collected, the students put the data on a chart in the classroom and draw line plots based on the data. Finally, they can display their work and draw conclusions based on the frequency that students selected each type of milk.

If strawberry milk is an option, you could add it in as a third possible option.

Project Outline

Chocolate or White Milk?

Part One: Data Collection

In this first part of the project, you will be collecting data on the choices of milk selected at your school. First, you will have a partner. Then you and your partner will tally the frequency that chocolate milk and white milk are selected. Fill out the data sheet. When everyone in the class has had a turn tallying, you will be ready to move on to Part Two.

Chocolate Milk

White Milk

Part Two: Line Plots

Now that you have data collected, combine your data with everyone else's data in the class. Next, design a line plot to show your data. Finally write a short paragraph to explain and draw conclusions based on the data. Be sure to back up your conclusions with numbers from the data set.

Portfolio Assessment

When assessing student work, here are some criteria to keep in mind.

Did the student complete the data sheet using tally marks?

Is the frequency of each milk choice clear?

Did the student gather data from all of the others in the class as well?

Is the line plot created accurately?

Does the line plot represent all of the data?

Is the conclusion written clearly?

Is it backed up by the data collected?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/Lessons/LineEmUp/LineEmUp-Programs.pdf –This is a reproducible from NCTM which uses line plots and graphing calculators. Students use technology in this activity.

http://www.schooltube.com/video/27ad1fffb73701b8ead9/Line-Plot-Tutorial –Here is a line plot tutorial from school tube. Could be used to help review constructing a line plot.

Bar Graphs

Lesson Project

In this project, students will be collecting data on milk. They will be collecting information on the number of students who choose white milk compared with chocolate milk. This project involves data collected over time. The students can use the provided data to complete the project or they can collect their own data. The best data is data that has meaning to the students. This is a project about white milk or chocolate milk. The outline provided is one that the students would use to keep track of the number of students who choose white milk or chocolate milk for two weeks.

To conduct this data collection, put the students in pairs. In each pair, one is assigned a white milk count and one is assigned a chocolate milk count. Each day the pairs change until everyone in the class has had a chance to collect the data.

After gathering all of the data, the students then use the data to create a bar graph.

Project Outline

White Milk or Chocolate Milk?

Part One: Data Collection

In this first part of the project, you will be collecting data on the choices of milk selected at your school. First, you will have a partner. Then you and your partner will tally the frequency that chocolate milk and white milk are selected. Fill out the data sheet. When everyone in the class has had a turn tallying, you will be ready to move on to Part Two.

Chocolate Milk

White Milk

Part Two: Bar Graphs

Now that you have your data and your partner's data, you will need to collect data from the other students in your class. Gather all of this information on white milk and chocolate milk. Once you have it collected, use the data to

create a bar graph. The bar graph will compare the number of students who chose chocolate milk with the number of students who chose white milk.

After completing the graph, write a paragraph that explains your findings. Be sure to use values from the actual data to support your conclusions.

Portfolio Assessment

When assessing student work, here are some criteria to keep in mind.

Did the student complete the data sheet using tally marks?

Is the frequency of each milk choice clear?

Did the student gather data from all of the others in the class as well?

Is the bar graph created accurately?

Does the bar graph represent all of the data?

Did the student choose a set of intervals that makes sense given the data?

Is the conclusion written clearly?

Is it backed up by the data collected?

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/reading-bar-graphs?playlist=Developmental+Math+2 –This is a Khan Academy video on reading bar graphs.

http://illuminations.nctm.org/LessonDetail.aspx?ID=U73 –This is a multi –lesson unit from NCTM on data representation. It can be differentiated given the age and grade of the students.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=204 –This is an interactive activity from NCTM. The Basic Data Grapher can be used to analyze data with bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.

Coordinates and Line Graphs

Lesson Project

In this project, students are going to investigate how the number of students who eat school lunch has changed over time. To do this, the students will need to figure out who at the school knows the information. Then they need to interview or ask the person and finally each students should use the data to create a line graph.

Project Outline

Line Plot Lunches

In this project, you will need to collect data for the past few years. This project will involve some investigating. First, you will need to find out who in your school's cafeteria knows how many students have eaten lunch over the past few years. These numbers will be approximate and not exact. That is okay. You are going to investigate how the numbers have changed over time.

Write down the number of students who have eaten school lunch for the past few years. Be sure to include the year.

Next, create a line graph that displays the data. You should be able to tell how the number of students eating lunch has changed over time.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the students figure out who has the most accurate information on school lunch numbers?

Were the students successful in finding out this information?

Did the students record the information for the past three years as assigned?

Did the student create a line graph to show how the data has changed or hasn't changed over time?

Is the line graph accurate?

Did the student create intervals that make sense given the data? Are the lines straight?

Did the student use a ruler to show the connections between the data over time?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/ActivityDetail.aspx?ID=204 –This is an interactive activity from NCTM. The Basic Data Grapher can be used to analyze data with bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.

http://www.youtube.com/watch?v=vCeAj4cLPIA –This is a youtube video from Math Made Easy on how to create a line graph.

http://www.khanacademy.org/video/misleading-line-graphs?playlist=Developmental+Math+2

This is a Khan Academy video on detecting a misleading line graph.

Circle Graphs and Choosing Displays

Lesson Project

In this project, students are going to be using a survey to create a display. The students will begin by identifying the top five lunch options offered at school. Then they will conduct a survey of all of the other students in their grade or if you are in a small school, the survey can be conducted of all of the students in the middle school. The students will count the number of students who select each of the five lunches. Then based on the number of students surveyed, create a circle graph to display the data.

Project Outline

Lunch Line - Up

Part One: Collecting Data

To begin this project, you will need to decide with your classmates what the top five lunch options are at your school. Once you have figured this out, conduct a survey, your teacher will tell you who to survey, to figure out which lunch option is the favorite of the students. Record your survey results below.

TABLE 4.2:

Top Five Lunches	Number of Students Who Chose it as #1
1.	
2.	
3.	
4.	
5.	

Part Two: Analyzing the Data

To create a circle graph, you will need to analyze the data. Use the questions below to help you with this task.

1. How many students were surveyed?
2. Write each result for the lunches as a fraction with the denominator being the total students surveyed.

$$\frac{number\ of\ students\ who\ chose\ this\ lunch}{total\ number\ of\ students\ surveyed}$$

You should have five fractions to represent each of the lunches.

Part Three: Circle Graph

Now you will create a circle graph. Because you haven't learned how to do the exact math for this yet, estimate each part of the circle graph and draw it in.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind?

Did the student follow directions?

Are the three parts of the assignment clear?

Did the students complete all three parts?

Did the student create five fractions to represent the data?

Did the student create a circle graph that estimates the data results?

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/reading-pie-graphs--circle-graphs?playlist=Developmental+Math+2 –This is a Khan Academy video on how to read a pie graph or a circle graph.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L199 –This is part of a multi –lesson unit from NCTM on examining and reading circle graphs and other data displays.

Mean, Median and Mode

Lesson Project

In this project, students are going to investigate their own grades. This will give the students an insight into how teachers figure out averages and assign grades. You can choose the scores that the students will work with. The scores could be quiz scores, test scores or homework grades. You will need to provide each student with his/her own scores so that he/she can complete the assignment.

You can complete the project by asking the students to estimate their final grade based on the work done so far.

Project Outline

Figuring Grades

Directions: Use the scores provided by your teacher to figure out the means of central tendency. This will give you some insight into your own grades and possibly your final grade. Use the list of scores to answer each question.

1. What is your average right now?
2. What is your median score?
3. Is there a mode? If so, what is it?
4. What is the range between your highest and lowest score?
5. Based on this analysis, what could you do to raise your final grade?

6. If your work continues at the same level, what is your estimate of a final grade?

Portfolio Assessment

Given this project, you will need to assess the accuracy of each student's work. Did the student figure out each measure accurately?

Use the information provided to individually coach each student. Can you help the student to bring his/her grade up to a new level?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/ActivityDetail.aspx?ID=160 –This is an interactive application from NCTM where students can explore measures of central tendency. This activity also connects to box-and-whisker plots.

http://illuminations.nctm.org/LessonDetail.aspx?id=L449 –This is an illuminations lesson from NCTM where students explore histograms while looking at mean, median and mode.

http://www.khanacademy.org/video/average-or-central-tendency–arithmetic-mean–median–and-mode?playlist=ck12.org+Algebra+1+Examples –This is a Khan Academy video on exploring the measures of central tendency.

4.3 Addition and Subtraction of Decimals

This third chapter *Addition and Subtraction of Decimals* introduces students to the Decimal System, covering addition and subtraction, rounding and estimation, and ordering decimals as well as introducing stem-and-leaf Plots. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Decimal Place Value
- Measuring Metric Length
- Ordering Decimals
- Rounding Decimals
- Decimal Estimation
- Adding and Subtracting Decimals
- Stem-and-Leaf Plots
- Use Estimation

Decimal Place Value

Lesson Project

This project is a hands–on one that uses play money. You will need play money in both dollar amounts and change amounts. To complete the project, the students are going to be given an unspecified amount of money. You randomly organize money denominations and put them in envelopes. Then hand them out to the students.

First, each student counts the money. Then they have to join - up with two other students to try to create the highest total in the room. Students can ask the sum another's money, but can't do the actual addition until they have chosen the person. Once they have chosen someone, they can't switch groups. Once the students have joined to create a group of three, they must actually count their money to figure out a sum.

Then each group shares and the class can figure out who has the greatest sum.

Project Outline

Summing It Up

Directions: The object of this game is to use estimation to try to create a group of three that will have the greatest sum of money in the room.

To do this, you will be given an envelope of money. Figure out how much you have.

Then walk around the room and find two other people to partner with you. The sum of the three of you will be entered into the class account. You want to find the most lucrative sum that you can.

As you think about selecting people, you can ask them for their total, but you aren't allowed to do any actual adding. You must figure out whether or not you want to team up with them based on their money amount.

The sum of the three persons in the group is only figured out at the end when the group has been selected.

Portfolio Assessment

The assessment of this project is done through observation. You can walk around and observe how the students chose their groups.

Figure out whether or not each student correctly added his/her sum of money.

Is the sum of the group accurate?

Did the students use random selection or was there a strategy to choosing a group? You can determine the answer to this question by having a discussion about the project at the end of the lesson.

Finish by asking the students if they would do anything differently if they did this assignment again.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Ck8FewtHpfU –This is a James Sousa video about understanding decimals and decimal notation.

http://www.youtube.com/watch?v=d1_1q1Dj1zY –This is a James Sousa video on how to write a number in decimal notation in words.

http://mindflight.plymouth.edu/standards/taa/webv1/moneyops.html –This is an activity designed by a teacher to help students work with decimals and operations. It is designed for sixth grade and could be a nice supplement to a class lesson or chapter.

Measuring Metric Length

Lesson Project

This project has a musical integration where students are assigned the task of writing a song about the metric system. To do this, students can listen to samples. There are some suggested links at the end of this section. Students can choose to write a rap song or any other type of song that they wish. The content is the metric system, and there are notes in the project outline.

Project Outline

Metric Sing Along

Directions: In this project, you will be writing a song about the metric system. To do this, you can write any type of song that you wish as long as the content about the metric system is included. It could be a folk song, a rap song or a country song. You can choose. Feel free to include music or instruments if you wish.

Here is the content to include:

Units from Small to Large

Millimeter

Centimeter

Meter

Kilometer

Conversions

1 cm = 10 mm

1 m = 100 cm

1 m = 1000 mm

1 km = 1000 m

You can use other conversions too if you wish. Be creative and have fun!

Portfolio Assessment

This is a creative assignment that really incorporates the musical intelligence. It allows students to differentiate and use many other skills while still communicating about the metric system. You can allow students to work in groups.

Grading of this project can be challenging because it is a creative project. You could use a grading rubric about the amount of content students are able to incorporate and that could be a way of grading without being subjective about student style.

For example:

5 = All units and conversions are included

4 = All of the units and some conversions included

3 = Some units and some conversions

2 = Missing either units or conversions or both

1 = Unsatisfactory due to lack of content

If you choose to use a grading rubric such as this one, be sure that you share it with the students when assigning to project.

Multimedia Links/Additional Activities

http://www.educationalrap.com/music-contents/Meters_Liters_and_Grams-m495_p473.html

This is a rap song on understanding the metric system. Can be used as a resource for students to hear prior to writing their own song.

http://www.songsforteaching.com/earthtone/metricsystemrap.htm –This is another rap song on the metric system.

There are many more that can be researched on youtube and used for students to hear prior to writing their own.

Ordering Decimals

Lesson Project

This project has the students research and investigate the Chinese numerals which were predecessors to the decimal system. This system of working with numbers uses characters. The students should use the internet or some other book resources to write an explanation of where the characters come from and how they were used. Students complete the project by drawing their own depiction of the characters.

Here is one chart from wikipedia: http://en.wikipedia.org/wiki/File:Shang_numerals.jpg . While you could give this to the students, it is much more valuable for them to research it and draw their own chart of characters.

You could also expand this project by giving the students different values and have them write or draw the Chinese characters for each number.

For example, what would 4250 look like?

Project Outline

Other Number Systems

There are many places where researchers believe our decimal system came from. In this project, you are going to do some research on the history of the decimal system. Specifically, you will be looking at the Chinese and how they chose to work with the number system. The Chinese also developed a base 10 system, but used characters instead of

simply numbers. These pictures can be researched.

In the first part of this lesson, use the internet or books to read about the Chinese system of numbers. Then create a chart depicting each of the characters and their different values. Finally, you will be asked to use what you have learned to draw some numbers. These numbers will be given to you at the end of the project.

Be sure to keep track of your resources.

Portfolio Assessment

When assessing student work, you can keep the following questions in mind.

Does the student understand how the Chinese system works?

Did the student research this number system?

Did the student create a chart that represents each character of the number system and each character's value?

Does the chart represent the student's best work and effort?

Can the student use the characters to represent numbers?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=whEp2TdYnII&feature=related –This is a video on how the Chinese number system of tens works.

http://www.youtube.com/watch?v=cw9RCCx9Rs8 –This is a James Sousa video on how to order decimals from least to greatest.

http://www.lessonpathways.com/Pathways/Detail?path=%2F05_Math%2FYear_4_Math_Guided_Journey%2F26Rounding_and_Comparing_Decimals –This website has many different pathways that students and teachers can use for learning how to order and compare decimals.

Rounding Decimals

Lesson Project

This project has the students use a large number line to round decimals to the nearest whole number. To prepare this activity, you will need colored electrical tape. It goes down on the floor easily and comes back up easily as well. To do this, you make a number line from 1 –10 on the floor. Be sure to space out the numbers so that you have room for decimals to be considered in between each whole number.

Once you have the number line created, you create decimals on cards for the students to use. Here are some examples.

3.45

6.89

2.12

3.09

4.9

5.67

3.34

You can make as many as you choose to make. Then the cards, one or more, are handed out to the students. The students take turns finding their place on the number line. Then they round to the nearest whole number and put their decimal card on that whole number.

If you want to expand the project, you can make teams and ask the students to see how fast they can round a whole stack of numbers. The first team to complete the stack accurately would be the winning team.

Project Outline

There isn't an outline necessary for this project.

Portfolio Assessment

When assessing this activity/project, you will need to keep in mind some of the following points.

Observation is the key way to assess student understanding.

Be sure that you write a wide range of decimals so that the students can practice all kinds of rounding.

Observe which students forget the rules of rounding and need reminders.

Have fun and enjoy this kinesthetic activity.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=qu4Y9DGqXlk –This is a James Sousa video about rounding decimals to the place indicated.

http://mathtrain.tv/play.php?vid=84 –This is a fun, engaging video about how to round decimals.

Decimal Estimation

Lesson Project

This lesson focuses on estimating sums and differences using rounding. To create a project around this content, you can use real world figures to help the students in their work. To do this, you will need a newspaper for everyone in your class. Multiple classes can use the same newspaper, but a newspaper number hunt is in the works.

Use the outline below to guide you and your students on the hunt. Students will be hunting down decimals and then working with those decimals.

Project Outline

Newspaper Number Hunt

Part One: Newspaper Number Hunt

To go on this newspaper hunt, you will need to search through the newspaper to find decimals and other evidence of mathematics. You will need to find at least ten different examples of how numbers are used in the newspaper. Three of these values need to have a decimal part represented.

TABLE 4.3:

Number Found	Section
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

TABLE 4.3: (continued)

Number Found	Section

Part Two: Estimating Sums and Differences

Choose two decimal numbers and estimate the sum of the two values. Use rounding to make your estimation.

Now take the third decimal and the sum and find the difference between this value and the sum. Be sure to use rounding to estimate your findings.

Write down at least three different ways that numbers are used in the newspaper.

How are these three ways connected to real –world events?

Portfolio Assessment

Here are some questions to think about as you assess student work.

Did the student follow the directions in the project?

Did the student find accurate examples of numbers in the newspaper?

Did the student record both the number and the section he/she found the number in on the worksheet?

Is the estimate of the sum accurate?

Is the estimate of the difference accurate?

Did the student answer the two final questions in complete sentences?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=BcnEzvbwhcI –This is a TenMark video on how to estimate sums that involve whole numbers and decimals.

http://mrrossimath.blogspot.com/2008/10/5th-grade-math-project-design-decimal.html –This is a fifth grade math project that could also be used in the sixth grade. This website has an outline for students to create a board game about decimals.

Adding and Subtracting Decimals

Lesson Project

This project uses the idea of a million dollars. Begin by reading David M. Schwartz's book "How Much is a Million." You can read it to the students in the class. After reading it, then talk about how much money a million dollars actually is and brainstorm ideas of what the students would purchase if they were suddenly given a million dollars. After brainstorming this list, provide the students with the project outline and let the fun begin.

You will need catalogs, magazines and newspapers for the students to use for research. They could also use the internet, but this should be supervised as the students will want to "surf" for things to purchase.

If a student is going to save for college, have him/her research the cost of the school and use that as a figure on his/her list.

Project Outline

A Million More

Directions: In this project, you are going to be given a million dollars and have to figure out how you would spend it. To do this, you will be using decimal addition and subtraction. You have one million to spend and you can't go over that amount.

To start, generate a list of things that you would like to do or purchase. Then research the cost of each of the things that you have written on your list. When finished with this first part, you should have a list of things and a cost for each thing.

Then begin subtracting the cost of each thing from $1,000,000. Be sure to include your total expenditures. Adjust anything that you need to if you find that you are going over your budget. Show all your work.

Portfolio Assessment

When assessing student work, you can use the following questions as a guide.

Did the student make a list with researched costs next to it?

Did the student accurately figure out the cost of all of his/her items?

Did the student figure out a total sum first and then subtract from one million?

Did the student subtract each item individually from one million?

Was the student able to spend his/her money without going over?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=weTZqwOisDU –This is a James Sousa video on adding decimals.

http://www.youtube.com/watch?v=t8r-S9Zr41c –This is a James Sousa video on subtracting decimals.

http://www.youtube.com/watch?v=CEHku41nb60 –This is a James Sousa video that focuses on adding and subtracting decimals.

Stem-and-Leaf Plots

Lesson Project

In this project students will be using sports figures to build stem-and-leaf plots. There will be data included in this section of the outline, but the data could be updated as well. If you choose to use the data provided, then the project will work. You could also use data from the internet by having students do their own research, or you could have students use data from local middle school or high school sports teams.

Here is a list of NF >quarterbacks and the number of touchdowns thrown in 2011.

Aaron Roger Green Bay 45

Drew Brees New Orleans 46

Tom Brady New England 39

Tony Romo Dallas 31

Matthew Stafford Detroit 41

Matt Schaub Houston 15

Eli Manning NY Giants 29

Matt Ryan Atlanta 29

Alex Smith San Fran. 17

Ben Roethlisberger PIT 21

Project Outline

Quarterback Statistics

Directions: You are going to analyze the statistics of the quarterbacks of the NFL for the 2011 football season. To do this, you will be using the statistics provided or researching the statistics given to you by your teacher. Using the handout of statistics or by using the ones that you have researched, complete the following instructions. All work should be done with a ruler and represent your best work. You may use colored pencils to illustrate your findings.

1. Organize the data by stems. What is the greatest stem? What is the smallest stem?
2. Now draw out the stem-and-leaf plot. When finished, answer the following questions.
3. What is the range of the data?
4. What is the interval with the most values in it?
5. What is the interval with the least values in it?
6. What is the mode of the data set?
7. What is the average number of touchdowns thrown?

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student create an accurate stem-and-leaf plot?

Is all of the data represented on the plot?

Does the display represent the student's best work?

Is it done with color and shows the use of a ruler?

Are the questions all answered?

Are they answered accurately?

Is there any feedback that you could give the student to have his/her work be better in the future?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/ActivityDetail.aspx?ID=220 –This is a technology tool from NCTM called the Advanced Data Grapher. The Advanced Data Grapher can be used to analyze data with box plots, bubble graphs, scatterplots, histograms, and stem-and-leaf plots. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L287 –This is a project where students collect data about the weather and use it to make a stem-and-leaf plot.

http://www.khanacademy.org/video/stem-and-leaf-plots?playlist=ck12.org+Algebra+1+Examples –This is a Khan Academy Video on stem-and-leaf plots.

Use Estimation

Lesson Project

In this project, students are going to pretend that they are the character from the story problem in the student edition of the text. A few days before you are going to do this project, ask the students to bring in clothing catalogs or magazine circulars from home. You will need to gather some up yourself from local department stores or you could print things out from the internet. The more resources you have the better.

Then use these visual resources, glue sticks, scissors and the information on the student sheet to have the students work on estimating expenditures and money. You can provide the students with large white construction paper or poster board to present their work.

Project Outline

The Shopping Spree

Directions: Pretend that you are Jose from the story problem in the text. You have been working all summer and have earned $2100.00. You are going to go shopping for new clothes. Using the catalogs and other resources in your classroom, cut and paste pictures of the items that you wish to purchase. Estimate the cost of each item. Then research the actual cost of the item.

Next, estimate the total cost of your purchases. Then figure out the actual cost of your purchases.

Here is what your final sheet should look like.

Column One –Picture of clothing item

Column Two –Estimated cost of item

Column Three –Actual cost of item

At bottom of page: Estimated total cost

Actual total cost

Portfolio Assessment

Here are some things to think about as you observe the students.

Does this student understand how much things cost and are the estimates reasonable?

Did the student find out the actual cost of the item?

Is the estimate of the total cost reasonable?

Did the student use rounding to estimate this total cost?

Is the actual total accurate?

Is the student work neat and represents the student's best work?

Did the student follow the directions presented in the outline?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=0C7JuLrUP10 –This is a James Sousa video on problem solving using whole number operations. The content is also relevant to using operations when estimating.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L713 –This is a project from NCTM using the book "One Grain of Rice." In it, the students use estimation and problem solving to figure out the grains of rice. This book also provides for an excellent integration of literature.

4.4 Multiplication and Division of Decimals

This fourth chapter *Multiplication and Division of Decimals* furthers a student's understanding of decimals by introducing methods of multiplication and division. Also covered are the Distributive Property and the use and conversion of metric units. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Multiplying Decimals and Whole Numbers
- The Distributive Property
- Multiplying Decimals
- Dividing by Whole Numbers
- Multiplying and Dividing Decimals by Powers of Ten
- Dividing by Decimals
- Metric Units of Mass and Capacity
- Converting Metric Units

Multiplying Decimals and Whole Numbers

Lesson Project

This project involves the preparation for a field trip. Students need to apply real world events to mathematics so that they can see the connections between the abstract concepts taught in the classroom and the real life. In this lesson, the students in the story problem are preparing to go on a field trip.

You are going to plan to take your students on a field trip as well. You can ask the students to research three different field trips in your area. One could be a science museum, one an art museum and one could be a play or live drama event. Brainstorm first and then allow students the chance to investigate one of the options.

The students should use the internet and complete their research. An outline for the project is provided below.

When finished, allow the students time to share their findings.

Project Outline

A Field Trip

Directions: This project requires that you apply what you have learned about multiplying decimals and whole numbers to figure out a class field trip. The three field trip options are written down on the board. Choose one and then complete the following questions to gather data for our field trip. You will have a chance to share your findings.

Field Trip Choice for Investigating:

Website visited:

Cost per Student:

Cost per Adult:

Are any chaperones free?

Number of students in the class:

Estimated Cost for both students and adults:

Total cost for the students:

Total cost for the adults:

Total cost for both students and adults:

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student complete the project outline?

Did the student complete the project totals accurately?

Is the estimate reasonable?

Does the estimate correspond to the data provided?

Does the actual cost make sense and use the data provided?

Did this help the students to make a decision about the field trip?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Zt_rFDQCi2I –This is a James Sousa video on multiplying decimals.

http://www.youtube.com/watch?v=EZ4KI0pv4Fk –This is a Ten Marks video on youtube which teaches students how to multiply decimals and whole numbers.

http://scratch.mit.edu/projects/DrSuper/471789 –This is a website where students can use an interactive technology program to multiply decimals.

The Distributive Property

Lesson Project

This project has an area/perimeter integration much like the example presented in the text. This project requires that the students follow directions and create the figured described.

You will need the following supplies:

Scissors

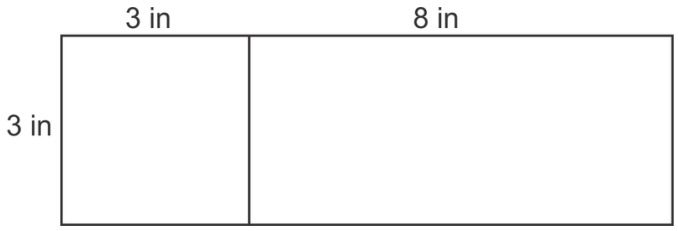
Different colors of construction paper

Glue sticks

Rulers

Pencils

When finished, students should turn in a copy of the project outline and a figure created out of construction paper with accurate measurements. Here is a picture of the figure they will make.



$3(3+8) = 3(11) = 33$ *square inches* = AREA

Project Outline

Figuring out Figures

Directions: Read the each step of this project carefully. You will be following each step to create a figure that uses the distributive property. This is similar to an example that was presented in your book. You will turn in a copy of this outline and the actual figure to your teacher at the end of the class.

1. The first figure has a length of 3 inches and a width of 3 inches. What shape is this figure?
2. Use a ruler and colored construction paper to draw this figure and cut it out. This is the first figure you will need.
3. The next figure has a length of 8 inches and a width that is the same as the first figure. What is the shape of this figure?
4. Use a ruler and colored construction paper to draw this figure and cut it out. This is the second figure you will need.
5. Glue the two figures together so that the length is equal to 11 inches. You may need a backing to glue them together.
6. Write an equation using the distributive property to show how you would figure out the area of this large figure.
7. What is the perimeter of figure 1?
8. What is the perimeter of figure 2?

Portfolio Assessment

When looking at student work, you can look for the following key things.

Did the student write an example for #6 which shows that he/she understands the distributive property?

Is the area accurate?

Are the perimeters accurate?

Does the figure created match the one in the project description?

Is the work neat and does it represent the student's best work?

Multimedia Links/Additional Activities

http://www.algebra.com/algebra/homework/Distributive-associative-commutative-properties/The-Distributive-Property.lesson –This is a simple video on the distributive property. It is presented in a blackboard format.

https://sites.google.com/site/bankamath7/classroom-news/extracreditdistributivepropertyposterproject –This is an extra credit project for students to create a poster on the distributive property.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L744 –This is an NCTM illuminations project using the distributive property factoring and area models.

Multiplying Decimals

Lesson Project

In this project, students are going to be designing rooms in a hotel. They will have specific measurements to work with, and these measurements will include decimals. To complete the assignment, students will need to draw a diagram of a hotel room using the scale provided. Then they will also have to figure out the area of the room using the given dimensions and figure out if the room fits the specified criteria. Finally, the students can include details and decorate their illustrations.

Students will need pencils, colored pencils, rulers and large sheets of paper.

The student drawing should be $15.5'' \times 20.5''$. The area of the room is 399.75 square feet.

Project Outline

Hotel Designs

Directions: You have been hired to design a hotel room for a new hotel being designed for your town. To do this, you will need to use creativity and mathematics. Using the given information, you will draw a design of your room. You can include furniture in the room as well.

The room must follow this criteria. It is $15.5\ feet \times 20.5\ feet$. You must use the scale to create your drawing. The average hotel room is between 300 and 400 square feet. Once you have drawn your design, figure out the area of the room and include this as part of your design. Every measurement in the design should be labeled and the scale needs to be included on the sheet as well.

Scale $= 1'' = 1\ foot$

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Is the drawing labeled with length, width and area?

Is the drawing done to the specified scale?

Is the scale included on the drawing?

Are the measurements accurate demonstrating an understanding of multiplying with decimals?

Does the drawing show creativity?

Does this drawing represent the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Zt_rFDQCi2I –This is a James Sousa video on multiplying with decimals.

http://www.shodor.org/interactivate/lessons/ –This website has a list of projects and activity ideas on all kinds of math concepts, including multiplying with decimals.

http://www.livingmath.net/ReaderLists/FractionsDecimalsPercents/tabid/367/language/en-US/Default.aspx –This website has a list of books including stories and picture books that use concepts associated with fractions, decimals and percents.

Dividing by Whole Numbers

Lesson Project

This project involves a mystery box. In this box, you are going to put a specified amount of money. You can use play money for this or you can write the amount on an index card. The students are going to have to figure out how much money is in the box based on the descriptions provided. You can use the amount of money here or use an amount of your own choosing. You can also choose to change or add to the clues provided.

The amount for this exercise is $143.50. This is the amount in the mystery box. Students are trying to figure out this amount of money.

Project Outline

Directions: You are a detective trying to figure out the amount of money in the mystery box. Use the clues provided to try to figure out the amount of money in the box. When you think you have it, write it in the space provided and bring it to your teacher.

1. This number is in the hundreds.
2. It is divisible by 22 evenly.
3. It has a four as one of the digits.
4. It ends in an odd number.
5. If it is evenly divided then each person would receive $6.52.
6. What is the number?
7. Use division of whole numbers and decimals to check your work.

Portfolio Assessment

When assessing student work, use these questions as guidelines.

Did the students use all of the clues provided?

Did the students understand that a number in the hundreds will have three digits?

Did the students make the connection between 22 and 6.52?

Did the students need more clues than the ones provided?

Did the student accurately divide 143.50 by 22?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=LkbSiL6uvtU –This is a James Sousa video on how to divide a decimal by a whole number.

http://www.coolmath.com/prealgebra/02-decimals/09-decimals-dividing-by-whole-number-01.htm –This is an engaging cool math lesson on how to divide a decimal by a whole number.

http://www.quia.com/rr/139818.html –This is an interactive math game titled "Rags to Riches" in which students will practice multiplying and dividing decimals by whole numbers.

Multiplying and Dividing Decimals by Powers of Ten

Lesson Project

This project is a research project combining astronomy with decimal powers of ten. Students will be researching a planet or other body of their choosing. They will need to research some facts about their choice and then use math to answer some questions. In addition, they are going to create a visual display of their findings to present to the class. In the visual display, the mathematical questions should be represented in a visual way.

Project Outline

Powers of 10 in Space

Directions: This is a research project. Just like the students in the story problem, you are going to explore astronomy and powers of ten. To work with this project, you will need to select a planet or other body in space to learn about. Then using a book or the internet, answer each question and apply mathematics where it is relevant.

After researching and figuring, create a poster about your planet or other body from space. You should include all of the data from these questions in the poster as well.

1. What is the name of the body or planet you have chosen?
2. Which sites or books are you using for research?
3. What is the diameter of your object?
4. What is its distance from earth?
5. What would the diameter be if it were 10 times as large?
6. One hundred times as large?
7. One hundred times as small?
8. One thousand times as small?

Portfolio Assessment

When looking at student work, you are looking to see that the student used the mathematics in the correct way. Be sure that multiplying and dividing by decimal powers of ten is represented on the poster and that the student was able to apply these skills when answering the questions. Once this is checked, you can look at the data the student collected and the artistic quality of his/her work. The key is that you want to be sure that the mathematics of the lesson is represented in the student display.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=TwZMeKipGXY –This is a James Sousa video on multiplying by powers of ten.

http://www.youtube.com/watch?v=OlbwHyzcQC0 –This is a James Sousa video which focuses on finding patterns when dividing by powers of ten.

http://www.math-drills.com/powersoften.shtml –This website has many resources for dividing and multiplying by powers of ten.

Dividing by Decimals

Lesson Project

In this lesson, students will be working backwards to figure out the dimensions of a mystery room. They will be given the area and one of the sides of the room. They will need to use division of decimals to figure out the missing side length of the room. Then they will use a given scale to draw a map of the room. Students can be creative in their decorations of the room.

Students will need colored pencils, rulers, pencils and large sheets of paper.

Missing side length = 9 feet

Diagram should be rectangle with a side length of 9 inches and width of 8.25 inches.

Project Outline

The Mystery Room

Directions: There is a mystery room. You will need to use all of the math that you have learned to figure out the dimensions of the room and the shape of it too. Then, you will need to use the given scale to draw a diagram of the room. You can decide what kind of room this mystery room is and share that by decorating the room in a particular way.

The length of one side of the room is 8.25 feet.

The area of the room is 74.25 square feet.

What is the missing side length of the room?

What shape is this room?

Draw a diagram of the room using the scale $1'' = 1\ foot$.

Portfolio Assessment

Here are some questions to keep in mind when assessing student work.

Did the student use division to figure out the missing side length?

Is the missing side length accurate?

Did the student draw a rectangle?

Does the drawing represent the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=AwM8WvmbfGM –This is a James Sousa video on how to divide a decimal by another decimal. It is the first of two videos on this subject.

http://www.youtube.com/watch?v=8D3CVf3HOLY –This is a James Sousa video on how to divide a decimal by another decimal. It is the second of two videos on this subject.

http://www.math-play.com/Decimal-Game.html –This is a math game with Santa that requires the students to add, subtract, multiply and divide decimals.

http://www.funbrain.com/football/ –This is a football math game where students practice operations in a fun, engaging format.

Metric Units of Mass and Capacity

Lesson Project

In this project, students are going to explore the mass and capacity of the earth. To do this, they are going to need to complete research using either books or the internet. Some of the information on the internet is slightly different, but the whole idea of the lesson is to explore the metric measures of mass and capacity as they are connected in a real world connection –the earth.

Mass of the earth $= 5.9722 \times 10^{24}$

Written out is 5972200000000000000000000

Volume of seawater on earth is 1.37 billion cubic kilometers

Project Outline

Directions: You are going to research mass and capacity as it is connected to the earth. To do this, you will be using either the internet or books or both. Be sure to keep track of the resources that you use in the resources section of this page. Include mathematics where it is relevant and be sure to answer all questions.

1. What is mass?
2. What is volume?
3. Which metric units are used to measure mass?
4. Which metric units are used to measure capacity?
5. True or false. Another word for capacity is volume?

6. What is the mass of the earth written in scientific notation?
7. Write out the mass of the earth without using scientific notation.
8. What is the volume of seawater on the earth?

Resources

Portfolio Assessment

When assessing student work, look at the following things.

Did the student answer all questions?

Did the student include measurement labels?

Did the student write the mass of the earth in scientific notation correctly?

Did the student write the mass of the earth in standard notation accurately?

Is volume represented in cubic units?

Did the student include all resources?

Is the work accurate?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=DQPQ_q59xyw&feature=related –This is a video from NASA on standard and metric units of mass and capacity.

http://www97.intel.com/en/ProjectDesign/UnitPlanIndex/MetricMadness –This is a great project on the metric system called Metric Madness. It is designed for middle school students.

Converting Metric Units

Lesson Project

Metric Matching game is the project for this lesson. You can either ask the students to create the game cards or use the list provided. The object of the game is to play a matching game where students try to find an equivalent measure for a selected card. This game is similar to concentration or memory. Students can either create the game themselves or the teacher can create the game for them. Either way, it is a valuable game for students since they will have to practice converting units as quickly as they can.

Project Outline

Here is a list of equivalent units to use for the creation of the game.

40 mm = 4 cm

200 cm = 2 m

4000 mm = 4 m

1.5 km = 1500 m

3000 g = 3 kg

5 kg = 5000 g

2 L = 2000 mL

6000 mL = 6 L

2500 mL = 2.5 L

1.5 g = 1500 mg

6000 mg = 6 grams

5 kg = 500 g

Portfolio Assessment

Assessment is completed through observation. You can walk around and observe each student's ability to convert units of measure. You will be able to see the speed and ease with which each student works.

Multimedia Links/Additional Activities

http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i17/bk8_17i2.htm –This website has a straightforward lesson on converting between metric units of measurements.

http://www.youtube.com/watch?v=cMFwpxkIFMY –This is a James Sousa video on converting between metric units of measurements.

4.5 Number Patterns and Fractions

This fifth chapter *Number Patterns and Fractions* covers prime and composite numbers and factorization. Students will also explore equivalent fractions, how to order fractions, and the relationship between fractions and decimals through methods of conversion. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Prime Factorization
- Greatest Common Factors
- Equivalent Fractions
- Least Common Multiple
- Ordering Fractions
- Mixed Numbers and Improper Fractions
- Changing Decimals to Fractions
- Changing Fractions to Decimals

Prime Factorization

Lesson Project

This activity takes the story problem from the lesson and has the students work with the content in a real way. The numbers for this activity will change based on the make –up of the school or the class that you have. There are two different scenarios possible. One is using only the number of boys and girls in your class. The other scenario uses the number of students in two different classes.

The activity is a math game day. Students are going to play four different games. Each game is going to have a mix of boys and girls or students from two different classes. Students are going to use what they have learned about factorization to figure out how many boys and how many girls or how many students from each class should be playing each game.

Some possibilities for games are shut the box, dice addition, fraction pizza, monopoly or other board games like Equate. Any games will do. This can be a very fun way to spend a Friday math class.

Here is an example of how this would work given boys and girls in one class.

There are 16 boys in one class and 12 girls in one class.

We can factor each into smaller parts.

16

1 16

2 8

4 4

12

1 12

2 6

3 4

Since there are four games, it makes sense that 4 boys and three girls play each game at one time.

Students can work in small groups to figure this out. They should write their findings on a piece of paper and share it with the class.

Project Outline

There isn't a project outline for this lesson. Students can work with the instructions of the teacher and the example can be explained on the board.

Portfolio Assessment

You can assess student learning by watching the student presentations. Each presentation will feature an explanation of how many girls and boys or how many students from each class should participate in each game group.

You can think about the following questions.

Did the students use factoring and greatest common factors to figure out the number of students in each group?

Were students able to explain their work during their presentations?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=i16N01IdIhk –This is a James Sousa video on divisibility rules.

http://www.youtube.com/watch?v=2K5pBvb7Sss –This is a James Sousa video on prime factorization.

http://www.beaconlearningcenter.com/Lessons/289.htm –This is a project developed by teachers in Florida. Students will create a prime factor mobile. This website has an outline for completing the project.

Greatest Common Factors

Lesson Project

The project for this lesson is a game called "Factor Four Corners." To play this game, students will be using the knowledge that they have about factors to figure out which corner of a room they move to. This game is kinethetic and requires students to do a lot of movement to be successful.

To play the game, you will need to create four corners of the classroom each with a different factor in them. You can choose any factors. Let's say you use.

2

6

8

12

Then you will call off numbers that either have or don't have those factors in them. Students then choose a corner that is a factor for the named number. Sometimes, there will be more than one possible answer.

For example: 24 is called.

Students can move to any of the four corners. Once the students have gone to a corner, you can ask them one by one how their number is a factor for 24.

A student in the 12 corner would say $12 \times 2 = 24$.

This also reinforces multiplication tables and is an excellent skill builder for students with special needs.

A variety of different numbers and factors can be used.

Project Outline

There isn't any written work for this lesson. It is an interactive project.

Portfolio Assessment

Student work is assessed through observation and listening. You can see the students as they move to each of the four corners in the room. You can also hear their responses to the question about whether or not their number is a factor. Some students will copy the responses of others and this is alright. We learn from each other. Notice if it becomes a pattern with some students because these students are probably weak in their tables or don't understand the assignment.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=g8QaKFuvAE0 –This is a James Sousa video on greatest common factors.

http://www.youtube.com/watch?v=v9dsxTCjdhE –This is a James Sousa video on factors. It could be seen prior to viewing the video on greatest common factors.

http://illuminations.nctm.org/activitydetail.aspx?id=12 –This is a fun interactive game from NCTM illuminations on finding factors.

Equivalent Fractions

Lesson Project

In this lesson students will create their own equivalent fraction bars. These can be made using heavy construction paper so that the students can use them as a manipulative. Each set should be made in different colors.

Students will need the following items:

Rulers

Construction paper

Pencils

Scissors

Tape

Project Outline

Directions: Begin by creating four different sets of fraction strips. Each set should be a different color. You will need to use a ruler so that each set is measured accurately. Each color base will be 24 inches. You will be dividing each strip into sections but each starts with 24 inches.

Set One –Create the following fraction strips. You should have six strips to represent each fraction. For example, the first strip is one half. So the first 24 inch strip is divided into one –half.

$\frac{1}{2}$

$\frac{1}{4}$

$\frac{1}{6}$

$\frac{1}{8}$

$\frac{1}{12}$

Set Two –Create a fraction set in thirds.

$\frac{1}{3}$

$\frac{1}{6}$

$\frac{1}{12}$

Set Three –Create a fraction set in Eighths.

$\frac{1}{8}$

$\frac{1}{24}$

Now that you have created fraction strips, make a list of any equal fractions that you can figure out. You can work with a partner to do this.

For example: $\frac{1}{2} = \frac{12}{24}$

You can have the fraction strip that is divided in one half and show 12 out of 24 on another fraction strip. These two measures are equal.

Portfolio Assessment

When assessing student work, here are some things to keep in mind.

Did the students divide each set of fraction strips correctly?

Did the students measure each section?

Did the students understand and follow the directions?

Were there any students who needed to work with a partner to be successful?

Were the students able to create a list of equivalent fractions?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=VRPSVEgo8uE –This James Sousa video uses a visual model to show how to find an equivalent fraction to a given fraction.

http://www.youtube.com/watch?v=jCMai8A3SwM –This is a James Sousa video that shows how to find an equivalent fraction to a specified denominator.

http://illuminations.nctm.org/activitydetail.aspx?id=80 –This is an NCTM game from illuminations on creating and working with equivalent fractions.

http://www.helpingwithmath.com/resources/games/fraction_game3/matching.html

This is a matching fractions game on equivalent fractions.

Least Common Multiple

Lesson Project

This project uses the information from the story problem in the lesson as a beginning point. Students are going to use the data from the chapter to create a visual display of a calendar that shows how each class is going to meet in the art room.

Assign students small groups to work with and assign each group a different month of the year when there is school. So there should be a group assigned to September, October, November... ending with the last month that you have school.

Students will need colored pencils, rulers, chart paper and probably a permanent marker of some kind.

Project Outline

The Art Room

Directions: Using the information in the lesson, you will be creating a calendar display to show how least common multiples can help Mr. Caron to figure out which group will be in his art class on which days. Let's look at the data from the lesson before we begin.

Since 6A meets every two days, two will be the first quantity.

Since 6B meets every three days, three will be the second quantity.

Now let's list the multiples of two and three. The common multiples will show the days that the students will both meet in the art room. The least common multiple will show the first day that the students will both meet in the art room.

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

3 6 9 12 15 18 21 24 27 30

The common multiples are 6, 12, 18, 24, 30.

The least common multiple is 6. The students will both be in the art room on these days.

Expand this to think about this next question.

If the students start the decoration committee on a Monday, what is the first day of the week that the students will both be in the art room?

We can make a list of days to figure this out.

Day 1 Monday

Day 2 Tuesday

Day 3 Wednesday

Day 4 Thursday

Day 5 Friday

Day 6 Monday –this the first day that both groups will be in the art room at the same time

To work on this calendar project, you will need to begin with the month assigned to your team. Create a calendar page with accurate days and dates for your month of this year.

6A is in one color

6B is in another color

Use the data from the lesson to create a calendar that shows which days 6A has art, which days 6B has art and which days they are both in the art room.

Portfolio Assessment

When assessing student work, here are some questions to think about.

Did the students follow directions?

Is the month created accurately with days and dates?

Did the students use the data from the lesson to show when 6A and 6B has art?

Students may need to work together to figure out the days if their month follows someone else's month.

Does the calendar show the best work and effort for the group of students who created it?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Tr75SIxNf80 –This is a link to a James Sousa video on least common multiples.

http://www.khanacademy.org/video/least-common-multiple–lcm?playlist=Developmental+Math –This is a Khan Academy video on least common multiples.

http://illuminations.nctm.org/LessonDetail.aspx?id=U165 –This is an NCTM project called "Paper Pool" it allows students the opportunity to learn about least common multiples, greatest common factors, ratios and proportions.

Ordering Fractions

Lesson Project

In this project, students are going to have an ice cream social. They had practice working on these types of fractions given the story problem in the text. Now it is time for them to experiment with the same content.

To have the ice cream social, you will need to organize the students into teams. Each team will keep track of the number of students who choose each type of ice cream and topping. One team will be in charge of money and two teams will be servers.

Ice cream choices

Chocolate

Vanilla

Strawberry

Toppings

Hot Fudge

Sprinkles

Caramel

When the ice cream social is over, the students can compile the data and write fractions based on the choices.

Students can then figure out which were more popular than others.

Project Outline

Directions: Here is a data sheet to help you to figure out the fractions to represent the results of the ice cream social.

Total number of students served:

This number is your denominator.

Number of students who chose chocolate

Number of students who chose vanilla

Number of students who chose strawberry

Number of students who chose hot fudge

Number of students who chose caramel

Number of students who chose sprinkles

You are trying to figure out the most popular and least popular items at the social. Based on these fractions, please write them in order from least to greatest.

What conclusions can you draw from the data?

Portfolio Assessment

Here are some questions to think about as you assess student work.

Did the students follow directions?

Are the fractions accurate?

If required, did the students simplify?

Did the students write the fractions accurately from least to greatest?

Was the student able to express any conclusions from the data?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=4CGEssgAIlA –This is a video on ordering fractions with unlike denominators.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L541 –This is a link to an NCTM activity using fraction strips. Students continue to work with fraction strips to compare and order fractions. This lesson builds on the work done with fraction relationships in the previous lesson. Students develop skills in problem solving and reasoning as they make connections between various fractions. You can find folders for each of the lessons before and after this one at the link listed.

http://www.khanacademy.org/video/ordering-fractions?playlist=Pre-algebra –This is a link to a Khan Academy video on ordering fractions.

Mixed Numbers and Improper Fractions

Lesson Project

This project is all about pizza. The students are going to use math to figure out how many pizzas and how to divide a pizza to feed the students in their class. Students use the project outline to figure out the math and then make pizza.

You will need the ingredients listed in the pizza recipe from allrecipes.com to help the students. Then make the pizza and eat. A great math lesson about pizza, fractions and measurement.

Project Outline

Directions: In this lesson you are going to use fractions to figure out how much pizza to make for your class to eat. Use the outline to figure out how many of each type of pizza you need.

You can cut a pizza in 10ths or 8ths. Let's start by doing the math for 10ths.

How many students are in your class?

How many pizzas will you need for each student to have one piece?

How many pizzas will you need for each student to have two pieces?

Now 8ths.

How many students are in your class?

How many pizzas will you need for each student to have one piece?

How many pizzas will you need for each student to have two pieces?

Make some decisions and make your pizzas. Remember, this recipe is for 1 pizza. You will need to increase it depending on how many pizzas you are going to make.

Ingredients

- 1 (.25 ounce) package active dry yeast
- 1 cup warm water (110 degrees F/45 degrees C)

- 2 cups bread flour
- 2 tablespoons olive oil
- 1 teaspoon salt
- 2 teaspoons white sugar

Directions

1. In a small bowl, dissolve yeast in warm water. Let stand until creamy, about 10 minutes.
2. In a large bowl, combine 2 cups bread flour, olive oil, salt, white sugar and the yeast mixture; stir well to combine. Beat well until a stiff dough has formed. Cover and rise until doubled in volume, about 30 minutes. Meanwhile, preheat oven to 350 degrees F (175 degrees C).
3. Turn dough out onto a well floured surface. Form dough into a round and roll out into a pizza crust shape. Cover with your favorite sauce and toppings and bake in preheated oven until golden brown, about 20 minutes.

Portfolio Assessment

When working on this project, you can assess student learning by looking at the accuracy of the answers on the student sheet. Each student should have accurate fractions represented.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=snPPwBp6tSQ –This is a James Sousa video on converting fractions and mixed numbers.

http://www.khanacademy.org/video/converting-mixed-numbers-to-improper-fractions?playlist=Developmental+Math –This is a Khan Academy video on converting mixed numbers to improper fractions.

http://illuminations.nctm.org/LessonDetail.aspx?id=L784 –This is a link to the NCTM game "Fractional Clothesline". In this lesson, a string will be stretched across the classroom and various points will be marked for 0, 1, 2, 3, and 4. This classroom number line will be used to show that all proper fractions are grouped between 0 and 1, and that improper fractions or mixed numbers are all grouped above 1. Students clip index cards with various proper fractions, improper fractions, and mixed numbers on the clothesline to visually see groupings. Students then play an estimation game with groups using the same principle. Encouraging students to look at fractions in various ways will help foster their conceptual fraction sense.

http://www.mathplayground.com/fractions_mixed.html –This is a game from math playground where students practice converting mixed numbers to improper fractions.

Changing Decimals to Fractions

Lesson Project

In this project, students are going to be converting decimals into fractions. The name of this activity is "Fraction Bank" and it is exactly that, a bank where fractions are how things are counted. Each student receives a card that has a decimal on it. One or two students are chosen as bankers. These two students sit at a pair of desks and wait for people to come and exchange their decimals for fractions. The two "tellers" also have cards with different fractions on them. They give a fraction for every decimal card given. Sometimes, they can be correct and sometimes, they may choose to give a fraction that has an incorrect conversion. The customer must catch the error at this point and ask for a new decimal. Catching these errors earns the customer a point. If the teller "gets away" with the incorrect conversion, the teller gets a point. Choose students who have strong abilities to be tellers because they will need to be able to "trick" customers.

Students can play for a whole class period or for a selected period of time.

Here are possible cards for decimals.

.25

.30

.40

.45

.50

.10

.05

.20

.01

.85

.62

.75

Possible fraction conversions.

$\frac{1}{4}$

$\frac{1}{5}$

$\frac{3}{10}$

$\frac{2}{5}$

$\frac{9}{20}$

$\frac{1}{2}$

$\frac{1}{10}$

$\frac{1}{20}$

$\frac{1}{5}$

$\frac{1}{100}$

$\frac{17}{20}$

$\frac{5}{9}$

$\frac{6}{7}$

You can also have the tellers make up their own conversions as an added challenge.

Project Outline

Fraction Bank

Directions: You are going to play a game called "Fraction Bank". In this game, all decimals are exchanged for fractions. Tellers at the bank exchange your decimals and will give you a fraction conversion to exchange for your decimal card. But beware, some of the cards may be incorrect. You must spot incorrect or correct conversions from the tellers. If you catch an incorrect one then you will earn a point. If the teller tricks you, then the teller will get a point.

Portfolio Assessment

You can sit behind the tellers and check on their work. This is the best way to observe the students as they try to spot correct and incorrect conversions. You can also offer advice to the tellers and the customers in this way. Students can discuss what worked and what didn't work by the end of the lesson.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=0yYQLZcTEXc –This is the first of two James Sousa videos on how to convert a decimal to a fraction.

http://www.youtube.com/watch?v=0F_f2Nidmtg –This is the second James Sousa video on how to convert a decimal to a fraction.

http://illuminations.nctm.org/activitydetail.aspx?ID=11 –This is an NCTM game where students investigate the properties of fractions, decimals and percents in an interesting way. Explore different representations for fractions including improper fractions, mixed numbers, decimals, and percentages. Additionally, there are length, area, region, and set models. Adjust numerators and denominators to see how they alter the representations and models. Use the table to keep track of interesting fractions.

Changing Fractions to Decimals

Lesson Project

This activity is a card game similar to the game "WAR" that is done with cards. The two types of cards that are used are fractions and decimals. Two students play at a time and each will need a pile of cards. One student is given the fractions and one is given the decimals. Then the students flip one card at a time. The greater card in the pair wins the pair. If an equivalent pair is flipped, then the students must continue to flip until the greater card wins.

The students will need to make the game. Here are some possible options for the fraction cards.

$\frac{1}{4}$

$\frac{1}{2}$

$\frac{1}{8}$

$\frac{3}{8}$

$\frac{1}{10}$

$\frac{6}{10}$

$\frac{6}{9}$

$\frac{2}{3}$

$\frac{3}{6}$

$\frac{4}{5}$

$\frac{9}{10}$

$\frac{8}{12}$

$\frac{4}{7}$

$\frac{1}{3}$

$\frac{7}{8}$

$\frac{10}{11}$

$\frac{12}{13}$

$\frac{6}{17}$

$\frac{8}{16}$

Possible Decimals

.25

.5

.74

.76

.15

.33

.60

.45

.90

.87

.52

.31

.22

.18

.19

.07

Students can also create other cards too.

Project Outline

There isn't a written section for this activity.

Portfolio Assessment

Assessment can be completed by observation. As the students are playing the game, you can walk around and observe the students. Through observation, you can see which students are successful in their conversions and which students struggle. This is a great practice game for reviewing the skills in a fun way.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=G-yjTj2ERII –This is a James Sousa video on converting fractions to decimals. It focuses on using a base ten denominator when completing a conversion. It is a first video for this content.

http://www.youtube.com/watch?v=L0DRrKG68fc –This is a James Sousa video on converting fractions to decimals when long division is involved. It could be used as a follow –up to the video listed above.

http://www.sheppardsoftware.com/mathgames/fractions/FractionsToDecimals.htm –This is a fun fruit game where students have to practice converting fractions to decimals. It has many different levels so students can challenge themselves as needed.

4.6 Addition and Subtraction of Fractions

This sixth chapter *Addition and Subtraction of Fractions* further develops a student's capability with fractions by introducing methods of addition and subtraction. Also covered are box-and-whisker plots, solving problems using diagrams, and basic problems involving time. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Fraction Estimation
- Adding and Subtracting Fractions with Like Denominators
- Adding and Subtracting Fractions with Different Denominators
- Adding and Subtracting Mixed Numbers
- Subtracting Mixed Numbers by Renaming
- Elapsed Time
- Box-and-Whisker Plots
- Problem-Solving Strategy-Draw a Diagram

Fraction Estimation

Lesson Project

This is a cooking project that involves working with pies of all different sizes. You will need to purchase pies for the students. The object of the project is to figure out how many slices to make so that each student receives one and then two pieces of pie. The number of slices will depend on how many pies are purchased. You can make this difficult or less challenging given the group of students.

For example: If there are 22 students in the class and you buy 2 pies, then the math is simple –each pie must be cut into 11 slices so that each student has one.

But if you buy 9 pies, then the math becomes more challenging.

Each pie must be cut into the same number of slices.

This is a mental math project because of the estimation. Students can talk about their choices, but they may not work out any math on paper.

Project Outline

Pie Slices

Directions: Your teacher has purchased a bunch of pies that you are going to share with everyone in the class.

In small groups, you are going to estimate how many slices to cut each pie into. You will need two numbers –one if each person receives one slice of pie and one if each student receives two slices of pie.

You may not figure out any numbers on paper. You are going to discuss and then verbally share your choices and your reasons.

**Keep in mind that each pie must be cut in exactly the same way!

Portfolio Assessment

When assessing student work, you will need to be up and around to observe and listen as students work in their groups. You will be able to hear their thinking and reasoning about the choices that they are making and why they are making these choices.

Then listen in the whole group discussion. What reasoning did the students use for making their choice? Were the students able to justify their choice of pie slices? Is every pie cut the same way?

You can also have the students draw a picture of their choices to see the fraction division in action.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Q3yiWwyuYiM –This is a youtube video on estimating fractions. It is a good beginning video on this concept.

http://illuminations.nctm.org/lessonDetail.aspx?id=L252 –This is an NCTM activity on estimating fractions. In this lesson, students develop number sense through a series of three hands-on activities. Students explore the following concepts: the magnitude of a million, fractions between 0 and 1, and the effect of decimal operations.

Adding and Subtracting Fractions with Like Denominators

Lesson Project

In this activity, students are going to be working on measuring out in eighths. Each team of two will be given a long strip of colored electrical tape and a permanent marker. Using a ruler, the students measure the entire length of the strip of electrical tape that they have been given. Then divide it up into eighths. Every time they have measured a whole, or eight –eighths, begin again with one –eighth. They are going to put a star on every five –eighths that you can.

The strip of electrical tape should be long –possibly the length of the room. You want the students to have a lot of practice adding and dividing the strip into fractions. Because they are dividing the strip into eighths, the addition will be natural and not require a lot of thought. Students will begin to think in terms of eighths in a natural way.

You will need:

Rolls of colored electrical tape (comes off the floor easily)

Permanent markers

Rulers

Project Outline

Measuring Madness

Directions: In this activity, you are going to be working on measuring out in eighths. You will be given a long strip of colored electrical tape and a permanent marker. Using a ruler, measure the entire length of the strip of electrical tape that you have been given. Then divide it up into eighths. Every time you have measured a whole, or eight –eighths, begin again with one –eighth. You are going to put a star on every five –eighths that you can.

Use a ruler and be sure that your work is accurate.

Portfolio Assessment

When assessing student work, you are going to be using observation and listening. Here are some questions to keep in mind as you watch students work.

Did the students measure the electrical tape as instructed?

Are the divisions equal showing that the students understand how fractions work?

Is the strip divided into eighths?

Do the students understand where one whole is so that they can stop at $\frac{8}{8}$ and begin again with one –eighth?

Are the stars in the correct place?

Multimedia Links/Additional Activities

http://www.nctm.org/standards/content.aspx?id=26975 –This is a game by NCTM. It was designed to be used by students in grades 3 –5, but could be used as a review or used with students who struggle with fraction operations. By working on this activity, students have opportunities to think about how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions.

http://www.youtube.com/watch?v=d53wePmJZFY –This is a James Sousa video on adding fractions with like and unlike denominators.

http://www.khanacademy.org/video/adding-fractions-with-like-denominators?playlist=Developmental+Math –This is a video from Khan Academy on adding fractions with like denominators.

http://www.khanacademy.org/video/subtracting–fractions?playlist=Developmental+Math

This is a Khan Academy video on subtracting fractions.

Adding and Subtracting Fractions with Different Denominators

Lesson Project

In this lesson, students are going to have a bake sale. However, this is a fraction bake sale, so everything made needs to be divided up into fractions. Cakes, brownies and pies are easy to do this with. You may also get a team that wants to do cupcakes. This can be an added challenge, but they will need to represent a fraction in some way with the cupcakes. One option could be with sprinkles or colored frosting.

For example, $\frac{3}{12}$ of the cupcakes have vanilla frosting and $\frac{2}{6}$ have sprinkles, etc.

Students should be given a different fraction. Each different fraction is how they are going to have to cut their baked item.

For example, if Susan receives "twelfths" then she must cut her cake into twelve slices.

When the bake sale occurs, the students use the project outline to keep track of how many of their fractions are sold.

After the sale is over, collect the data and add all of the fractions of the items sold. Each student or team should have a list of fractions. Then you can practice adding all of the fractions that have different denominators.

You will need to prepare the fraction options for the students or teams. You want many different options so that the students will practice adding fractions with unlike denominators after the sale is complete.

Project Outline

Bake Sale

Directions: This is a sheet to keep track of the fractions of your baked item that are sold at the bake sale. Keep this because you will need to use it in class when the sale is over.

My fraction: _____

Fractions of Baked Item Sold

Portfolio Assessment

When assessing student work, there are three different parts that you will be looking at and observing. Here are some questions to keep in mind.

Did the student divide his/her baked good into the assigned fractional parts correctly?

Did the student keep track of the fraction of pieces sold?

Are the number of pieces sold written as fractions on the project outline?

Are the students able to find the sum of all of the items sold?

Did the students come up with any other sums besides the total sum of items sold? For example, sum of all of the fractions of brownies versus cake slices.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=LaSrytQ0pjw –This is a James Sousa video on adding and subtracting fractions with different denominators. This is the first video of two on the topic.

http://www.youtube.com/watch?v=tN3u-V3yseQ –This is the second James Sousa video on the topic of adding and subtracting fractions with different denominators.

http://illuminations.nctm.org/LessonDetail.aspx?ID=U125 –This is a project from NCTM that has multiple lessons to it. The interactive paper pool game provides an opportunity for students to develop their understanding of ratio, proportion, greatest common denominator and least common multiple. This investigation includes student resources for the Paper Pool project, preparation notes, answers, and a holistic-by-category scoring rubric with guidelines for how it can be used to assess the project. Samples of two students' work and a teacher's comments accompany the suggested rubric.

Adding and Subtracting Mixed Numbers

Lesson Project

This project is a continuation of the bake sale from the last lesson. If you did not do the last lesson with your class, then here is the outline for it.

In this lesson, students are going to have a bake sale. However, this is a fraction bake sale, so everything made needs to be divided up into fractions. Cakes, brownies and pies are easy to do this with. You may also get a team that wants to do cupcakes. This can be an added challenge, but they will need to represent a fraction in some way with the cupcakes. One option could be with sprinkles or colored frosting.

For example, $\frac{3}{12}$ of the cupcakes have vanilla frosting and $\frac{2}{6}$ have sprinkles, etc.

Students should be given a different fraction. Each different fraction is how they are going to have to cut their baked item.

For example, if Susan receives "twelfths" then she must cut her cake into twelve slices.

When the bake sale occurs, the students use the project outline to keep track of how many of their fractions are sold.

After the sale is over, collect the data and add all of the fractions of the items sold. Each student or team should have a list of fractions. Then you can practice adding all of the fractions that have different denominators.

You will need to prepare the fraction options for the students or teams. You want many different options so that the students will practice adding fractions with unlike denominators after the sale is complete.

Once the students have done all of the adding of unlike denominators. You may have numbers like this:

$\frac{14}{10}$ of brownies were sold. This means that 14 slices of brownies were sold and there were two pans of brownies both divided into tenths.

Next, students are going to take the totals and convert them to mixed numbers.

Project Outline

Bake Sale

Directions: This is a sheet to keep track of the fractions of your baked item that are sold at the bake sale. Keep this because you will need to use it in class when the sale is over.

My fraction: _____

Fractions of Baked Item Sold

Sums of Items Sold

Convert Each Sum to a Mixed Number

Note: Sometimes you won't be able to do this, but sometimes you will.

Portfolio Assessment

When assessing student work, there are three different parts that you will be looking at and observing. Here are some questions to keep in mind.

Did the student divide his/her baked good into the assigned fractional parts correctly?

Did the student keep track of the fraction of pieces sold?

Are the number of pieces sold written as fractions on the project outline?

Are the students able to find the sum of all of the items sold?

Did the students come up with any other sums besides the total sum of items sold? For example, sum of all of the fractions of brownies versus cake slices.

Did the students write the sums accurately on their project outline sheet?

Were the students able to convert improper fractions to mixed numbers when necessary?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=f5a28iu-V6E –This is a James Sousa video on adding mixed numbers. He has several other videos on adding mixed numbers as well, but this one is very comprehensive.

http://www.youtube.com/watch?v=tVrelLu6K6k –This is a James Sousa video on subtracting mixed numbers. The content is detailed and complete.

http://www.nctm.org/publications/article.aspx?id=22085 –This is a link to the NCTM journal "Teaching Children Mathematics" which focuses on mixed numbers and on teaching children how to work with mixed numbers. It is a very interesting article for anyone teaching this concept to children.

Subtracting Mixed Numbers by Renaming

Lesson Project

This is a game where students are going to practice subtracting mixed numbers and renaming numbers. You will need to divide the class in half. Half of the students is going to hold whole numbers and half of the class is going to hold mixed numbers. Each whole number needs to be larger than the largest mixed number.

Put the two lines facing each other so that everyone has a partner. Use desks so that the students have a surface to work on.

Then the fun begin.

Each pair must subtract the mixed number from the fraction in the allotted amount of time. Then when you ring a bell, the student in the mixed number line moves one spot to the right. Now there is a new whole number to subtract from. Each student keeps their work sheet and you can go over the answers at the end of the class.

You will need to prepare enough whole numbers for half of the class and enough mixed numbers for the other half of the class.

Project Outline

Subtraction Scramble

Directions: You have been given either a mixed number or a whole number. Use this sheet to keep track of the subtraction that you do in each spot of the game. You need to write a subtraction problem and solve it as quickly as possible. You can work with the person in the desk across from you so that the two of you are working together.

Whole Number Mixed Number Difference

Portfolio Assessment

When assessing student work, use observation and the answers on the student sheet to evaluate student learning. Here are some questions to keep in mind.

Were the students able to rename each mixed number?

Were the students able to find a difference in the time allotted?

Did you need to adjust your time given the work of the students?

Were there any students who were unable to complete the assignment and needed more help?

Were the problems and answers on the student sheet accurate?

Multimedia Links/Additional Activities

http://mathflix.luc.edu/NCTM_cat/NumberOperation/Fractions/010215/SubtractFractionsOne/index.html –This is a video based on the NCTM standards which teaches students how to subtract a fraction from one. This could be used as an introduction to working with mixed numbers and renaming because it begins with one of the most basic ideas in renaming.

http://www.khanacademy.org/video/subtracting-mixed-numbers-2?playlist=Developmental+Math –This is a Khan Academy video on subtracting mixed numbers.

Elapsed Time

Lesson Project

This project focuses on The Boston Marathon. You can begin by reading the paragraph on the race in the project outline. Read it aloud with the students.

Then put the students into small groups. After doing this, the students are going to work on problem solving elapsed time and The Boston Marathon

You could also add a research piece and have students compare data from other races in this lesson project.

Project Outline

Directions: This project is about the Boston Marathon. Let's begin by reading about this race. he **Boston Marathon** is an annual marathon hosted by the U.S. city of Boston , Massachusetts , on Patriots' Day , the third Monday of April. Begun in 1897 and inspired by the success of the first modern-day marathon competition in the 1896 Summer Olympics , [1] the Boston Marathon is the world's oldest annual marathon and ranks as one of the world's best-known road racing events. It is one of five World Marathon Majors .

Today, the Boston Athletic Association (B.A.A.) manages this event. Amateur and professional runners from all over the world compete in the Boston Marathon each year, braving the hilly New England terrain and varying weather to take part in the race.

The event attracts 500,000 spectators each year, making it New England's most widely viewed sporting event. [2] Though starting with 18 participants in 1897, the event now attracts an average of about 20,000 registered participants each year, with 26,895 people entering in 2011. [3] The all-time record for the world's largest marathon ever run was the Centennial Boston Marathon in 1996 with 38,708 entrants. [2]

Courtesy of wikipedia.

Now you are going to work with a small group to figure out the elapsed time. Research and find out when the start time of the race was in 2010.

If TeybaErkesso won with a time of 2:26:11, what time did he finish?

Who was the female winner of that year? What time did she finish given the start time for women compared with men?

Portfolio Assessment

To assess student work, you can think about the following questions.

Did the students complete the research accurately?

Is the elapsed time correct?

Did the students calculate the finish time for both the male and female winners?

Is the work accurate?

Multimedia Links/Additional Activities

http://www.dailymotion.com/video/xeajqg_how-to-find-elapsed-time_tech –This is a Ten Marks video on how to calculate elapsed time.

http://illuminations.nctm.org/LessonDetail.aspx?id=U115 –This is a multi –unit project from NCTM on planning a class trip. In it, students use problem solving to figure out the details of planning a trip including calculating elapsed time. Each unit is presented in a clear format and could be a nice supplement to any lesson.

Box-and-Whisker Plots

Lesson Project

This project asks the students to create a box-and-whisker plot using statistics from the NFL and specifically from the New England Patriots. Now any set of scores could be used. You could choose a team from your local area or from a high school team as well. Here the scores for the Patriots have been listed and could be used for this project.

38	35	31	31	30	20	17	20	37	34	38	31	34
	41	27	49									

Project Outline

Directions: You will be creating a box-and-whisker plot using the scores from the New England Patriots.

38	35	31	31	30	20	17	20	37	34	38	31	34
	41	27	49									

Here are the scores. If you can't remember how to build a box-and-whisker plot, here are the steps.

1. List the data from the least to the greatest on a number line.
2. Identify the median, lower quartile, upper quartile, lower extreme and upper extreme
3. Put a box around the quartiles
4. Draw in the whiskers

Portfolio Assessment

To assess student learning, keep the following questions in mind.

Did the student follow the directions?

Are all of the scores represented in the data display?

Is the data display neat and is it made with a ruler with care?

Does the student understand the vocabulary associated with the lesson?

Was the student able to use the vocabulary accurately in the context of the lesson?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/LessonDetail.aspx?id=L737 –In this lesson, students use information from NBA statistics to make and compare box and whisker plots. The data provided in the lesson come from the NBA, but you could apply the lesson to data from the WNBA or any other sports teams or leagues for which player statistics are available.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=220 –This is an interactive application from NCTM. The Advanced Data Grapher can be used to analyze data with box plots, bubble graphs, scatterplots, histograms, and stem-and-leaf plots. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation. (To analyze data with bar graphs, line graphs, pie charts, or pictographs, try the Data Grapher .)

http://www.khanacademy.org/video/box-and-whisker-plot?playlist=ck12.org+Algebra+1+Examples –This is a Khan Academy video on how to create a box-and-whisker plot using a set of data.

Problem-Solving Strategy-Draw a Diagram

Lesson Project

This project is a creative writing project that should be done after the students have completed the lesson. The directions are explained below and should be straightforward for the students to follow.

Project Outline

Writing a Word Problem

Directions: In this project, you are going to work with a partner to write a story problem that uses fractions and can be solved using a diagram or drawing. You can look back at the examples in the student edition to solve your problem. Your problem should have three fractions in it and use either addition, subtraction or both. Be sure to create a solution key to your problem on another sheet of paper.

When finished, you will be exchanging papers with another team. Then your team will be solving their problem and they will be solving yours.

Portfolio Assessment

You can use the following rubric to grade student work.

Student has included 3 fractions 5

Students has included one operation 4

Student has included two operations 5

Student has used creativity to write an interesting, fun problem 5

Student has used a diagram to solve the problem in the solution key 5

Student has successfully solve another teams problem by using a diagram 5

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/word-problem-solving-strategies?playlist=ck12.org+Algebra+1+Examples –This is a video from Khan Academy on problem solving strategies that can be used and how to implement them.

http://www.mathtv.org/ –This is a site that has problem solving videos for students.

http://pred.boun.edu.tr/ps/ps1.html –This site has a set of problems and discusses how to work with your students to solve them. Each of the steps of the problem involve using diagrams and is excellent practice for any student.

4.7 Multiplication and Division of Fractions

This seventh chapter *Multiplication and Division of Fractions* covers the methods needed to multiply and divide fractions and mixed numbers. Students are also introduced to using and converting customary units. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Multiplying Fractions and Whole Numbers
- Multiplying Fractions
- Multiplying Mixed Numbers
- Dividing Fractions
- Dividing Mixed Numbers
- Customary Units of Weight and Capacity
- Converting Customary Units
- Problem-Solving Strategy: Choose an Operation

Multiplying Fractions and Whole Numbers

Lesson Project

This is a project about animals in the rainforest. The students are going to use math to figure out the total weight of a group of giant armadillos. There are a couple of websites listed below for research or students could use books on the giant armadillo. Ask the students to write down some facts about the animal and then solve the problem written on the project outline.

Answer: $12\frac{1}{4} \times 8 = 98$ *pounds*

Project Outline

Directions: You are going to research animals in the rainforest, specifically the giant armadillo. There are armadillos of all size out there, but in the rainforest, the giant one is especially interesting. Begin by reading about the armadillo and writing down some facts. Then solve the problem related to the armadillo and include a drawing of one.

http://en.wikipedia.org/wiki/Giant_Armadillo

http://www.sandiegozoo.org/animalbytes/t-armadillo.html

1. What kind of food does the giant armadillo eat?
2. In what countries are they commonly found?
3. What is the size of the armadillo?
4. How much does one usually weigh?
5. How long does one live?

Math Question

You have found a group of giant armadillos. One of them weighs $12\frac{1}{4}$ *pounds*. If there are 8 in the group, what is the total weight of the group?

Include a drawing of a giant armadillo.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student answer all of the questions on the giant armadillo?

Did the student complete the math question accurately?

Does the drawing of the armadillo show the student's best work?

Multimedia Links/Additional Activities

http://msteacher.org/epubs/math/math14/fractions.aspx –This is a resource page for teachers on teaching the multiplication and division of fractions to students in the sixth grade.

Multiplying Fractions

Lesson Project

This is a project where students will use what they have learned about multiplying fractions and whole numbers to investigate a fact about the giant armadillo. To begin, students can complete some research about the armadillo and write a few paragraphs about the animal. Then they can move on to the problem solving part of the activity. The activity includes the math, but also a drawing to illustrate the student's findings.

Students will need large poster paper to complete their work.

Here are the answers that you are looking for:

Armadillo 35 inches long with $\frac{2}{5}$ of that being the tail.

$35 \times \frac{2}{5} = 14$ *inches* –this is the tail measurement

$35 - 14 = 21$ *inches* –this is the body measurement

Using the scale, the drawing should have a body that is $10\frac{1}{2}$ *inches* long and a tail that is 7 inches long.

Project Outline

Directions: You are going to learn about the giant armadillo, an animal common to the rainforest. Just like the students in the story problems in the text, you are going to research some facts about the animal. Begin by completing research using either the internet or books and write two paragraphs about the giant armadillo. Then move on to the second part of this assignment which includes mathematics.

Part Two: A giant armadillo is about 35 inches long. Two –fifths of that length is the tail of the armadillo. First, figure out how long the tail is given these dimensions. Then use a piece of paper and draw an accurate diagram of the armadillo. Be sure to show the fractional parts that are the body and tail by using the given measurements. Label your work.

Scale $= 1$ *inch* $= \frac{1}{2}$ *inch*

Using a scale will help you to fit the armadillo on the paper provided.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the students write two creative, neat paragraphs about the armadillo?

Is the writing done in complete sentences?

Did the students complete the arithmetic accurately?

Is the drawing done to scale?

Does this drawing show the student's best work and effort?

Is the drawing labeled with the actual measures?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=_Lhw85H8i0Q –This is a James Sousa video on multiplication involving fractions.

http://www.youtube.com/watch?v=szzJvv16UIs –This is a James Sousa video on multiplying three fractions together.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L338 –This is a multi –unit project from NCTM on working with fractions and fraction operations. Students use twelve eggs to identify equivalent fractions. Construction paper cutouts are used as a physical model to represent various fractions of the set of eggs, for example, $\frac{1}{12}, \frac{1}{6}$, and $\frac{1}{3}$. Students investigate relationships among fractions that are equivalent.

Multiplying Mixed Numbers

Lesson Project

In this lesson, students are going to use some fractional facts to figure out the weight of a backpack that for themselves. Students will pretend that they are going on a hike in the rainforest. You will need a scale for students to weigh themselves.

Example: How much weight can a 90 pound student carry?

$\frac{15}{100}$ *or* 15% of body weight. We will use the fraction.

Create equivalent fractions.

$$\frac{15}{100} = \frac{x}{90}$$

$13\frac{1}{2}$ *pounds* is the answer.

Some students may choose to use decimals. This is alright because we want the students to think fluidly with numbers and this is an exercise that could lend itself to work with fractions or decimals.

You can use this example to introduce the concept to the students.

Project Outline

Directions: You are going on a hike in the rainforest. To prepare for this hike, you must figure out how much weight you can carry in your backpack. A student shouldn't carry more than 15% or $\frac{15}{100}$ of his/her weight in a backpack.

1. Weigh yourself.

2. Use equivalent fractions or decimals to figure out how much weight you can carry.

3. How much weight can you carry?

Portfolio Assessment

When assessing student work, you can use observation as your best tool. Look at the answers that the students include on their sheets.

Did the students complete the project outline?

Did the student use equivalent fractions?

Did the student use decimals?

Did the student arrive at the correct weight for the backpack?

Did the student write his/her answer as a fraction or decimal?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=dsgLHlxFwEU –This is the first of two James Sousa videos on multiplying mixed numbers.

http://www.youtube.com/watch?v=_HxMTQmOjwY –This is the second James Sousa video on multiplying mixed numbers.

http://www.shodor.org/interactivate/lessons/MultiplyingFractions/ –This is a web link provided by NCTM for teachers on multiplying fractions and mixed numbers. There are many different activities to choose from which can help students in learning these skills.

Dividing Fractions

Lesson Project

This lesson project involves a hands–on demonstration of how to divide different fractions. There is some preparation that you will need to do before beginning. You will need to cut a bunch of strips of construction or poster board. Have these be as long as possible so that the students have a lot of paper to work with, but keep the strip in an even measurement. Then name each strip with a different fraction and write it on one side of the strip.

Here are some possible options.

$\frac{1}{2}$

$\frac{3}{4}$

$\frac{5}{6}$

$\frac{7}{8}$

Plan for one strip for every two students.

Next, you will hand out the strips. Based on the number of students in the class, the pairs will need to figure out how to divide each of the given strips so that each student in the class gets a section.

For example, if a pair has been given $\frac{1}{2}$ and there are 20 students in the class, then they will need to divide $\frac{1}{2}$ into 20 sections.

This fractional amount is represented on the strip. Students use rulers to divide the piece of cardboard evenly.

The project ends with everyone sharing their fraction strip and showing how they divided it up for the class of students.

Project Outline

Directions: You are going to practice dividing fractions.

Each pair is given a fraction strip. On one side, your teacher has told you what fraction you have been given. Your task is to take the fractional strip and divide it by the number of students in your class so that each student can get an equal part of your fraction strip.

Once you have done the math, you will need to use a ruler and actually divide the strip up into even sections.

Mark each section with the appropriate fraction.

Example: You have been given $\frac{1}{2}$ as a fraction strip.

There are 20 students in your class.

Divide $\frac{1}{2}$ by 20 = $\frac{1}{40}$

Divide the fraction strip into 40 even sections and label each section $\frac{1}{40}$.

Every person in the class will receive $\frac{1}{40}$ of the fraction strip.

Portfolio Assessment

Use the outline for assessment purposes. You are looking to be sure that each part of the assignment has been completed.

Did the student divide the fraction by the whole number accurately?

Is the fraction strip divided accurately?

Did the students label each section with a fraction?

Can the students share about their process and how they solved this problem?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=3ahgPUBdanE –This is a James Sousa video on dividing fractions.

http://www.nctm.org/publications/article.aspx?id=20371 –This is a journal article from the NCTM journal "Mathematics Teaching in the Middle School". This article illustrates students' efforts to resolve an apparent discrepancy between their self-generated solutions and the answer obtained using the division algorithm for fractions. The activity described in this article happened in a college classroom, but teachers will learn how to transfer this knowledge and ideology to the middle school classroom to help students explore the division of fractions.

http://www.aaamath.com/fra66ox2.htm –This is an interactive lesson on dividing fractions and whole numbers.

Dividing Mixed Numbers

Lesson Project

This project is about measurement and cooking. You can either use a real recipe or simply practice with the students by completing the mathematics. Use the project outline and allow students a chance to work in small groups or pairs.

Answers: $14\frac{1}{2}$ cups of squash

$\frac{3}{4}$ cup needed per bread

$14\frac{1}{2} \div \frac{3}{4} = \frac{29}{2} \div \frac{3}{4} = \frac{29}{2} \times \frac{4}{3} = \frac{58}{3} = 19\frac{1}{3}$

The students can make 19 loaves of bread.

There will be squash left over because $\frac{1}{3}$ is not a whole loaf.

Project Outline

Squash Bread

Directions: You are going to be making squash bread. You have $14\frac{1}{2}$ cups of squash arranged in plastic bags. You will need $\frac{3}{4}$ of a cup of squash for each bread. Given these measures, how many loaves of bread can you make? Is there any squash left over?

Be sure to show all of your work and illustrate your findings.

Portfolio Assessment

When assessing student work, keep the following questions in mind.

Did the student set up the division problem correctly?

Did the student remember to invert and multiply?

Did the student write the fraction in simplest form?

Did the student realize that 19 loaves can be made?

Did the student realize that $\frac{1}{3}$ is not a whole loaf?

Did the student illustrate his/her work?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=x1coIlZoFag –This is a James Sousa video on division involving mixed numbers. It also includes some work with whole numbers too.

http://www.youtube.com/watch?v=jPx6D2eShmc –This is an extensive video on multiplying and dividing mixed numbers. It is by James Sousa and shows students how to make connections between the two operations.

Customary Units of Weight and Capacity

Lesson Project

This project asks the students to figure out capacity amounts. The students are going to pretend that they are going on a hike in the rainforest. They will need to carry water. First, they will need to figure out how much water and then they will need to choose a water bottle and figure out how many water bottles they will need to carry so that they have enough water for the day.

Conversion Information–answer key

$16\ ounces = 1\ cup$

$2\ cups = 1\ pint$

$4\ cups = 2\ pints = 1\ quart$

$4\ cups \times 16\ ounces = 1\ quart$

$64\ ounces = 1\ quart$

$64\ ounces \times 2 = 2\ quarts$

$128\ ounces = 2\ quarts$

Project Outline

A Rainforest Hike

Directions: You are going on a hike in the rainforest. To do this, you will need at least 2 quarts of water per day. Use this information to answer all of the following math questions about capacity. Then you will need to choose a water bottle and answer questions about the water bottle too.

1. If you need 2 quarts of water per day, how many cups of water is that?
2. How many ounces is 1 quarts of water?
3. How many ounces is 2 quarts of water?
4. If a water bottle can carry 96 fluid ounces, how many will you need for the day?
5. If a water bottle can hold 32 ounces of water, how many will you need for the day?

Portfolio Assessment

Use the project outline for assessment. This is a project that requires accurate calculations. These accurate calculations are all that is needed to assess student learning.

Multimedia Links/Additional Activities

http://www.dailymotion.com/video/xeajzj_how-to-measure-weight-in-customary_tech –This is a Ten Marks video on how to measure weight using customary units.

http://www.khanacademy.org/video/application-problems-involving-units-of-weight?playlist=Developmental+Math –This is a video on applying different units of weight when problem solving.

http://www.linkslearning.org/Kids/1_Math/2_Illustrated_Lessons/6_Weight_and_Capacity/index.html –This is a video on measuring weight and capacity.

Converting Customary Units

Lesson Project

In this lesson, students are going to research facts about the rainforest and use these facts to answer questions and convert customary units of measurement.

Project Outline

Directions: Research some facts about the rainforest. You will need this information to answer the questions presented below on the project sheet.

1. Giant bamboo plants can grow 9 inches per day. How many feet is that in one week?
2. Draw a picture of a giant bamboo plant.
3. The rainforest can receive between 68 –78 inches of rain per year. How many feet of rain is that?
4. There are many types of trees and layers of trees in the rainforest. What is the emergent layer of trees?
5. If this layer is 100 –240 feet high, how many inches is that?
6. The Amazon River can hold barges that carry 200 tons of weight on them. How many pounds is that?

Research and write down five more facts about the rainforest. Find facts that use measurement in them. List the fact and the measurement associated with it below.

Portfolio Assessment

Student assessment is completed through the project outline. The questions are straightforward and should be answer accurately. Be sure that the students have included facts that involve measurement in the last section.

You can either correct these sheets yourself or use a peer review system. After correcting answers, discuss any discrepancies and have students correct and errors.

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/application-problems-involving-units-of-weight?playlist=Developmental+Math –This is a Khan Academy video on working with customary units of weight when problem solving.

http://www.teachingideas.co.uk/maths/contents_measure.htm –This website has many teaching projects listed for teaching measurement and measurement units to students of different ages.

http://www.nctm.org/profdev/content.aspx?id=23510 –This is an NCTM article about integrating measurement across the curriculum. An excellent resource article for teachers.

Problem-Solving Strategy: Choose an Operation

Lesson Project

In this project, students will be combining creative writing and mathematics. Students will be using the key words that they have learned in the lesson to write word problems about different animals in the rainforest.

You may want to review these key words when you begin the lesson. Each student must write four different problems. One problem must require addition to solve it, one subtraction, one multiplication and one division.

Project Outline

Directions: You will be writing your own word problems in this assignment. To do this, you will be combining the key words that you learned in the lesson with facts that you research about animals in the rainforest. You will begin by doing some research. Here is a great website with a lot of information about rainforest animals.

http://www.rainforestanimals.net/

After completing your research, write four different word problems. Each problem must require a different operation to solve it. One must use addition, one subtraction, one multiplication and one division.

When finished, you will be exchanging papers with a friend. They will solve your problems and you will solve their problems.

Portfolio Assessment

In assessing student work, here are some questions to keep in mind.

Did the student complete the research?

Did the student write four different problem?

Does each problem require the use of a different operation to solve it?

Did the student use the key words from the lesson in his/her story problems?

Did the student solve another student's problems correctly?

Multimedia Links/Additional Activities

http://www.teachervision.fen.com/math/teaching-methods/48952.html –This website has teaching tips and strategies for teaching children problem solving. This link focuses on problem solving and the strategy choosing an operation.

www.eduplace.com/math/mw/problem/4/4_9.pdf –This is a resource page for students on problem solving and choosing an operation. It has examples that are useful for students to work on.

4.8 Ratios, Proportions and Percents

This eighth chapter *Ratios, Proportions and Percents* introduces students to ratios and rates, basic uses of proportions including understanding scale drawings, and percents. Also explored is the relationship among percents, decimals, and fractions. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Ratios
- Rates
- Solving Proportions
- Proportions and Scale Drawings
- Understanding Percent
- Percents, Decimals and Fractions
- Finding a Percent of a Number
- Problem-Solving Strategy: Use a Proportion

Ratios

Lesson Project

This project is a hands–on project of students working with ratios. You will need magazines of all kinds for the students to cut up. Students are going to make a picture of different ratios. They can use sports or games or other pictures to create ratios with images.

For example, a student could cut out pictures of three dogs and then a picture of two cats and write the ratio 3:2 underneath the pictures to show the ratio.

Students must use different ratios. They should also write each ratio in three different ways.

Project Outline

Magazine Ratios

Directions: You are going to create a poster of ratios. Each poster is to have at least eight different ratios. You are going to use pictures from magazines to create your ratios. Each ratio must be different.

For example: If you use the ratio 3 to 2, then you can't use it again.

For each ratio of pictures, you are to write the ratio which describes it in three different ways. Be sure that your ratios are in simplest form.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student follow directions?

Are there eight different ratios represented through pictures?

Did the student write the ratios in three different forms?

Did the student write each ratio in simplest form?

Does this poster show the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=-YLWlPVEpbQ –This is an introductory video by James Sousa on ratios.

http://www.youtube.com/watch?v=zUGLLvymVag –This is a James Sousa video on how to write a ratio as a simplified fraction.

http://illuminations.nctm.org/LessonDetail.aspx?id=L722 –This is an NCTM project on bean counting and ratios. It uses large sample sizes and can be used with upper level students. By using sampling from a large collection of beans, students get a sense of equivalent fractions, which leads to a better understanding of proportions. Equivalent fractions are used to develop an understanding of proportions. This lesson can be adapted for lower-skilled students by using a more common fraction, such as $\frac{2}{3}$. It can be adapted for upper grades or higher-skilled students by using ratios that are less instinctual, such as $\frac{12}{42}$ (which reduces to $\frac{2}{7}$). Scaffold the level of difficulty in this lesson by going from a simple ratio such as $\frac{2}{3}$ to more complicated ratios such as $\frac{2}{7}$ or $\frac{5}{9}$.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=178 –This is an interactive online game where students look at bicycles, ratios and fractions.

Rates

Lesson Project

This project is about rates, receipts and the supermarket. Using the example from the story problem in the text, students are going to write a receipt that shows a rate and a total.

For example: Peanuts are 3.99 per pound

The receipt would say: Peanuts 3.99 per pound @ 4 pounds = 15.96

Each receipt is to have at least ten different things on it. At the bottom of the receipt, students are going to show the total cost for the shopping trip.

The receipt can be illustrated as well.

You will need colored pencils and roll paper.

Project Outline

Rates, Receipts and Supermarkets

Directions: In this project, you are going to create a shopping receipt. You can create the receipt with any items that you would like as long as you can show a rate a number of pounds or items purchased and a total cost for that item.

For example: Peanuts are 3.99 per pound

The receipt would say: Peanuts 3.99 per pound @ 4 pounds = 15.96

There should be at least ten different things on your receipt. You may illustrate your receipt for extra credit.

Be sure to include the total cost of the shopping trip at the bottom of the receipt.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student follow the directions?

Are there ten different items, rates and totals on the receipt?

Did the student calculate each total accurately given the rate?

Did the student remember to include a total for the shopping trip?

Does this show the student's best work and effort?

Did the student illustrate the receipt for extra credit?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=jlEJU-l5DWw –This is a James Sousa video on rates and on understanding rates.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L511 –This is the third lesson in a multi –lesson unit from NCTM on how to calculate rates. The title of the unit is called "Measuring Up."

http://illuminations.nctm.org/LessonDetail.aspx?id=L378 –This project from NCTM uses rates and connects them with percents and taxes. This lesson could be used with advanced students or by students who are looking for an additional challenge.

Solving Proportions

Lesson Project

This is an project where students in small groups need to find a way to teach other students how to solve proportions. The students can use any method that they would like to teach. They can play a game, make a puzzle, sing a song, create a skit, there are many options.

Project Outline

Teaching Proportions

Directions: Your assignment is to teach the class how to solve proportions. You have been assigned a small group to work in and together you will decide how you can teach the class to solve proportions. You can use any method that has been taught in the lesson in the book. You may use any method that they would like to teach. You can play a game, make a puzzle, sing a song, create a skit, there are many options. Whatever you choose, you should have some kind of visual display with your presentation.

Be creative, have fun and be prepared.

Portfolio Assessment

Here is a possible grading rubric for student presentation. If you decide to use a rubric, present it to the students before giving out the assignment.

Problem solving method is clear 5

Explanation of a Proportion 5

Explanation of how to solve a proportion 5

Creativity of presentation 5

All members were involved in the presentation 5

Students used a hand –out or visual display 5

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=j-va9qeYyOI –This is a James Sousa video on proportions and on how to understand and share proportions.

http://illuminations.nctm.org/LessonDetail.aspx?id=L721 –This is a project from NCTM where students are going to do work just like the work of scientists. In this lesson, students experience an application of proportion that

scientists actually use to solve real-life problems. Students learn how to estimate the size of a total population by taking samples and using proportions. The ratio of "tagged" items to the number of items in a sample is the same as the ratio of tagged items to the total population.

http://illuminations.nctm.org/lessondetail.aspx?id=L284 –This is an NCTM project where students use real-world models to develop an understanding of fractions, decimals, unit rates, proportions, and problem solving. The three activities in this investigation center on situations involving rational numbers and proportions that students encounter at a bakery.

Proportions and Scale Drawings

Lesson Project

In this project, students are going to be creating scale drawings of their desks. They are going to measure each part of the desk and then create a drawing to scale. This is a project that should be done after all of the content to the lesson has been taught. It could be used as an assessment tool as well. By observing students as they work as well as the diagram that is created, you will be able to evaluate student understanding and progress.

Project Outline

The Desk Design

Directions: In this lesson, you are going to draw your desk. You will need to measure each part of the desk by using a ruler or a tape measure. Then you will choose a scale and draw the desk to scale.

For example, if the length of the desk is 14 inches, then you could choose as scale of $\frac{1}{2}'' = 1\ in$ so the length of the desk in the drawing will be 7 inches.

Choose a scale that works based on your paper size.

First Part: Draw a sketch of the desk. Be sure to include all of the parts that you will be measuring.

Part 2: Accurately measure each part of the desk and make a note of these actual measurements.

Part 3: Choose a scale.

Part 4: Draw your desk to scale.

Portfolio Assessment

When assessing student work, look to see that the student followed directions and then evaluate the accuracy of the measurements and use of scale.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Cv7_CVD6_Yk&feature=related –This is a video from NASA on how to use scale modeling and proportions to understand different situations in science. This is an interesting engaging video that will integrate science and mathematics for students.

http://illuminations.nctm.org/lessondetail.aspx?ID=L652 –This is an NCTM lesson where students are going draw a two –dimensional drawing of a clubhouse using graph paper. It is part of a multi-lesson unit called "Junior Architects."

Understanding Percent

Lesson Project

This project is a trip to a shopping mall. To organize this project, you will work in stages. The first stage is to create a bunch of % off cards. These will be distributed to each student so that each student receives a discount card prior to beginning the project.

Some possible % cards are:

10%

15%

20%

30%

You can create as many different ones as you wish.

Next you will need catalogs or magazines or circulars for different items. Electronic ones are great because the students will love the idea of purchasing electronics.

Next students "go shopping". They cut out pictures of their purchases and complete the data sheet on the project.

You will need:

Glue sticks

Scissors

Colored pencils

Magazines, circulars and other advertisements

Project Outline

Go Shopping

Directions: You are going to go shopping in our classroom. You have a budget of $500.00 to use. You have also been given a discount card that you can use. Cut out pictures of your purchases, glue them to this project outline and complete the math work that is required. Be sure to total your purchases at the end of the sheet.

ITEM PURCHASE PRICE % OF DISCOUNT AMOUNT OF DISCOUNT

FINAL COST TOTAL SPENT

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student follow directions?

Is the work neat and complete?

Did the student calculate the discount and final cost accurately?

Did the student calculate the total amount spent?

Did the student stay within the budget?

Are there corrections that need to be made?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=z1p9o6ymteI –This James Sousa video is an introduction to percent and to understanding percents.

http://illuminations.nctm.org/LessonDetail.aspx?id=L249 –This is an NCTM lesson on understanding percent. In this lesson, students use a 10×10 grid as a model for solving various types of percent problems. This model offers a means of representing the given information as well as suggesting different approaches for finding a solution.

http://www.brightstorm.com/math/algebra/pre-algebra/percents/ –This is a Brightstorm video on percents and on

understanding percents.

Percents, Decimals and Fractions

Lesson Project

This is a team game where you will divide the students into three teams. One team will be the percents, one team the decimals and one the fractions. Then the students will play the game. You will say a percent or a decimal or a fraction. Then the team writes the number accurately and then the other two teams must convert the number called out into their form. Then each group shares their answer.

Accurate responses earn the team a point.

For example: You call out 15%. The percent team writes it out in words correctly. The fraction team writes it out as a fraction and the decimal team writes it out as a decimal.

You will need to vary the numbers you call out so that each team is required to practice their conversions. This is a game played as a whole class. It is a great review game before a quiz or a test.

Project Outline

To play this game, students will need to use scraps of paper to write their answers on. There isn't a project outline sheet for this game.

Portfolio Assessment

Assessment for this activity is completed through observation of students as they play the game. The number of points that a team scores will help you to evaluate each student's work.

Multimedia Links/Additional Activities

www.khanacademy.org/video/percent-and-decimals –This is a Khan Academy video on how to express percents as decimals and decimals as percents.

http://www.youtube.com/watch?v=aUJ-4oD9Oe8 –This is a first of two videos by James Sousa on relating fractions, decimals and percents.

http://www.youtube.com/watch?v=SphqHJVVMak –This is a second video by James Sousa on relating fractions, decimals and percents.

http://illuminations.nctm.org/LessonDetail.aspx?id=L779 –In this lesson, students begin by breaking down a typical summer day into a variety of activities and the amount of time they spend on each. They then translate their activity times into a simplified fraction, a decimal, and a percent. Students create a pie chart for this information that is unique to them. Students who struggle with the calculations will have the opportunity to practice these conversions by playing a game that can easily be differentiated for various levels of learners.

Finding a Percent of a Number

Lesson Project

In this project, students are going to use clip art to design a number and a percent of a number. Here is how the assignment will work. Each student is assigned a number and a percent. You will need to prepare cards or slips of paper with given numbers and percents. There should be one for each student in the class, and they should all be different.

For example: 30 and 20%

In this case, a student would need 30 pieces of clip art to create a visual display that would show 20%. The student needs to complete the math associated with finding the percent of the number and find a way to represent the answer in a visual way.

What is 20% of 30?

20% of 30 is 6.

Out of the 30 clip art items, six of them need to be distinguished in some way. One possible way of doing this is through color.

You can have this be a cut and paste assignment or a technology driven assignment by having the students work on it in the computer lab.

Project Outline

Representing Percents

Directions: You will be creating a visual display of a percent of a number. You will be assigned a number and a percent. Next you will use clip art to show your number and then find a way to represent the % of that number in some way.

For example: 30 and 20%

In this case, a student would need 30 pieces of clip art to create a visual display that would show 20%. The student needs to complete the math associated with finding the percent of the number and find a way to represent the answer in a visual way.

What is 20% of 30?

20% of 30 is 6.

Out of the 30 clip art items, six of them need to be distinguished in some way. One possible way of doing this is through color.

Be creative and have fun! There are many possible options.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the student represent the given number accurately?

Was the student able to identify the answer?

Is the visual display accurate?

Does it show creativity and the student's best effort?

Are there any corrections that need to be made?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=_SpE4hQ8D_o –This is a Khan Academy video on finding a percent of a number.

There are many James Sousa videos on this content which can be found at mathispower4u. They can be selected based on the tutorials needed or wanted in the classroom.

http://www.purplemath.com/modules/percntof.htm –This website has many options for teaching students percent problems. There are many examples and solutions presented for students.

Problem-Solving Strategy: Use a Proportion

Lesson Project

This lesson requires that students do a little research. The internet is the simplest way to accomplish this research since students will be learning about the tube –lipped nectar bat. Once the research is finished, students simply need to complete the project worksheet.

Project Outline

Directions: Complete some research about the tube –lipped nectar bat. Then solve the problem using a proportion.

1. Where does this type of bat live?
2. What does it eat?
3. How long does it live?
4. How big or small is it?
5. Draw a picture of this bat.

The tube –lipped nectar bat has a tongue that is 1.5 times longer than its body. If you had a tongue that was 1.5 times longer than your body, how long would it be? Use a proportion to solve this problem. You will need to measure yourself and keep everything in inches. The first ratio on the bat is set up for you.

$$\frac{3.4 \textit{ inch tongue}}{2.3 \textit{ inch body}}$$

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student complete the research?

Was it done with good effort?

Did the student create a proportion for the second part?

Was the proportion solved correctly?

Does this work demonstrate that the student understands proportions and how to solve them?

If not, what other demonstration will you need to be sure of the student's understanding?

Multimedia Links/Additional Activities

http://www.purplemath.com/modules/ratio4.htm –This website moves from solving simple proportions to solving word problems with proportions.

http://www.mathleague.com/help/ratio/ratio.htm –This page has many resources for tutorials on all of the different lessons presented in this chapter.

http://www.youtube.com/watch?v=vnB1mh5X5cA –This is a James Sousa video on using proportions in problem solving. It has several word problem examples that are relevant for students.

4.9 Geometric Figures

This ninth chapter *Geometric Figures* covers basic principles of geometry. Students are introduced to angles, triangles, quadrilaterals, and polygons. Also explored are congruent and similar figures, line symmetry, and strategies for using Venn Diagrams to solve problems. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Introduction to Geometry
- Classifying Angles
- Classifying Triangles
- Classifying Quadrilaterals
- Classifying Polygons
- Congruent and Similar Figures
- Line Symmetry
- Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Introduction to Geometry

Lesson Project

This project is about using geometric terms to create a word search. Students will need graph paper to work with so that each letter can be placed in a box. Words can be written forward, backwards and diagonally.

Project Outline

Geometry Word Search

Directions: You are going to use graph paper to create a word search of geometric terms. Each letter should be placed in a box and the words can be written backwards, forwards or diagonally. These are the words for the word search.

Point

Line

Ray

Line Segment

Intersecting

Geometry

Parallel

Angle

Vertex

Protractor

Portfolio Assessment

When assessing student work, you will be looking at each word search. The student should have used each word in the search and the search should be challenging and show effort in its creation.

You can use these word searches as a homework assignment or an extra credit project.

Multimedia Links/Additional Activities

http://www.onlinemathlearning.com/basic-geometry.html –This website has basic geometry terms and videos to accompany the given definitions.

Classifying Angles

Lesson Project

This project involves a trip to a local skatepark. The students are going to be hunting for all of the angles that they can find at the skatepark.

Students need two examples of each of the following angles.

Acute angles

Obtuse angles

Straight angles

Supplementary Angles

Complementary Angles

If the angles are acute or obtuse, the students also need to estimate the number of degrees in the angle.

Each example should be a drawing with a description so that it is easy for others in the class to identify where the identified angle was located.

Project Outline

Angle Inventory

Directions: In this assignment, you are going to be analyzing angles at the skate park. Just like the students in our story, you are going on an angle hunt and your task is to inventory the angles that you can find at the skate park. To complete this inventory, you will draw at least two examples of each type of angle. If your angles are acute or obtuse, you must also estimate the angle measurement of each angle.

Begin by defining each type of angle.

1. Acute
2. Straight
3. Obtuse
4. Complementary Angles
5. Supplementary Angles

Now complete the inventory at the skatepark.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student define each term correctly?

Was the student focused and on task at the skate park?

Did the student give two examples for each type of angle?

Did the student estimate angle measures for acute and obtuse examples?

Are the drawings clear?

Does this represent good effort and the student's best work?

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/angle-basics?playlist=Geometry –This is a Khan Academy video on angle basics.

http://www.khanacademy.org/video/measuring-angles?playlist=Geometry –This is a Khan Academy video on measuring angles using a virtual protractor.

http://www.shodor.org/interactivate/activities/ –There are several interactive activities on this page for students that involve angles. You will need to scroll down to the geometry section to find these activities.

Classifying Triangles

Lesson Project

In this project, students are going to use art to create a design of all types of triangles. Students are going to draw these triangles using rulers and outlines. All of the triangles should be outlined and not colored in. You can give students a sheet of paper and ask them to fill the page with triangles. Every type of triangle should be represented in the design. The student provides a color key.

For example: Color all scalene triangles RED

After the students are finished with the design, collect them and make copies. The next day, hand out the designs randomly and have the students color them according to the key provided.

Project Outline

Triangle Tangle

Directions: You are going to use art to create a design of all types of triangles. You are going to draw these triangles using rulers. All of the triangles should be outlined and not colored in. You can use the attached sheet of paper to fill the page with triangles. Every type of triangle should be represented in the design. When finished, provide a color key.

For example: Color all scalene triangles RED

Part One: Define each type of triangle.

1. Acute Triangle
2. Right Triangle
3. Obtuse Triangle
4. Equiangular Triangle
5. Scalene Triangle
6. Isosceles Triangle
7. Equilateral Triangle

Now create your diagram and don't forget the key.

Portfolio Assessment

You can assess this project on a few different levels. First, you can analyze the diagrams that the students create.

Look to see if all of the different triangles have been included in the diagram and that they have been accurately drawn.

Next, you can look and see if the triangles have been colored correctly. When the students are asked to color someone else's diagram, you will be able to assess their understanding by looking at the accuracy of their work.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=uMcEgALdvgk –This is a Khan Academy video on classifying triangles according to sides and angles.

http://www.youtube.com/watch?v=vjcyzDmigVI –This is a math made easy video on types of triangles.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=142 –This is an illuminations interactive computer activity where students design and classify triangles.

http://illuminations.nctm.org/lessondetail.aspx?ID=L656 –This is an activity from NCTM that can be done in a classroom where students attempt to classify the triangles formed in a plane when a randomly selected point is connected to the endpoints of a given line segment.

Classifying Quadrilaterals

Lesson Project

This is a project where students are going to write clues to describe different quadrilaterals. Then the clues are used to play a game. Here is how it works. Students are assigned a small group of two or three people to work with. Then the students begin the creation part of the game. Without using the name of the quadrilateral, they have to describe it.

For example: I have four congruent sides. Who am I?

The answer is a square.

The students write clues to describe each type of quadrilateral.

Next, the students meet up with another team and play the game. Each team takes turns giving clues to the other team. Then the students have to try to guess the solution. Each correct clue is worth a point.

Teams can continue to play until they run out of clues.

Project Outline

Who am I? Quadrilateral Puzzles

Directions: You are going to work in a small group on writing puzzles about different quadrilaterals. These word clues or puzzles are going to be used in a game that you will play against another team. In this game, you will try to guess using their clues, and they will try to guess using your clues.

For example: I have four congruent sides. Who am I?

The answer is a square.

Be tricky! Correct answers are worth a point for the other team.

Use part one of this sheet to write down facts about each figure. Use part two to write your clues.

Part One: Define each term.

1. Quadrilateral
2. Trapezoid
3. Rectangle

4. Parallelogram
5. Square
6. Rhombus

Part Two: Write clues for each term. You can write more than one clue with the same answer. Be tricky and use math vocabulary. End each clue with the words "Who am I?"

Portfolio Assessment

Student assessment is completed by reading through each set of clues and by observing the students as they play the game.

Multimedia Links/Additional Activities

http://www.mathsisfun.com/quadrilaterals.html –This website has the definitions for all of the different types of quadrilaterals. It also has a computer application where students can manipulate and move different quadrilaterals to explore each figure.

http://illuminations.nctm.org/LessonDetail.aspx?id=L655 –This is an activity from NCTM on quadrilaterals and on understanding quadrilaterals. Using an applet, students explore certain characteristics of diagonals and the quadrilaterals that are associated with them.

http://www.nctm.org/profdev/content.aspx?id=23748 –This article talks about classroom dialogue as it relates to mathematics. It is a useful article for professional development.

Classifying Polygons

Lesson Project

In this project, students are going to use the vocabulary that they have learned about polygons to create a crossword puzzle. Students will need pencils, pens and graph paper.

Project Outline

Polygon Puzzle

Directions: You are going to use the terms from this lesson to create a crossword puzzle. Different from a word search, you will need to include clues. It is recommended that you create a puzzle answer key first, and then go back and make one without the answers included in it. Use part one to define each term and part two to create the crossword puzzle.

Part One: Define each term.

1. Polygon
2. Pentagon
3. Hexagon
4. Heptagon
5. Octagon
6. Nonagon
7. Decagon
8. Regular Polygon
9. Irregular Polygon
10. Congruent

Part Two: Design your crossword puzzle.

Portfolio Assessment

As you assess student work, keep the following questions in mind.

Did the student define each term correctly?

Did the student write clues to go with the crossword puzzle?

Did the student create an answer key and an empty puzzle?

Does the puzzle work?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/lessonDetail.aspx?id=L270 –This is a game from NCTM called "Polygon Capture". In this lesson, students classify polygons according to more than one property at a time. In the context of a game, students move from a simple description of shapes to an analysis of how properties are related.

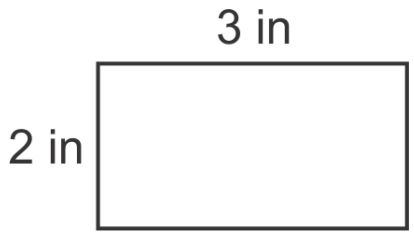
http://illuminations.nctm.org/lessonDetail.aspx?id=L277 –In this activity from NCTM, students identify and classify polygons according to various attributes. They then sort the polygons in Venn Diagrams, according to these attributes. Extensions to fundamental ideas about probability and statistics are also included.

Congruent and Similar Figures

Lesson Project

Similar figures are connected to work with ratios and while many students understand congruent figures, there is some difficulty when it comes to working with similar figures. In this activity, students are going to draw figures that are similar write corresponding side lengths as a proportion.

For example:



If this was a figure drawn, the students could draw any figure similar to it and then write the corresponding side lengths as a proportion.

If the new figure has a width of 4 inches and a length of 6 inches, then the following proportion would be created.

$$\frac{2}{4} = \frac{3}{6}$$

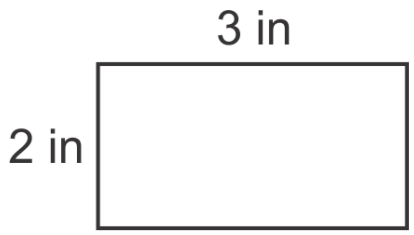
You can specify how many figures you would like the students to draw. You can also vary the difficulty of the assignment. Sometimes, students can be required to draw a figure that is a quadrilateral or other polygon. Then the side lengths would form numerous ratios and proportions.

You can also use graph paper to help students in their work.

Project Outline

Directions: Similar figures are connected to work with ratios and while many students understand congruent figures, there is some difficulty when it comes to working with similar figures. In this activity, you are going to draw figures that are similar write corresponding side lengths as a proportion.

For example:



If this was a figure drawn, the students could draw any figure similar to it and then write the corresponding side lengths as a proportion.

If the new figure has a width of 4 inches and a length of 6 inches, then the following proportion would be created.

$$\frac{2}{4} = \frac{3}{6}$$

Create at least five different pairs of similar figures. Write the proportion that is connected with each pair of figures.

Portfolio Assessment

When assessing student work, you can keep the following questions in mind.

Did the student follow directions?

Did the student create five pairs of figures?

Are the figures similar?

Do the side lengths form a proportion?

Did the student write the proportion correctly by comparing the values associated with corresponding sides?

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/similar-triangles?playlist=Geometry This is a Khan Academy video on similar triangles.

http://www.brightstorm.com/math/geometry/similarity/properties-of-similar-polygons/

This is a Brightstorm video on understanding similar polygons.

Line Symmetry

Lesson Project

In this project, students are going to use pattern blocks to create symmetrical patterns. When you begin, ask the students to draw a vertical line down the center of a piece of paper. Then using pattern blocks, the students need to create a pattern of some kind on the left side of the paper.

After creating it with blocks, the students draw in the pattern. Then they will use blocks to create a symmetrical pattern on the right side of the paper. Students sketch in both sides and color them.

To add a level of difficulty, you can have the students draw a line vertically and horizontally to create four squares. Then the students create a pattern in one of the boxes and then they must create mirror images in the other three boxes.

You will need pattern blocks like the ones pictured here:

http://www.eaieducation.com/Category/71_1/Pattern_Blocks.aspx

Project Outline

This project is explained in the project description above. Students are going to use pattern blocks, paper and colored pencils to create the symmetrical patterns. There isn't a project outline for this project.

Portfolio Assessment

When assessing student work, you will be looking for the accuracy of the symmetry in the student designs. You also want to evaluate whether or not the student has done his/her best work in the design of the patterns.

Multimedia Links/Additional Activities

http://illuminations.nctm.org/activitydetail.aspx?id=24 –This is a mirror tool from NCTM. Students can use this tool to investigate symmetry. You can rotate, flip, or reflect a figure across a line.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L556 –This is an activity from NCTM where students explore and count the lines of symmetry. This is one lesson in a multi-lesson unit on geometry and art.

Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Lesson Project

In this project, students are going to use geometric shapes to create patterns represented by an equation. Students should take each equation and create a pattern of shapes that corresponds to the pattern.

Project Outline

Shapes and Patterns

Directions: Use shapes or pattern blocks to create patterns which represent each of the equations listed below. You will need to show three steps of the equation.

For example: $x+1$

Now you are going to create your own patterns for the following equations.

$$2x+1$$
$$x+3$$
$$2x-1$$
$$5x$$
$$3x+2$$

Portfolio Assessment

When assessing student work, here are some of the key points to keep in mind.

Does the pattern show steps?

Did the student use geometric shapes or patterns?

Does the pattern progress in the right way?

Is the equation clear in the visual pictures?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/lessonDetail.aspx?id=L277 –In this activity from NCTM, students identify and classify polygons according to various attributes. They then sort the polygons in Venn Diagrams, according to these attributes. Extensions to fundamental ideas about probability and statistics are also included.

http://www.shodor.org/interactivate/lessons/VisualPatterns/ –Here is an activity from Interactivate which uses patterns and tessellations.

4.10 Geometry and Measurement

This tenth chapter *Geometry and Measurement* further covers geometric principles through methods for finding the area of parallelograms, triangles, and circles. The student is also introduced to the concepts of solid figures, surface areas, and volumes of prisms and cylinders.

In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

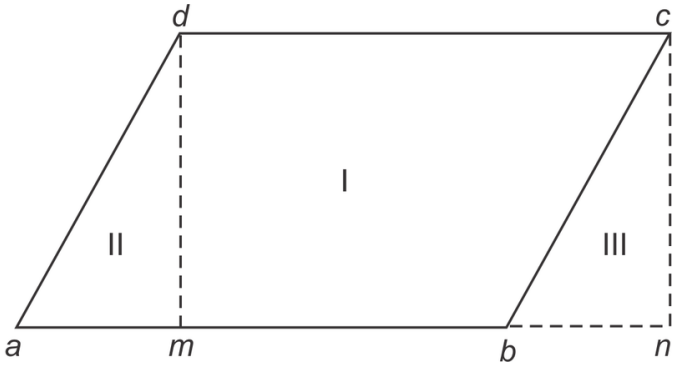
The following lessons are part of this chapter.

- Area of Parallelograms
- Area of Triangles
- Circumference of Circles
- Area of Circles
- Classifying Solid Figures
- Surface Area and Volume of Prisms
- Surface Area and Volume of Cylinders
- Problem-Solving Strategy: Solve a Simpler Problem

Area of Parallelograms

Lesson Project

There are many different ways that you can create projects with the area of parallelograms. In this project, students are going to practice finding the area of different parallelograms. You will need to enlarge the figure on the student outline.



Project Outline

Figuring Out Parallelograms

Directions: You will need paper,ruler, scissors and glue for this exploration. You have been handed a parallelogram from your teacher. Now take the parallelogram and measure all of its parts. Next, cut the sections two and three off of the parallelogram and reglue the figure into the shape of a rectangle. Now answer the following questions.

1. What was the measure of the base of the parallelogram?
2. What was its height?
3. What is the width of the rectangle?
4. What is the length of the rectangle?
5. How does the width and length of the rectangle compare with the height and base of the parallelogram?
6. What conclusions can you draw about the connections between these two figures?

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student follow directions?

Did the student measure the base and height of the parallelogram?

Did the student measure the length and width of the rectangle?

Is the student able to make connections between a rectangle and a parallelogram?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=npY54EKjWaA –This is a James Sousa video on how to find the area of a parallelogram.

http://www.youtube.com/watch?v=TN4tm_rONNc –This is a James Sousa video on how to find the area of a rectangle.

http://illuminations.nctm.org/LessonDetail.aspx?id=L578 –In this lesson from NCTM, students will use their knowledge of rectangles to find the area of a parallelogram.

http://illuminations.nctm.org/activitydetail.aspx?id=21 –This is an interactive activity from NCTM where students can click on different figures and explore the shape of those figures as well as the area of those figures.

Area of Triangles

Lesson Project

In this project, students are going to create a pattern of triangles. The triangles need to go from smaller to larger with the base measurement varying by small increments. For example, the first triangle is a right triangle with a base of 2 inches and a height of 2 inches. The student begins by drawing this triangle. Then the student must figure out the area of the triangle. Next, the student needs to draw four other triangles. In each new triangle, the base should increase by a small amount. The student can choose the pattern for enlarging each base. Then the students will find the area of each triangle.

The final step of the project is a class discussion where the students talk about how the area of each triangle changed when the base was increased. Observations and conclusions should be noted by the teacher.

Project Outline

Changing Triangles

Directions: In this project you are going to create a pattern of triangles. The triangles need to go from smaller to larger with the base measurement varying by small increments. For example, the first triangle is a right triangle with a base of 2 inches and a height of 2 inches.

Draw this triangle first. Now figure out the area of the triangle.

Next, you will draw four other triangles. The height of each triangle should remain the same as the first triangle, but the base should get gradually larger. You can choose the increments that you increase the length of the base. Then

figure out the area of each triangle in the pattern.

Portfolio Assessment

When assessing student work, here are some questions to help lead the discussion.

How did the area change when the base changed?

What pattern of change did you use?

What connections can we make between the length of the base of a triangle and the area?

Do you think the height has as much impact on the area? Why or why not?

What if we decreased the length of the base? Do you think we would have seen the same change in area?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=yEV8-sc094c –This is a James Sousa video on how to figure out the area of a triangle.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L577 –In this lesson, students develop the area formula for a triangle. Students find the area of rectangles and squares, and compare them to the areas of triangles derived from the original shape.

http://illuminations.nctm.org/activitydetail.aspx?id=108 –This is an interactive area tool from NCTM. Use this tool to determine how the length of the base and the height of a figure can be used to determine its area. Can you find the similarities and differences between the area formulas for trapezoids, parallelograms, and triangles?

Circumference of Circles

Lesson Project

This project is an outdoor project. You will need string, rulers, yard sticks and chalk. Each child is going to practice drawing different size circles. Begin with a discussion about the parts of a circle. Then ask the children to measure a length of string. This string will be the diameter of the circle. The ideal situation is when children choose different lengths of string so that the size of each circle is different. The relationship between the circumference and the diameter because clear in these types of experiments. Children then connect the chalk to the end of the string. One child holds the other end of the string, and the other child draws the circumference of the circle with the chalk.

After everyone has had a chance to draw a few, walk around and look at them. Then ask the children to bring a list of the lengths of each string into the classroom. This is when you can use the string lengths to determine the actual circumference of each circle.

Project Outline

Directions: Use this page to write down the length of each string that you use to draw a circle.

Portfolio Assessment

When assessing student work, here are some questions to think about.

Does this student understand the connection between diameter and circumference?

Is the student familiar with all of the terms associated with circles?

Did the student draw the circle correctly?

Did the student keep track of the length of each string he or she used?

Is the student able to connect the drawing of the circle to the formula for circumference?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=sHtsnC2Mgnk –This is a James Sousa video on how to determine the circumference of a circle.

http://www.khanacademy.org/video/circles–radius–diameter-and-circumference?playlist=Geometry –This is a Khan Academy video on understanding the radius, diameter and circumference of circles.

http://illuminations.nctm.org/activitydetail.aspx?id=116 –This is an interactive tool for NCTM where students can explore the relationships in a circle. How do the area and circumference of a circle compare to its radius and diameter? This activity allows you to investigate these relationships in the Intro and Investigation sections and then hone your skills in the Problems section.

Area of Circles

Lesson Project

This project builds on the lesson project for the lesson on finding the circumference of a circle. You can do this project with that one or they can be done separately. In this lesson, students are going to use a string to create different size circles. The string represents the diameter of the circle. They are to use one string, but can use different lengths of string. The string can't be longer than 12 inches. Then the students use a pencil and draw different size circles. After completing this step, the students use the formula for area to find the area of each circle. Connections can be made between the length of the diameter and the area of the circle.

Project Outline

Part One: Drawing Circles

Directions: In this lesson, you are going to draw circles of different sizes. Begin by cutting a length of string. This will be the diameter for all of the circles that you create. You can vary the length of the string so that you will create one large circle and several different sized circles. Choose your beginning string then attach a pencil to one side of the string. This will make a compass to draw circles. You will need five different circles to work with.

Part Two: Working with Area

Now that you have five different circles, use the length of the diameter for each circle to figure out the area for that circle. How does the area of the circle change based on the diameter?

Portfolio Assessment

When assessing student work, you can keep the following questions in mind.

Did the student follow directions?

Did the student draw five different circles?

Does the student understand how to use the formula to find the area of a circle?

Does the student see a connection between the length of the diameter and the area of the circle?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/LessonDetail.aspx?ID=L574 –This is a lesson from NCTM where students explore finding the area of a circle through different relationships. Using a circle that has been divided into congruent sectors, students will discover the area formula by using their knowledge of parallelograms. Students will then calculate the area of various flat circular objects that they have brought to school. Finally, students will investigate various strategies for estimating the area of circles.

http://www.youtube.com/watch?v=SIKkWLqt2mQ –This is a James Sousa video on how to find the area of a circle.

http://www.khanacademy.org/video/area-of-a-circle?playlist=Developmental+Math –This is a Khan Academy video which uses word problems to help students find the area of a circle.

Classifying Solid Figures

Lesson Project

To complete this project, students will be making a set of solid figures. There are many resources on the internet for printing out nets of solids. Students will create each solid figure and then complete the project outline on the different parts of each solid figure. You can specify each type of solid that you want the students to build. This project could also be stretched out and done in combination with learning about surface area and volume. For example, students create a rectangular prism and then figure out the surface area and volume for the figure that they created.

http://www.senteacher.org/wk/3dshape.php –This website has many different nets for creating solid figures.

http://gwydir.demon.co.uk/jo/solid/ –This is website on solid figures and their characteristics.

Project Outline

Creating Solids

Directions: After building each solid figure, complete the chart on each type of solid.

Name of Solid Faces Edges Vertices

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student create a solid of each figure that was assigned?

Did the student complete the project outline?

Does the students understand the vocabulary associated with each solid figure?

Does the student understand how to categorize each figure?

Did the student accurately count the faces, edges and vertices of each figure?

Additional Resources

http://www.learner.org/interactives/geometry/about.html –*Geometry 3-D Shapes* is an interactive Web site where students can learn about three-dimensional shapes, calculate surface area and volume, and discover some of the mathematical properties of shapes.

http://illuminations.nctm.org/activitydetail.aspx?id=70 –This tool allows you to learn about various geometric solids and their properties. You can manipulate and color each shape to explore the number of faces, edges, and vertices, and you can also use this tool to investigate the following question:

For any polyhedron, what is the relationship between the number of faces, vertices, and edges?

http://illuminations.nctm.org/LessonDetail.aspx?id=L237 –In this activity, students learn the names of solid geometric shapes and explore their properties. At various centers, they use physical models of simple solid shapes, including cubes, cones, spheres, rectangular prisms, and triangular prisms.

Surface Area and Volume of Prisms

Lesson Project

This project has a couple of different parts to it. The first thing that needs to happen is that the students must bring in boxes from home. You are looking for all different kinds of boxes. Then assign each student a number to put on his/her box. Finally, you can put all of the boxes on a table in front of the room.

Next, the students are going to write down the numbers of the boxes that they think have the greatest surface area. Every box is included in the list, but the numbers of the boxes are organized from the greatest surface area to the smallest surface area. For example, a student might think that #8 has the greatest surface area, he/she would write this one first on their list.

Next, the student repeat the process except this time they are thinking about volume.

After the lists have been completed, organize the students in small groups. Now they will actually have to figure out the surface area and volume of each box and revise their lists. Bring in a huge bag of packing peanuts for the students to work with. This is a fun way of exploring volume.

Finally, conduct a whole class discussion on the project.

Project Outline

Box Sort

Directions: You have each brought in a box from home. Label each box with the number that you have been assigned. Your task is to organize the boxes in two different ways. The first way is to organize the boxes so that they are numbered so that the box with the greatest surface area is a listed first and the box that you think has the least amount of surface area is listed last. This is part one.

Then you must list the boxes according to which box has the greatest volume. You will list the boxes from the greatest volume to the smallest volume. Be careful, volume is tricky, sometimes a box that you think has the greatest volume actually doesn't.

When finished, you will discuss your estimates in small groups. Then you will figure out if you were accurate or not.

Portfolio Assessment

Student work can be assessed through the project outline and through observation of group discussions. Many students will be surprised when their estimates are or are not accurate. This can be a real hands–on learning experience for the students. Provide the students with feedback and observe the efforts of each group.

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/solid-geometry-volume?playlist=Geometry –This is a video from Khan Academy on how to find the volume of a prism.

http://www.khanacademy.org/exercise/solid_geometry –This is an interactive exercise from Khan Academy on how to find the surface area of a cube.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L262 –In this lesson from NCTM students investigate the surface area of various different rectangular prisms.

Surface Area and Volume of Cylinders

Lesson Project

To do this project, the students will have an estimation part and an experiment part. It would be wise to do this outside. The students are going to be moving water. Each group will be given the same amount of water. You can choose 5 gallon or 10 gallon containers. Local water companies have empty bottles that you can get empty and fill up with tap water. You will also need cylinders of different sizes for the students to move the water.

Each team will be assigned a cylinder. Then the teams will estimate how many trips it will take to move all of their water from the container to a new bucket or small kiddie pool. The estimate should be written on a piece of paper and turned in before the students begin.

Then the students actually move the water. The team that comes closest to their original estimate wins.

Project Outline

The Moving Project

Directions: here is an overview of your project.

To do this project, the students will have an estimation part and an experiment part. It would be wise to do this outside. The students are going to be moving water. Each group will be given the same amount of water. You can choose 5 gallon or 10 gallon containers. Local water companies have empty bottles that you can get empty and fill up with tap water. You will also need cylinders of different sizes for the students to move the water.

Each team will be assigned a cylinder. Then the teams will estimate how many trips it will take to move all of their water from the container to a new bucket or small kiddie pool. The estimate should be written on a piece of paper and turned in before the students begin.

Then the students actually move the water. The team that comes closest to their original estimate wins.

Write down your estimate on a piece of paper and hand it in to your teacher.

Portfolio Assessment

To assess student work, ask the students to write up their experiment and the results from their experiment. Here are questions that you can use as a guide.

What was your estimate?

Draw a picture of your cylinder?

How many trips did it take for you to move your water?

Were you surprised by this number?

Why or why not?

What did you learn about the volume of cylinders?

Is there anything that you would do differently if you were to do this experiment again?

Why or why not?

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/cylinder-volume-and-surface-area?playlist=Geometry This is a video from Khan Academy on how to find the volume and surface area of a cylinder.

http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AS-Cylinders.pdf –This is a pdf from NCTM Illuminations which is a project about figuring out the volume of a cylinder that has been made from paper.

Problem-Solving Strategy: Solve a Simpler Problem

Lesson Project

This project takes the story problem topic from the lesson and connects it with a student exploration of quilts. Students begin by researching some facts about quilts and finally design their own drawing of a quilt using geometric shapes. The project can stop there with the design and drawing, or can be expanded into the students actually sewing their own quilts.

Project Outline

Quilts and Mathematics

Directions: There are many ways that quilts and quilt making connects to mathematics. Through the story problems in the chapter, you have been able to see some of these connections. Now you are going to do some research on quilts and finally end up designing your own quilts using geometric shapes and patterns.

Part One: Learning about Quilting –use the following website to learn about the history of quilting.

http://en.wikipedia.org/wiki/History_of_quilting

1. When did quilting actually begin?
2. What is a doublet, and who wore it?
3. Which country has the oldest sample of a quilt?
4. Name three different types of quilts and describe each quilt.
5. Now do a search on the internet and discover the definition of a star

quilt.

Part Two: Designing a Quilt

In this chapter, you have been learning all about different geometric shapes and measurements associated with geometry. Now you are going to apply what you have learned by drawing and designing a quilt. You will use a ruler and colored pencils. Your measurements should be accurate and show care and effort. Here are a few guidelines.

Your quilt will be $54'' \times 80''$. However, you can't draw it that big. You will need to use a scale.

$\frac{1}{4}'' = 1''$

Now you can figure out the new dimensions of the quilt. Remember that a quilt consists of squares, so you will need to break down the dimensions of the quilt to figure out how many squares and how big the squares will be that you use for your quilt.

Portfolio Assessment

Assessment is done through observation and through the work done on the project outline. When working with the students, you may want to allow students the chance to work in small groups. This will be especially helpful when and if students are struggling with the problem solving.

Multimedia Links/Additional Activities

http://www.mathtv.org/ –This is an interactive program from math playground where students solve word problems.

http://www.thefutureschannel.com/problem_solving/problem_solving_movies.php –This website has a bunch of videos that involve problem solving.

4.11 Integers

In this eleventh chapter, *Integers*, students will learn about positive and negative numbers, adding numbers, sums and differences of integers, multiplication, division, coordinates to name locations, transformations, and all about data displays. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Comparing Integers
- Adding Integers
- Subtracting Integers
- Multiplying Integers
- Dividing Integers
- The Coordinate Plane
- Transformations
- Surveys and Data Displays

Comparing Integers

Lesson Project

This project is an extension to the project that is the focus of the story problem in the text. This project deals with a comparison of temperatures. To complete the project, the students will first need to record the outside temperature for one month. Students should also record it at home on the weekends and then complete a chart in the classroom. To do this, you will need access to an outside thermometer and a chart for the students to track the temperature.

When finished with the tracking, the students will need to figure out the average temperature for the month. Then each student should pick a place somewhere in the United States to compare temperatures with. The students complete this part of the research using the internet.

Use the project outline to help keep track of the data. Students create a poster to share their comparisons as a final part of the project.

Project Outline

Temperature Totals

In the story problem in the text, the students in Mrs. Harris' class are comparing temperatures. You are going to do the same thing and then expand upon the data that you gather. The first step is to gather data on the temperature in your area. You will do this for one month.

TABLE 4.4:

Day/Date	Temperature

What was the average temperature for the month?

Part Two: Comparing Temperatures

Now choose a city somewhere in the United States to compare your temperature with the average temperature for last month.

Part Three: Chart

Now create a display to show your data. Include the temperature chart and the comparisons between your city and the city of your choice. Write the final comparison as a comparison between integers.

Portfolio Assessment

Here are some questions to keep in mind when you assess student work.

Did the student complete the chart?

Are the temperatures accurate?

Did the student find the average temperature?

Was the average calculated accurately?

Does the chart show an integer comparison?

Does it show the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=04pURxo-iu0 –This James Sousa video focuses on an introduction to integers including comparing integers.

http://www.khanacademy.org/video/ordering-negative-numbers?playlist=Pre-algebra –This is a Khan Academy video on ordering negative numbers from least to greatest.

http://illuminations.nctm.org/LessonDetail.aspx?id=L819 –This is a game from NCTM to help students to understand the relationship between integers. Positive and negative numbers become more than marks on paper when students play this variation of the card game, Rummy. Engaged in a game involving both strategy and luck, students build understanding of additive inverses, adding integers, and absolute value.

Adding Integers

Lesson Project

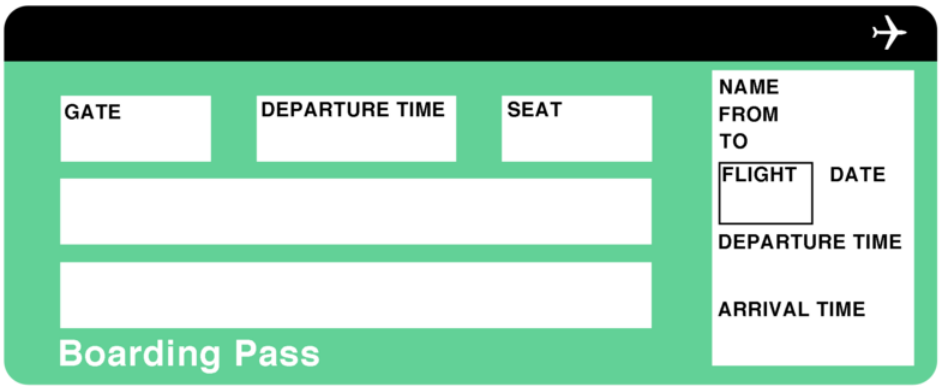
In this project, students are going to pretend that they are traveling to New Zealand. They are going to be calculating time zones and writing addition sentences and explanations to show how time changes as they travel. To complete this project, you will need to make up some tickets to help the students to have fun. The tickets should have a day and time and location of departure. Then the arrival time is left blank. The students must calculate the time that they will arrive in New Zealand.

You could choose any global city to travel to. In fact, this could be an additional extension of the project. However, using New Zealand links the project to the story problem from the text which creates an integrated unit for the students.

The project outline is the ticket template for the students. You can fill them out and print them for the students.

Project Outline

Time Travel



Portfolio Assessment

When assessing student work, the ticket is your assessment tool. You can collect the tickets from the students and they will have filled in the time that they are going to arrive in New Zealand. You can also add in an assessment piece for students to help each other. Once you have collected the tickets, redistribute them to other students. These students help by correcting the work of a peer. The ticket is either accepted or declined.

These tickets could also make a fun display for the classroom. A couple of students could create this display as part of an extra credit project.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=Js9sVSFpyFE –This is a James Sousa video on adding integers using Algebra tiles.

http://www.youtube.com/watch?v=204uFu0DRWE –This is a James Sousa video on adding integers using a number line.

http://www.khanacademy.org/video/adding-integers-with-different-signs?playlist=Developmental+Math+2 –This is a Khan Academy video on adding multiple integers with different signs.

http://illuminations.nctm.org/LessonDetail.aspx?id=L819 –This is a game from NCTM to help students to understand the relationship between integers. Positive and negative numbers become more than marks on paper when students play this variation of the card game, Rummy. Engaged in a game involving both strategy and luck, students build understanding of additive inverses, adding integers, and absolute value.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L734 –In this lesson, students will adapt expressions that add or subtract two signed integers. This lesson builds on the previous lesson in the unit, where students add/subtract just a positive integer from a signed integer. Then, students will play a game that practices adding and subtracting with signed integers. The game will enforce the skill of arithmetic with integers, but will also get students thinking about maximizing their sum, and ordering signed numbers.

Subtracting Integers

Lesson Project

This project involves students practicing the addition and subtraction of integers on a number line. The number line will not be on a piece of paper, however, but it will be on the floor. You can use colored electrical tape to make an integer number line on the floor. Be sure to mark 0. The other numbers don't need to be specifically marked as long as you indicate where they are by using a small piece of tape at each interval.

Next, the students write a number sentence using addition and subtraction of integers. The sentence must also

require the use of positive and negative numbers. All students write a number sentence without a solution on a piece of paper. Then these are collected and the game begins.

Now you can divide the students up into pairs. Each pair has a turn trying to move the number sentence on the number line. You will need to be one step ahead of the students by having the solutions ready. If necessary, you could have them write number sentences one day, you solve them that night, and then do the project the next day.

Each team gets a turn on the number line and points are recorded for correct answers. This could end up as a fun competition where two teams are left standing –much like a spelling bee.

Project Outline

The activity itself does not have a project outline.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the student write a number sentence that has both addition and subtraction in it?

Does the number sentence require the use of negative and positive numbers?

Do the students understand how to add and take away on the number line?

Is the student able to "think on his/her feet" when solving problems?

Are the answers accurate?

Is there any review needed given the ability or difficulty that students had with this project?

Complete this project with an in class discussion. Students should talk about their experience and what they learned by completing this project.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=CWynE4T4-5w –This is a James Sousa video on subtracting integers and on the basics for learning to subtract integers.

http://www.youtube.com/watch?v=ciuIKFCtWWU –This is a James Sousa video which focuses on an example of subtracting two digit integers.

http://www.youtube.com/watch?v=rywucgsvDjU –This video is an example of a real –world application using the example of an overdrawn checking account.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L734 –In this lesson, students will adapt expressions that add or subtract two signed integers.

This lesson builds on the previous lesson in the unit, where students add/subtract just a positive integer from a signed integer. Then, students will play a game that practices adding and subtracting with signed integers. The game will enforce the skill of arithmetic with integers, but will also get students thinking about maximizing their sum, and ordering signed numbers.

Multiplying Integers

Lesson Project

In this project, students are going to research Mount Washington one of the highest peaks in the United States. Mount Washington looms over the skies of New Hampshire and in this exploration, students will do research and then use that research to solve math problems.

Project Outline

Mount Washington Magic

Directions: Use this first part to complete research on Mount Washington. You can use the internet or other books to complete each section.

1. In which state is Mount Washington located?
2. How many feet is it from the base to the summit?
3. About how many hikers climb it each year?
4. When was the first summit of Mount Washington?
5. What is one of the things that the mountain is known for?
6. Where is Tuckerman's Ravine?
7. What important building can be found on the summit of Washington?
8. True or false. You can only summit Washington by climbing it.
9. What is the change in temperature from the base of the mountain to the summit?
10. If there is a 2° change every 10 feet on Washington, what would be the temperature difference between the base and the summit?
11. How many trails are there that ascend Washington?
12. Is Mount Washington part of the Appalacian Trail?
13. If it is 65° at the base of Washington, what temperature is it at the summit?
14. Write this as an equation.

Now take what you have learned and create a poster about Mount Washington. Use pictures and be sure to include arithmetic in your work.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the student complete all of the questions?

Are the answers accurate?

Did the student write a number sentence and a solution for number 14?

Does the poster show the student's best work and effort?

Allow time for students to share and present their work in class.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=YR6BzEjLx6k –This is a James Sousa video which focuses on multiplying integers.

http://www.youtube.com/watch?v=rx8F9SPd0HE –This video focuses on examples on how to multiply three or more integers together.

http://illuminations.nctm.org/LessonDetail.aspx?id=L285 –This is an NCTM game which focuses on helping students to understand the multiplication of integers. In this lesson, students experience beginning-algebra concepts through discussion, exploration, and videotaping. The concept of multiplication of integers is presented in a format which encourages understanding, not simply rote memorization of facts.

Dividing Integers

Lesson Project

In this project, students are going to create a football game. The football game should have negative yards and positive yards and should have at least ten plays in it. Each student draws a football field. They can draw pictures of

players and a football as well. Many students will really enjoy doing this because they can use their favorite teams. If you have students who aren't familiar with football, you can begin by sharing with them the basic rules of yardage and touchdowns.

Each team should have a series of plays recorded on the football field. The students use integers to show the yardage gained and lost. Allow time for the students to present their work to the class. See the project outline for more details.

Project Outline

Directions: You are going to create your own football game. First, you will need to draw a football field. Use color and choose two teams who are going to play against each other. You may draw players or simply draw a football and show how the football moves on the field with each play. The game is to have at least ten plays. You may choose more plays if you wish and there should be yards lost and gained.

After drawing out the game, you will need to write integer sentences to describe the plays of each team. Choose two different colors. One color for one team, and one color for the other team. These colors should be used when you write your integer sentences so that someone looking at your poster can follow the action.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student follow directions?

Do the integer sentences match with the action drawn on the football field?

Does this poster demonstrate an understanding of integer operations?

Can the student explain his/her work?

Does this poster represent the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=o6zh558w8R4 –This is a James Sousa video that explains the basics of dividing integers. It also provides several examples.

http://www.youtube.com/watch?v=z5ZFiyLi5Y0 –This is a James Sousa video which provides additional examples on dividing integers.

http://teams.lacoe.edu/documentation/classrooms/amy/number/6-8/teacher/teacher.html

This website has helpful math resources for teachers. There are several links as well as ideas for building competencies in the area of mathematics.

The Coordinate Plane

Lesson Project

This project has the students create a map of their school. To do this, they will be using a coordinate plane. In the story problem in the lesson, the students used a coordinate plane to create a map of a town. In this project, students will be doing something similar, except that it will be the school.

You may want to purchase large sheets of graph poster/chart paper. This can be bought at any office supply store. Then you should create a map yourself so that you can figure out a goal for the students. Their maps may be different from yours, but you can discuss this in the creative realm of the school. You don't have to choose to put every single thing on the map, but could focus on the places where the students actually have classes or eat lunch.

Here is a beginning plan to get you started:

First, brainstorm all of the places where the students have classes or participate throughout the day. Make a list of these things on the board.

Next, either pair the student up into teams or small groups. Give each group a large sheet of chart paper, pencils, colored pencils, and yardsticks or rulers.

Ask the students to draw a coordinate grid on their paper. Based on your school, decide what should be (0,0). For example, if the gym is in the center of the school, this would be at (0,0). You should identify the place at the origin before letting the students work on their own.

Once you have the place labeled at the origin, you can let the students work on their own. You may need to take a walk around the school to figure out where things actually are located. You would be surprised how many students will be challenged by just remembering.

Project Outline

There isn't a project outline sheet for this project.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student follow directions?

Is this map an accurate representation of our school?

Are there any corrections to be made?

Can you clearly see where each place in the school is located?

Does this map represent the student's best work and effort?

Did everyone in group contribute to the map of the school?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/LessonDetail.aspx?id=L280 –This is an NCTM activity that uses the coordinate plane and directions. Students explore two-dimensional space via an activity in which they navigate the coordinate plane. This NCTM Publication-Based Lesson Plan is adapted from the article, "Tips for Teaching Cartesian Graphing: Linking Concepts and Procedures" by Cathy G. Schloemer, which appeared in Teaching Children Mathematics , Vol. 1, No. 1 (September 1994) pp. 20-23.

http://www.khanacademy.org/video/quadrants-of-coordinate-plane?playlist=Developmental+Math+2 –This is a Khan Academy video on quadrants of the coordinate plane.

Transformations

Lesson Project

In this project, students will each be given a different two –dimensional figure to use. You can make these figures as simple or complex as you choose. Then you will hand out three coordinate grids to each student. The students are going to practice drawing their figure as each of the different transformations. Each coordinate grid should represent the following transformations.

Translation

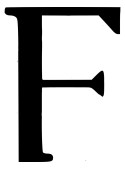
Reflection

Rotation

When working on a reflection or a rotation, students are going to work in four quadrants.

http://donnayoung.org/math/c-grids.htm –This website has coordinate grids that you can print for the students.

Any figure could be used for the translations, or the students could use the first letter of their name written in block letters. Here is an example that came from a clip art collection.



Project Outline

There isn't a project outline for this project.

Portfolio Assessment

When assessing student work, here are some questions to think about.

Did the student follow the directions?

Are there three different transformations?

Is one on each grid?

Did the student complete each transformation correctly?

Is the rotation done in four quadrants?

Is the reflection done in four quadrants?

Is it clear through the student's work that he/she understands each transformation and how to complete them?

Multimedia Links/Additional Activities

http://www.brightstorm.com/math/geometry/transformations/transformations-and-isometries/ –This is a Brightstorm video on transformations and on understanding transformations.

http://www.onlinemathlearning.com/math-transformation.html –This website has a tutorial and video on transformations. There are examples with worked solutions on the site.

http://illuminations.nctm.org/LessonDetail.aspx?id=U157 –This is a project from NCTM on transformations. In this unit, students create a shape sorter and consider all possible moves that will return a shape to its original position. They investigate the results when two of these moves are performed consecutively, to learn about the commutative and associative properties.

Surveys and Data Displays

Lesson Project

In this project, students are going to conduct a survey like the one that is done in the lesson. Students will use the question "What time do students in our class go to bed?" Then conduct a survey.

Here are the survey times:

9 pm

9:30 pm

10 pm

10:30 pm

Other

Students should survey the students in their class or in other classes too. The data is combined to show totals.

Then the students are going to create two different data displays. They are to create a circle graph and a bar graph to show the data.

All students will hand in three sheets: one survey, one circle graph and one bar graph.

Project Outline

Bedtime Survey

Directions: Like the students in the story problem, you are going to conduct a survey to figure out what time students are in bed. First, conduct the survey and gather data. Then move on to the data displays.

Part One: Survey

Question: What time do students go to bed on school nights?

9 pm

9:30 pm

10 pm

10:30 pm

Other

Part Two: Data Displays

Now create a circle graph to represent the data. You will need to convert each value on the survey to a fraction and then a percent. Use the space below to do this work.

Next, create a bar graph. Because you aren't comparing with another group, a single bar graph will do. Choose intervals and draw your graphs.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the student complete the survey?

Are the values accurate?

Did the student change each value to a fraction and a percent?

Did the student take these values and create an accurate circle graph?

Does this work show care and good effort?

Did the student create an accurate bar graph?

Do the intervals make sense?

Does the bar graph show the student's best work?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/activitydetail.aspx?id=204 –This is an interactive application from NCTM. This is the basic data grapher. There is also an advanced data grapher. The Basic Data Grapher can be used to analyze data with bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.

http://mathflix.luc.edu/NCTM_cat/DataAnalysis/Surveys/nctm-data-analysis-surveys-math-videos.html –This website has many math videos that students can watch on different surveys and different data displays.

4.12 Equations and Functions; Probability

In this twelfth chapter *Equations and Functions; Probability*, students will begin learning all about expressions and equations. They will also learn about solving equations, single-variable equations, graphing functions, probability, and how to calculate the probability of independent events. In this flexbook, Enrichment, you will find resources for making each chapter and lesson rich and vibrant. The project outlined can be completed on its own or as a part of a portfolio. You will see hints for portfolio/alternative assessment included in each lesson.

Lessons

The following lessons are part of this chapter.

- Writing Expressions and Equations
- Solving Equations Using Addition and Subtraction
- Solving Equations Using Multiplication and Division
- Functions
- Graphing Functions
- Introduction to Probability
- Finding Outcomes
- Probability of Independent Events

Writing Expressions and Equations

Lesson Project

This project encourages the use of math vocabulary so that students can better understand word problems. The students will work with cards that have key words on them. Then the students will take these key words and use them to write expressions and equations. You can do this as a basic activity or you can make it more creative by introducing a theme to the students.

For example, you could use the theme of going to an amusement park. This theme was introduced through the story problem in the lesson. Now the students can write short word problems about going to an amusement park. The one criteria of the short word problems is that they need to use some of the key words presented in the lesson. Be sure to designate how many word problems the students will need to write.

These key words can be written on index cards. Students can work on the creative piece in groups of three or four. Then the key words can be distributed to each group. You will need one set of cards per group based on the number of students in your class. Here are the key words.

Addition –sum, plus, altogether, in all, and

Subtraction –Difference, subtract, take away, less than, minus

Multiplication –product, times, grouping

Division –split up, divided, quotient

Project Outline

There isn't a project outline for this lesson.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the students follow the instructions?

Did the group incorporate the key words into each word problem?

Do the word problems make sense?

Are the word problems written correctly so that they can be solved?

Did the student use any unknowns?

Could you incorporate unknowns into each problem?

Is there a way to expand this exercise for homework?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=NI95DpVZX3Q –This is a James Sousa video on writing variable expressions.

http://www.brightstorm.com/math/algebra/graphs-and-functions/writing-an-equation-to-describe-pictures/ –This is a Brightstorm video on how to write equations.

Solving Equations Using Addition and Subtraction

Lesson Project

In this project, students will expand upon the skills that they learned in the last lesson. In the last lesson, the students worked on writing variable expressions and equations using key words. They interpreted and wrote these expressions and equations. This lesson in the text teaches the students to solve equations using addition and subtraction.

Instead of simply solving the equations, the students are going to think about what each equation means or is asking. They are going to write stories about the equations.

For example, $x+1=13$

Students would write a story about how some quantity added one and got 13. You can use the amusement park theme to encourage students to write an illustrate each story. The story needs to illustrate both a problem and a solution.

When students are finished, allow them time to share their stories with the class.

An extension of this project would be to have the students include two - step equations.

Project Outline

Directions: You are going to write a story about an equation. In a word problem, an equation is a number sentence used to describe a situation. In this project, you are going to write a story problem about something that happens at an amusement park. You can be as creative as you would like, but you must include the equation and its solution in your story. Your story should also be illustrated.

Write an amusement park story about $x+14=30$

Be creative and have fun!

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student include the equation in the story?

Did the student include a solution in the story?

Does the use of math in the story make sense?

Is the arithmetic accurate?

Did the student include the amusement park theme as instructed?

Did the student illustrate his/her work?

Does this demonstrate the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=E2povst-oYs –This is a James Sousa video on solving one –step equations using addition and subtraction.

http://www.youtube.com/watch?v=pa3FrW7HAhc –This is a James Sousa video on how to solve a one –step equation when the variable is on the right of the equal sign.

http://illuminations.nctm.org/ActivityDetail.aspx?id=10 –This is an interactive tool from NCTM where students can use a pan balance to work with expressions. This interactive pan balance allows numeric or algebraic expressions to be entered and compared. You can "weigh" the expressions you want to compare by entering them on either side of the balance. Using this interactive tool, you can practice arithmetic and algebraic skills, and investigate the important concept of equivalence. Two other tools, Pan Balance tendash Numbers and Pan Balance tendash Shapes , are natural extensions.

Solving Equations Using Multiplication and Division

Lesson Project

In this project, students will expand upon the skills that they learned in the last two lessons. In an earlier lesson, the students worked on writing variable expressions and equations using key words. They interpreted and wrote these expressions and equations. Then students learned how to solve equations using addition and subtraction. This lesson in the text teaches the students to solve equations using multiplication and division.

Instead of simply solving the equations, the students are going to think about what each equation means or is asking. They are going to write stories about the equations.

For example, $2x = 16$

Students would write a story about two times some quantity equals 16. You can use the amusement park theme to encourage students to write an illustrate each story. The story needs to illustrate both a problem and a solution.

When students are finished, allow them time to share their stories with the class.

An extension of this project would be to have the students include two - step equations.

Project Outline

Directions: You are going to write a story about an equation. In a word problem, an equation is a number sentence used to describe a situation. In this project, you are going to write a story problem about something that happens at an amusement park. You can be as creative as you would like, but you must include the equation and its solution in your story. Your story should also be illustrated.

Write an amusement park story about $5x = 45$

Be creative and have fun!

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student include the equation in the story?

Did the student include a solution in the story?

Does the use of math in the story make sense?

Is the arithmetic accurate?

Did the student include the amusement park theme as instructed?

Did the student illustrate his/her work?

Does this demonstrate the student's best work and effort?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=HEMxTOdhVgg –This James Sousa video teachers students how to solve one –step equations by using multiplication and division.

http://illuminations.nctm.org/ActivityDetail.aspx?id=10 –This is an interactive tool from NCTM where students can use a pan balance to work with expressions. This interactive pan balance allows numeric or algebraic expressions to be entered and compared. You can "weigh" the expressions you want to compare by entering them on either side of the balance. Using this interactive tool, you can practice arithmetic and algebraic skills, and investigate the important concept of equivalence. Two other tools, Pan Balance tendash Numbers and Pan Balance tendash Shapes , are natural extensions.

http://www.mathplayground.com/howto_solvevariable.html –This is a math playground video on how to solve a variable equation. It reviews addition, subtraction, multiplication and division.

Functions

Lesson Project

In this project, students are going to look at how money and hours are connected through functions. If a student is paid hourly, then the amount of money he/she earns is a function of the number of hours that he/she works. This is the work of this project.

The selected activities are dog walking and lawn cutting. You can determine an amount of money per hour that seems reasonable given the age of your students and the area in which you live. Then you can tell the students that they are going to choose one of the occupations.

The students will need to figure out how many hours they will need to work to earn $1000.00. This can be shown through an equation and also a table.

Be sure that the students show the work in a table even if they find the correct answer through another method. Students will use this table to graph a function in the next lesson.

There is a blank table provided in the project outline.

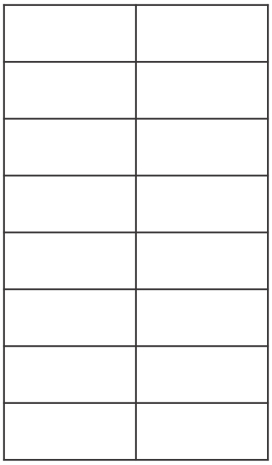
Project Outline

Job:

Hourly rate:

Goal: Earn $1000.00

You will need to figure out how many hours it will take you to do this.



Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student complete the table accurately?

Did the student figure out the number of hours he/she will need to work to earn $1000.00?

Did the student use arithmetic first or the table?

Does the student understand how this example represents a function?

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=uWXmfnpAirA –This is a yourteacher.com video which teaches students all about functions.

http://www.khanacademy.org/video/introduction-to-functions?playlist=Algebra

This Khan Academy video is an introduction to functions.

http://illuminations.nctm.org/LessonDetail.aspx?id=L269 –In this activity, students investigate properties of perimeter, area, and volume related to various geometric two- and three-dimensions shapes. They conjecture, test, discuss, verbalize, and generalize patterns. Through this process they *discover* the salient features of the pattern, *construct* understandings of concepts and relationships, develop a language to talk about the pattern, *integrate*, and *discriminate* between the pattern and other patterns. When relationships between quantities in a pattern are studied, knowledge about important mathematical relationships and functions emerges.

Graphing Functions

Lesson Project

In this project, students are going to use the data from the last project in this flexbook. If your students did not complete this project, then they can complete it first. Here is the outline for the project.

Part One: Data about Money

In this project, students are going to look at how money and hours are connected through functions. If a student is paid hourly, then the amount of money he/she earns is a function of the number of hours that he/she works. This is the work of this project.

The selected activities are dog walking and lawn cutting. You can determine an amount of money per hour that seems reasonable given the age of your students and the area in which you live. Then you can tell the students that they are going to choose one of the occupations.

The students will need to figure out how many hours they will need to work to earn $1000.00. This can be shown through an equation and also a table.

Be sure that the students show the work in a table even if they find the correct answer through another method. Students will use this table to graph a function in the next lesson.

Part Two: Graphing

Students now take the data and create a graph in one quadrant. They must graph the function on the graph.

Project Outline

Part One: Data about Money

Job:

Hourly rate:

Goal: Earn $1000.00

You will need to figure out how many hours it will take you to do this.

Part Two: Graphing

Now create a graph in one quadrant and graph the function. Be sure to use color to show your results.

You will need to choose the values for the x and y axes. Be sure to use a ruler so that your work is neat.

Portfolio Assessment

When assessing student work, here are some questions to keep in mind.

Did the student complete the table accurately?

Did the student figure out the number of hours he/she will need to work to earn $1000.00?

Did the student use arithmetic first or the table?

Does the student understand how this example represents a function?

Did the student choose values for the two axes that made sense?

Did the student use color to graph the data?

Does the graph match the table?

Does the student understand how his/her graph represents a function?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/ActivityDetail.aspx?id=215 –This is an interactive application from NCTM where

students can explore the functions and their graphs.

http://illuminations.nctm.org/LessonDetail.aspx?id=L282 –This lesson is a bit advanced, but could be used for advanced students. This lesson focuses on having students make connections among different classes of polynomial functions by exploring the graphs of the functions. The questions in the activity sheets allow students to make connections between the $x-$intercepts of the graph of a polynomial and the polynomial's factors. This activity is designed for students who already have a strong understanding of linear functions, some knowledge of quadratic functions, and what is meant by a polynomial function.

http://blog.thinkwell.com/2010/07/6th-grade-math-graphing-functions.html –This is a sixth grade video on graphing functions. It does not go into more advanced concepts.

Introduction to Probability

Lesson Project

In this project, students are going to use some of the ideas presented in the lesson to explore probability. This is a chance to have a hands–on exploration of the material where students really engage in and work with the concepts presented. Students will work on two experiments. Allow room for creativity if students want to design a different experiment. That is fine and sometimes better than using the ones presented. Just be sure that they still include the data required.

You will need material for students to make spinners or you can buy them ahead of time. There are places on the internet to print out spinners. You will also need dice.

1. Spinner Game –create an investigation of probability using a spinner. You may not use any experiments from the lesson.
2. Dice Game –create an investigation of probability using one or two dice. You may not use any experiments from the lesson.

Each experiment must include:

- a write up of the experiment
- A prediction
- The results
- A written conclusion
- A probability written as a fraction, decimal and percent.

Project Outline

There isn't a project outline for this project.

Portfolio Assessment

When assessing student work, you can use the following questions as a guide.

Did the student follow directions?

Is each write up clear?

How could the students improve his/her work?

Multimedia Links/Additional Activities

http://www.khanacademy.org/video/basic-probability?playlist=Probability

This Khan Academy video is an introduction to probability.

http://illuminations.nctm.org/activitydetail.aspx?ID=79 –This is an interactive application from NCTM where students can change a spinner and practice calculating probability.

Finding Outcomes

Lesson Project

This project involves the students planning an ice cream social. You can do this simply as an exploration or you can actually plan to host an ice cream social as a fundraiser. To prepare, the students are going to decide a couple of different things. Then they will take these options and create a tree diagram to show all of the different combinations possible at the ice cream social. This group decides the different options and then each small group prepares a tree diagram.

- Vote on four different ice cream flavors
- Vote on three different sauces
- Vote on three or four additional toppings

Students should create poster displays for the ice cream social. These displays must use tree diagrams. Once the tree diagrams have been created, the students can use them to advertise the ice cream social by hanging them up around the school.

Students will need large poster paper, colored pencils or markers and rulers.

Project Outline

There isn't a project outline for this project.

Portfolio Assessment

When assessing student work, here are a few questions to keep in mind.

Did the students create a tree diagram that includes all of the options?

Is the poster eye-catching and does it show creativity?

Is the tree diagram accurate?

Does the student understand how to calculate outcomes using this tool?

Multimedia Links/Additional Activities

http://illuminations.nctm.org/LessonDetail.aspx?id=L377 –This lesson plan presents a classic game-show scenario. A student picks one of three doors in the hopes of winning the prize. The host, who knows the door behind which the prize is hidden, opens one of the two remaining doors. When no prize is revealed, the host asks if the student wishes to "stick or switch." Which choice gives you the best chance to win? The approach in this activity runs from guesses to experiments to computer simulations to theoretical models. This lesson was adapted from an article written by J. Michael Shaughnessy and Thomas Dick, which appeared in the April 1991 issue of the *Mathematics Teacher* .

http://illuminations.nctm.org/LessonDetail.aspx?id=L290 –In this lesson, students analyze the fairness of certain games by examining the probabilities of the outcomes. The explorations provide opportunities to predict results, play the games, and calculate probabilities. Students should have had prior experiences with simple probability investigations, including flipping coins, drawing items from a set, and making tree diagrams.

http://www.onlinemathlearning.com/probability-tree-diagrams.html –This website has a tutorial and video on calculating outcomes using tree diagrams.

Probability of Independent Events

Lesson Project

This project is a presentation on one of the probability topics of the past few lessons. Students can choose probability notation, independent events, compound events, theoretical probability, experimental probability or finding outcomes using tree diagrams and other methods. You can allow students to choose their topics or you could assign them.

The students, in pairs or groups of three, are going to prepare a presentation on their topic. The presentation is designed to teach the rest of the class. The presentation should use visuals of some kind and have a hand –out. It can also have an experiment or demonstration if this is necessary.

Students can use creativity to design and present their material.

Project Outline

Directions: You have been assigned or have chosen a probability topic. You along with a partner or small group are going to prepare a presentation on your topic. The presentation is designed to teach the rest of the class. The presentation should use visuals of some kind and have a hand –out. It can also have an experiment or demonstration if this is necessary.

You can use creativity to design and present their material in a way that is engaging and interesting to you. You will be graded on the presentation as well as the clarity with which the topic has been presented.

Portfolio Assessment

Here is a grading rubric for the project. Be sure to show this to the students prior to the assignment.

5 Students followed directions.

5 Students used creative visuals in their presentation.

5 Students used an experiment or demonstration in their presentation.

5 Students understood the topic and could talk about it clearly.

5 The students in the class benefitted by listening to the presentation.

Multimedia Links/Additional Activities

http://www.youtube.com/watch?v=P-oFpsQlt-A –This is a math video from yourteacher.com which focuses on finding the probability of independent events.

http://www.khanacademy.org/video/compound-probability-of-independent-events?playlist=Probability –This is a Khan Academy video on the compound probability of independent events.

http://illuminations.nctm.org/LessonDetail.aspx?ID=U190 –This is a two lesson unit from NCTM on exploring probability. Students will explore theoretical and experimental probability and the relationship between them. Students will also graph an experiment to further explore the relationship according to the law of large numbers.

CHAPTER 5 Grade 6 - Problem Solving

Chapter Outline

5.1 Number Sense and Variable Expressions

This first chapter *Number Sense and Variable Expressions* takes students from whole number operations into the world of beginning Algebra. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Operations with Whole Numbers
- Whole Number Estimation
- Powers and Exponents
- Order of Operations
- Variables and Expressions
- A Problem-Solving Plan
- Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Operations with Whole Numbers

Word Problem –Elephant Issues

Jonah got the knack of measuring and ordering seal food and in no time at all he was in charge each and every week. But then, just when he was feeling very confident about the seal food, he was transferred to the elephants. Now elephants are much bigger than seals. Jonah had no idea how much food they ate, but he was assigned the ordering right away.

"How much does an elephant eat?" Jonah asked Mr. Hodgkins his mentor.

"Well, the average elephant eats between 125 and 175 pounds of food. This includes hay, fruit, vegetables and nutritional pellets," Mr. Hodgkins explained.

"Per day?" Jonah asked astonished.

"Yes sir, per day. Now you are going to do the hay order, so take the low end and figure it out for the month. Okay?" Mr. Hodgkins asked.

"Alright," Jonah said.

After Mr. Hodgkins left Jonah looked at his pad of paper and then at the elephants that he could see through the glass window.

"Wow, this is quite a job!" Jonah thought to himself.

Jonah's problem has a lot of math in it. Jonah will need to calculate how much hay to order for the month. To do this, he will need to use whole number operations. You can help Jonah by figuring out the amount of hay that he needs to order. The order has to be placed right away, so it is time for you to get to work.

Solution –Elephant Issues

Here is the original problem once again. Reread the problem and underline the important information in the problem.

Jonah got the knack of measuring and ordering seal food and in no time at all he was in charge each and every week. But then, just when he was feeling very confident about the seal food, he was transferred to the elephants. Now elephants are much bigger than seals. Jonah had no idea how much food they ate, but he was assigned the ordering right away.

"How much does an elephant eat?" Jonah asked Mr. Hodgkins his mentor.

"Well, the average elephant eats between 125 and 175 pounds of food. This includes hay, fruit, vegetables and nutritional pellets," Mr. Hodgkins explained.

"Per day?" Jonah asked astonished.

"Yes sir, per day. We have 5 elephants. Now you are going to do the hay order, so take the low end and figure it out for the month. Okay?" Mr. Hodgkins asked.

"Alright," Jonah said.

After Mr. Hodgkins left Jonah looked at his pad of paper and then at the elephants that he could see through the glass window.

"Wow, this is quite a job!" Jonah thought to himself.

Jonah's problem has a lot of math in it. Jonah will need to calculate how much hay to order for the month. To do this, he will need to use whole number operations. You can help Jonah by figuring out the amount of hay that he needs to order. The order has to be placed right away, so it is time for you to get to work.

Jonah is going to figure out the amount of hay to order for one month.

The low end of pounds per day that an elephant eats is 125 pounds.

First, we can figure out the amount needed for 1 week or 7 days.

$125 \times 7 = 875$ *pounds*

There are four weeks in a month that's 28 days. We have to figure 2 or 3 more days for a 30 or 31 day month.

$875 \times 4 = 3500$ *pounds of hay* **for four weeks**

250 or 375 additional pounds depending on 30 or 31 days.

Jonah will need to order 3750 or 3875 pounds per elephant.

There are five elephants at the zoo.

$3750 \times 5 = 18,750$ *pounds of hay*

OR

$3875 \times 5 = 19,375$ *pounds of hay*

How many tons is that?

9.38 tons

Or

9.68 tons

Whole Number Estimation

Word Problem –Elephant Estimation

After Jonah had switched to the elephants, he went and had lunch with his friend Sarah once again. They sat down to eat and Jonah had a huge sneaky smile on his face. Sarah looked at him inquisitively.

"What's up? You've got quite a look on your face," she said.

"I am working in the elephants. Let's see you figure this one out without a piece of paper," Jonah challenged.

"Alright, give me the figures. I am just going to write them down so that I can keep track," Sarah said smiling.

"125 pounds per day. Five elephants in the pen. How much food for the month?" Jonah said.

Sarah wrote down 125 pounds, five elephants and 30 days. She didn't say anything for a few minutes and then smiled.

"Not as easy as you think is it?" Jonah said.

"About 19,500 pounds," Sarah said.

Jonah gasped with his mouth wide open.

"How did you do that?" Jonah asked.

Sarah simply smiled.

Solution –Elephant Estimation

Here is the original problem once again. Reread it and underline all of the important information.

After Jonah had switched to the elephants, he went and had lunch with his friend Sarah once again. They sat down to eat and Jonah had a huge sneaky smile on his face. Sarah looked at him inquisitively.

"What's up? You've got quite a look on your face," she said.

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"Not as easy as you think is it?" Jonah said.

"About 19,500 pounds," Sarah said.

Jonah gasped with his mouth wide open.

"How did you do that?" Jonah asked.

Sarah simply smiled.

Sarah used estimation to figure out the solution to the problem. Let's look at how she did that. Remember that estimates don't have to be exact, but they do have to make sense.

Sarah smiled at Jonah.

"First, I rounded 125 up to 130. Remember it is always easier to work with a number with a zero at the end," Sarah said.

"Then I multiplied 13×5 **and got 65, since it is 130 really, I added a zero to the product."**

Sarah wrote 650 down on the paper.

"Next, I multiplied 65×3 **by dropping both zeros in 650 and 30, the math is simpler."**

$65 \times 3 = 195$

"Finally, I added the two zeros. 19,500 pounds is the final answer," Sarah smiled and took a bite of her sandwich.

Powers and Exponents

Word Problem –Elephant Spaces

Mr. Hodgkins showed Jonah a huge drawing on his third day in the elephant habitat. It was a drawing of a whole new place for the elephants.

"Wow," said Jonah. "This is beautiful!"

"It's the new elephant habitat. The Association of Zoos and Aquariums says that we have to have $1800 ft^2$ for every elephant, but we want to have more than that. Each elephant will have 60^2 $feet$ in this habitat," Mr. Hodgkins explained.

"For each one?" Jonah asked.

"Yes, that is why it is so big."

Jonah began to think about the size of the habitat. If each elephant had 60^2 $feet$, how much space would the new habitat actually be?

This is where you come in. Use what you have learned about exponents and whole number multiplication to figure out the space of the new elephant habitat.

Solution –Elephant Spaces

Here is the original problem once again. Reread it and underline all of the important information.

Mr. Hodgkins showed Jonah a huge drawing on his third day in the elephant habitat. It was a drawing of a whole new place for the five elephants.

"Wow," said Jonah. "This is beautiful!"

"It's the new elephant habitat. The Association of Zoos and Aquariums says that we have to have 1800 ft^2 for every elephant, but we want to have more than that. Each elephant will have 60^2feet in this habitat," Mr. Hodgkins explained.

"For each one?" Jonah asked.

"Yes, that is why it is so big."

Jonah began to think about the size of the habitat. If each elephant had 60^2 $feet$, how much space would the new habitat actually be?

This is where you come in. Use what you have learned about exponents and whole number multiplication to figure out the space of the new elephant habitat.

To figure this out, Jonah will first need to figure out the value of 60^2.

$60^2 = 60 \times 60 = 3600\ ft^2$

Now there are five elephants, so we multiply this product by five.

$3600 \times 5 = 18{,}000\ ft^2$

This is our answer.

Order of Operations

Word Problem –Helping the Reptiles

While Keisha worked in the bird rescue, her best friend Jana was working in the reptile hospital. Reptiles can be ill

for all kinds of reasons and when they are healed, they return to the reptile house.

Here is a typical day in the reptile house.

Jana arrived at 8 am and looked at the clipboard status and saw that there were 135 reptiles in the hospital. Then at noon, five of them were cleared by the doctor to return to general population. Then at three o'clock in the afternoon, two groups of three reptiles, one group of snakes and one groups of lizards came in for check–ups. At five o'clock, when Jana went to leave, she filled in the status report.

She wrote 396 reptiles.

Then she stopped and looked at the number again. It didn't seem correct. Is it right? Did Jana make an error? Why or why not?

Solution –Helping the Reptiles

Here is the original problem once again. Reread it and then underline all of the important information.

While Keisha worked in the bird rescue, her best friend Jana was working in the reptile hospital. Reptiles can be ill for all kinds of reasons and when they are healed, they return to the reptile house.

Here is a typical day in the reptile house.

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She wrote 396 reptiles.

Then she stopped and looked at the number again. It didn't seem correct. Is it right? Did Jana make an error? Why or why not?

To figure this out, you have to write a number sentence and follow the order of operations. Here is the number sentence.

$135 - 5 + 2 \times 3$

Jana wrote down 396 reptiles. However, she did not follow the order of operations. She simply worked from left to right. You must follow the order of operations when there is more than one operation in a number sentence.

$135 - 5 + 2 \times 3$

First we multiply.

$2 \times 3 = 6$

Now we can add/subtract in order from left to right.

$$135 - 5 + 6$$
$$130 + 6$$
$$136$$

There were 136 reptiles in the hospital at the end of the day. This is the correct answer.

Variables and Expressions

Word Problem –50% Off

Just when Joshua was in the swing of things counting people and calculating ticket sales, the zoo anniversary arrived. The zoo was officially ten years old and a huge celebration was scheduled. In honor of the anniversary, every admission was to be half or 50% off.

An adult ticket is usually $7.00 so this would drop it to $3.50.

A child ticket is usually $5.00 so this would drop it to $2.50.

On the big day, Joshua counted all of the people entering the zoo. The crowd was large, but he was confident that he counted every adult and child who entered.

He counted 200 adults and 300 children.

Joshua sat down and began trying to calculate how much money the zoo earned on that day. Then he tried to figure out how much they would have earned if it had been a full price day.

Using what you have learned about variables and expressions, can you help Joshua figure out these two amounts. You will need to use variables and expressions and whole number operations to figure the two totals.

Solution

Here is the original problem once again. Reread it and underline all of the important information.

Just when Joshua was in the swing of things counting people and calculating ticket sales, the zoo anniversary arrived. The zoo was officially ten years old and a huge celebration was scheduled. In honor of the anniversary, every admission was to be half or 50% off.

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Using what you have learned about variables and expressions, can you help Joshua figure out these two amounts. You will need to use variables and expressions and whole number operations to figure the two totals.

First, Joshua needs to write an expression to show adult prices and child prices.

$$3.5x + 2.5y$$

To make the whole number operations easier, Joshua left out the zeros.

Notice that he used an x to represent the unknown number of adults and a y to represent the unknown number of children.

Then he filled in those values.

$$3.5(200) + 2.5(300)$$
$$700 + 750 = \$1450.00$$

This was the amount earned on that day.

To figure out what it would have been on a regular price day with those numbers of people, you can take the total and double it.

$1450 \times 2 = \$2900.00$

This is the amount the zoo would have earned if it had been a regular day.

A Problem-Solving Plan

Word Problem –The Orangutan Adoption –part two

Do you remember Tyler and the orangutans? Well, here is a part of the original problem to help you remember.

Tyler loves to visit the orangutans at the city zoo. The orangutans are one of four living generations of great apes. They are reddish-orange in color and swing and climb all around. Tyler thinks that they are very social as the orangutans often come up to the glass to peer at him when he visits. Tyler could stay at the orangutan exhibit for hours.

In his last visit, Tyler saw a sign about orangutan adoption at the zoo. This peaked his interest and so he investigated more about it.

At many zoos, including the city zoo in Tyler's town, you can adopt a specific animal or species of animal. Any money donated goes directly to the care of this species of animals. You can adopt an animal for any amount from $35 to $1000.

Tyler has decided to use the money from his summer job to adopt an orangutan.

Tyler is working this summer doing yard work for his neighbors. Because of his excellent work ethic, he has many clients. Tyler figures out that he will make $125.00 per week on yard work.

There are different adoption pledge levels:

Bronze = $ 35 - $100

Silver = $100 - $500

Gold = $500 - $1000

Tyler was sure that he would earn $125.00, but he ended up earning $165.00 per week.

If Tyler did this for 8 weeks, how much money would he earn?

What would be his new pledge level?

Solution

Here is the original problem once again. Reread it and then use the problem solving plan to answer the questions at the end.

Do you remember Tyler and the orangutans? Well, here is a part of the original problem to help you remember.

Tyler loves to visit the orangutans at the city zoo. The orangutans are one of four living generations of great apes. They are reddish-orange in color and swing and climb all around. Tyler thinks that they are very social as the orangutans often come up to the glass to peer at him when he visits. Tyler could stay at the orangutan exhibit for hours.

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Tyler was sure that he would earn $125.00, but he ended up earning $165.00 per week.

If Tyler did this for 8 weeks, how much money would he earn?

What would be his new pledge level?

Tyler earned $165.00 per week.

He worked for 8 weeks, so we can multiply 165.00×8 **to figure out the total amount that he earned.**

$165 \times 8 = 1320$

Because we are talking about money, let's turn that into a money amount.

$1320.00 for 8 weeks of work.

Tyler earned so much money that he is beyond a Gold level of adoption. So Tyler decided to spend $1000.00 on the adoption at the Gold level. Then he took the other $320.00 and put it into his savings account.

Problem-Solving Strategies: Guess, Check and Revise; Use Mental Math

Word Problem –Hippo Info

There are two hippopotamus' at the city zoo, and one is a male and the other is a female.

A male is larger than a female. While a male can range in weight from 1500–3200 kilograms, the female doesn't usually weigh more than 1500 kilograms.

Because hippos are such large creatures, the weight of them can also be a lot. The zoo keepers weigh the hippos just like they weigh the elephants.

Tara Jonsen has been a zoo keeper for many years. Today, she is going to weigh Zingo the male hippo and then Kumen the female hippo. Zingo weighs 1200 more pounds than Kumen does.

Their combined weight is 4200 kilograms.

Tara assigns her assistant, Ren the job of writing the weights down in the log book. Ren goes back to do this, but when he arrives he can't remember everything.

"I should have taken notes," Ren thinks to himself.

He remembers that Zingo weighs 1200 more kilograms than Kumen and that their combined weight is 4200 kilograms.

Given this information, can Ren figure out what each hippo weighed?

Solution –Hippo Info

Here is the original problem once again. Reread it and then use problem solving strategies to figure out the solution.

There are two hippopotamus' at the city zoo, and one is a male and the other is a female.

A male is larger than a female. While a male can range in weight from 1500–3200 kilograms, the female doesn't usually weigh more than 1500 kilograms.

Because hippos are such large creatures, the weight of them can also be a lot. The zoo keepers weigh the hippos just like they weigh the elephants.

Tara Jonsen has been a zoo keeper for many years. Today, she is going to weigh Zingo the male hippo and then Kumen the female hippo. Zingo weighs 1200 more pounds than Kumen does.

Their combined weight is 4200 kilograms.

Tara assigns her assistant, Ren the job of writing the weights down in the log book. Ren goes back to do this, but when he arrives he can't remember everything.

"I should have taken notes," Ren thinks to himself.

He remembers that Zingo weighs 1200 more kilograms than Kumen and that their combined weight is 4200 kilograms.

Given this information, can Ren figure out what each hippo weighed?

Now Ren knows some valuable information. One hippo weighs 1200 kilograms more than the other and their total weight is 4200 kilograms.

We have two unknowns here and we know that one is 1200 more than another and the total is 4200.

$$x + 1200 + y = 4200$$

This is a possible equation where x is the weight of the male hippo and y is the weight of the other one.

Now it makes sense to subtract 1200 from 4200 because this is the added weight. We can figure out what Kumen weighs first.

$$x + y = 3000$$

If they both weighed the same, then they would each weigh 1500 kilograms. However, this is not the case so we can say that Kumen weighs 1500 kilograms.

Kumen weighs 1500 kilograms.

Zingo weighs 1200 kilograms more than Kumen

$1500 + 1200 = 2700$ *kilograms*

The sum of their weights is 4200 kilograms.

Let's see if we add these weights together if they equal 4200 kilograms.

$1500 + 2700 = 4200$

Our work is accurate!

5.2 Statistics and Measurement

In this second chapter *Statistics and Measurement* students become familiar with measurements. Also covered are finding the perimeter and area of rectangles, frequency, creating and understanding graphs, and an introduction to the mean, median, mode, range, and central tendency. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Measuring Length
- Perimeter and Area
- Scale Drawings and Maps
- Frequency Tables and Line Plots
- Bar Graphs
- Coordinates and Line Graphs
- Circle Graphs and Choosing Displays
- Mean, Median and Mode

Measuring Length

Word Problem –Summer Squash

Once Tania has figured out the tomato situation, she moved onto summer squash. She chose a golden egg variety of summer squash. They are called that because when grown they look like a large egg.

Tania read the package before planting and discovered that this squash was a direct sow kind of seed. She spread out her seeds and waited.

Each day Tania visited the garden and waited to see growth. Pretty soon she could see the results of her hard work. The summer squash has a 41 day maturation and fully grown measures 5 inches across.

On day 20, Tania took her ruler to the garden and measured $2\frac{1}{4}$ *inches* across one of the squash plants. Based on this figure, how much more does Tania's plant need to grow to be at full growth.

Tania looked at her ruler and began to figure this out.

Solution

Here is the original problem once again. Reread it and then answer the question at the end.

Once Tania has figured out the tomato situation, she moved onto summer squash. She chose a golden egg variety of summer squash. They are called that because when grown they look like a large egg.

Tania read the package before planting and discovered that this squash was a direct sow kind of seed. She spread out her seeds and waited.

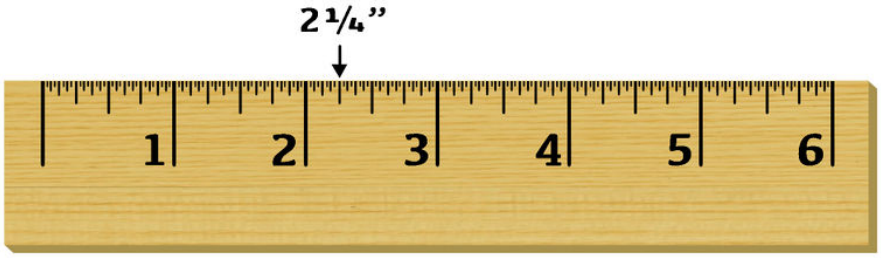
Each day Tania visited the garden and waited to see growth. Pretty soon she could see the results of her hard work. The summer squash has a 41 day maturation and fully grown measures 5 inches across.

On day 20, Tania took her ruler to the garden and measured $2\frac{1}{4}$ *inches* across one of the squash plants. Based on this

figure, how much more does Tania's plant need to grow to be at full growth.

Tania looked at her ruler and began to figure this out.

We know that Tania is expecting each squash plant to measure 5 inches across by day 41. On day 20, she measured $2\frac{1}{4}$ on one of the plants. Let's look at a ruler and figure out where $2\frac{1}{4}$ *inches* is located.



Next, Tania looked at where the 5 inch mark is on the ruler. She figured out that the plant needs to grow $2\frac{3}{4}''$ more to read the desired 5 inch mark.

Perimeter and Area

Word Problem –Spacing Out Squash

After Tania had sowed all of the squash seeds, she went back and read the package once again. She went to her brother Alex with some concerns.

"Hey Alex, I think I put too many plants in that first row," she said.

"Really, what has you say that?"

"Well, they spread to 30 inches each," Tania said.

"That's an easy solution. After they start growing, you can thin the plants out," Alex told her.

"How?"

"Well, you use the length of the plot and figure out how many plants will fit in one row. Then you can move the others around," Alex said.

Tania thought she understood exactly what to do. She wrote down the length of the square plot and the length of the rectangle plot. Since Alex combined both plots to be irregular in shape, she would need to add them up to find the total length of the garden.

Square plot = 9 feet

Rectangle plot = 12 feet

Total length = 21 feet

"Now what?" thought Tania.

Tania is stuck. Given the information about the squash plants and the length of the plot, how many plants can Tania grow in one row? Use what you have learned to figure this out. You will need to remember measurement and whole number operations.

Solution

Here is the original problem once again. Reread it and underline all of the important information.

After Tania had sowed all of the squash seeds, she went back and read the package once again. She went to her brother Alex with some concerns.

"Hey Alex, I think I put too many plants in that first row," she said.

"Really, what has you say that?"

"Well, they spread to 30 inches each," Tania said.

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Square plot = 9 feet

Rectangle plot = 12 feet

Total length = 21 feet

"Now what?" thought Tania.

Tania is stuck. Given the information about the squash plants and the length of the plot, how many plants can Tania grow in one row? Use what you have learned to figure this out. You will need to remember measurement and whole number operations.

Tania knows that the length of the garden plot is 21 feet. Each squash plant will spread to 30 inches. She needs to divide 21 feet by 30 inches.

$21 \div 30 =$ ____

Wait a minute, Tania can't divide feet by inches. She will need to change all the measurements to be the same. To do this, Tania needs to change feet to inches.

There are 12 inches in 1 foot, so she can multiply 21×12

$21 \times 12 = 252$ *inches*

Now she can divide by 30

$252 \div 30 = 8.4$

Tania can thin out the plants so that there are 8 plants in each row. Then each plant will have room to grow.

Scale Drawings and Maps

Word Problem –The Squash Plan

Now that Tania has figured out how many plants she will grow in one row, she has decided to draw a plan of the garden. She is going to use a scale and a drawing to figure it all out. She has decided to plant two rows of the squash given the measurements that she has figured out.

Tania begins with the following information.

The garden plot is 21 feet long.

Each squash plant needs 30 inches for the spreading.

Tania will need to convert inches to feet so that she can use a scale that is in feet. There are 12 inches in one foot, so Tania divides 30 by 12.

$30 \div 12 = 2.5$

There will be a new squash plant every 2 feet.

Now Tania is ready to draw, or is she? She needs to decide on a scale. If she uses the scale $1'' = 1\ foot$, at which inch marks will she draw each plant? What will be the length of the drawing?

If she uses $\frac{1}{2}'' = 1\ foot$, at which inch marks will she draw her squash plants? What will be the length of the drawing?

Solution

This problem has a lot of parts to it. Reread it carefully and then work on answering all of the questions at the end.

Now that Tania has figured out how many plants she will grow in one row, she has decided to draw a plan of the garden. She is going to use a scale and a drawing to figure it all out. She has decided to plant two rows of the squash given the measurements that she has figured out. Tania begins with the following information.

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Tania will need to convert inches to feet so that she can use a scale that is in feet. There are 12 inches in one foot, so Tania divides 30 by 12.

$30 \div 12 = 2.5$

There will be a new squash plant every 2 feet.

Now Tania is ready to draw, or is she? She needs to decide on a scale. If she uses the scale $1'' = 1\ foot$, at which inch marks will she draw each plant? What will be the length of the drawing?

If she uses $\frac{1}{2}'' = 1\ foot$, at which inch marks will she draw her squash plants? What will be the length of the drawing?

First, let's figure out the answers to the problem where $1'' = 1\ foot$. With a length of 21 feet, the length of Tania's drawing will be $21''$. She will draw a plant every $2\frac{1}{2}$ *inches*.

If Tania uses a scale of $\frac{1}{2}'' = 1\ foot$, the length of the drawing will be 10.5 inches. Every plant will be drawn in at $1\frac{1}{4}''$.

Frequency Tables and Line Plots

Word Problem –Watering

Tania and Alex have so many volunteers that they have lost track of the watering of the garden. This hasn't been a problem so far, but a heat wave is expected, so Alex and Tania decide to track the watering in the garden.

For two weeks they keep track of the garden watering. Here are the number of times that the garden was watered over two weeks.

2, 1, 2, 1, 2, 1, 2, 1, 1, 2, 1, 2, 2, 2

"We are going to have to water twice a day during the heat wave," Alex said to Tania after seeing that on some days the garden is only watered once.

Tania agrees. They decide to create a line plot of the data to show the volunteers. Given the data, what will this plot

look like?

Solution

Here is the original data. Reread the problem and create a line plot to show the data.

Tania and Alex have so many volunteers that they have lost track of the watering of the garden. This hasn't been a problem so far, but a heat wave is expected, so Alex and Tania decide to track the watering in the garden.

For two weeks they keep track of the garden watering. Here are the number of times that the garden was watered over two weeks.

2, 1, 2, 1, 2, 1, 2, 1, 1, 2, 1, 2, 2, 2

"We are going to have to water twice a day during the heat wave," Alex said to Tania after seeing that on some days the garden is only watered once.

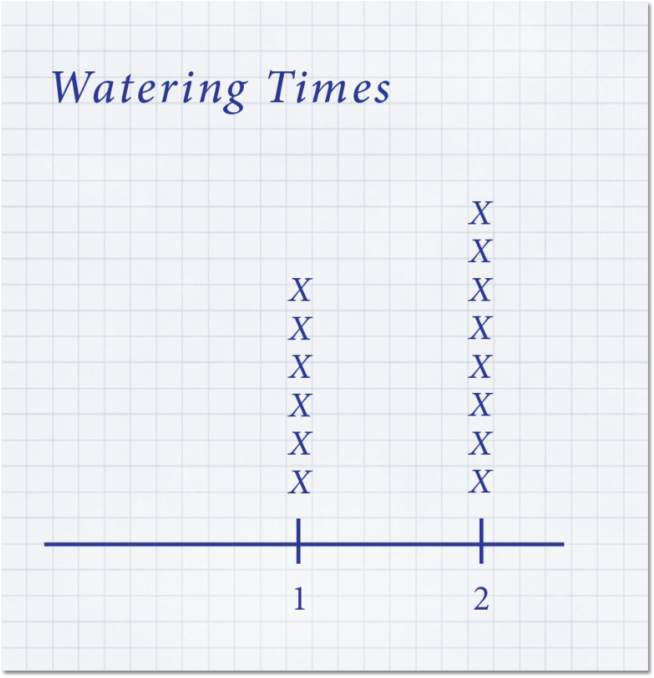
Tania agrees. They decide to create a line plot of the data to show the volunteers. Given the data, what will this plot look like?

To do this, first we can look at the frequency of each of the two watering times.

The garden was watered once per day at total of 6 times.

The garden was watered twice 8 times.

Here is the line plot of the data.



Bar Graphs

Word Problem –September Growth Explosion

Tania and Alex can't believe the difference in vegetable growth from August to September. By the end of the month, they were overwhelmed with vegetables. Look at the explosion of growth that happened from August to September.

August

60 carrots

20 tomatoes

30 zucchini

25 squash

20 potatoes

September

120 carrots

80 tomatoes

60 zucchini

50 squash

50 potatoes

Now take this data and use what you have learned to create a double bar graph.

Solution

Here is the original problem once again. Reread it and then create the double bar graph.

Tania and Alex can't believe the difference in vegetable growth from August to September. By the end of the month, they were overwhelmed with vegetables. Look at the explosion of growth that happened from August to September.

August

60 carrots

20 tomatoes

30 zucchini

25 squash

20 potatoes

September

120 carrots

80 tomatoes

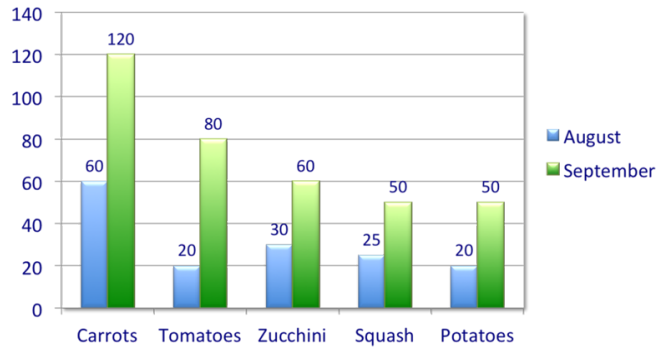
60 zucchini

50 squash

50 potatoes

Now take this data and use what you have learned to create a double bar graph.

By using this data, we can build the following graph.

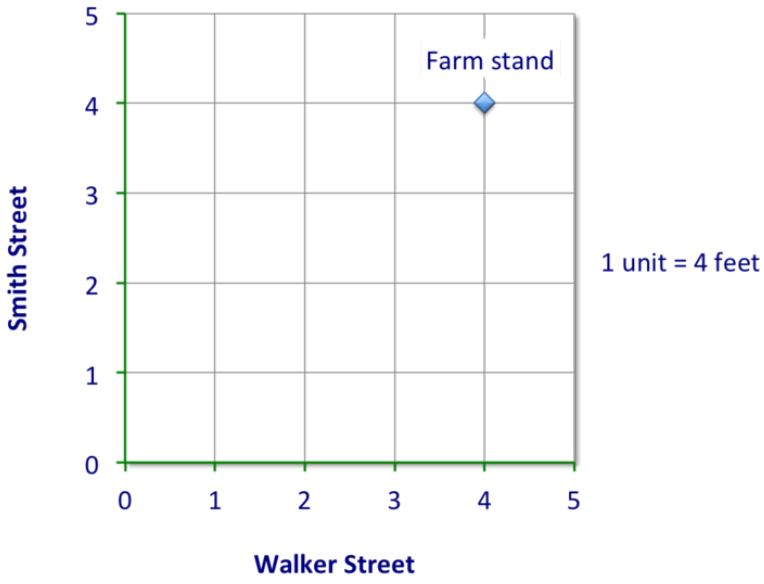


The bar graph really shows off the growth the garden. Tania and Alex are proud and pleased.

Coordinates and Line Graphs

Word Problem –Signs

Now that Alex has figured out where to put the farm stand, he wants to make a few signs to advertise it. He figures that since the two neighboring streets are just a 16 feet away that he can put signs on both streets. Here is the original plan that he created. Smith Street is the vertical axis and is 16 feet away from the farm stand. Walker Street is the horizontal axis and is also 16 feet away from the farm stand. Here is the plot.

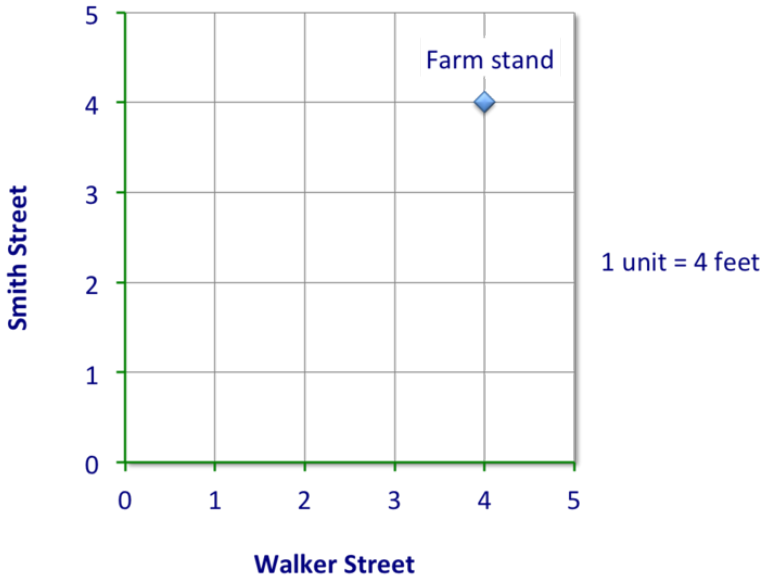


Alex wants to put each sign exactly 16 feet from where the farm stand is located. Each unit or box on the grid represents 4 feet. If he does this, what will be the coordinates of the sign on Walker? What will be the coordinates of the sign on Smith? Figure this out and then draw in the signs on the coordinate grid.

Solution

Here is the original problem once again. Reread it and then figure out the answers to the two questions. Finally, plot the points on the grid.

Now that Alex has figured out where to put the farm stand, he wants to make a few signs to advertise it. He figures that since the two neighboring streets are just a 16 feet away that he can put signs on both streets. Here is the original plan that he created. Smith Street is the vertical axis and is 16 feet away from the farm stand. Walker Street is the horizontal axis and is also 16 feet away from the farm stand. Here is the plot.

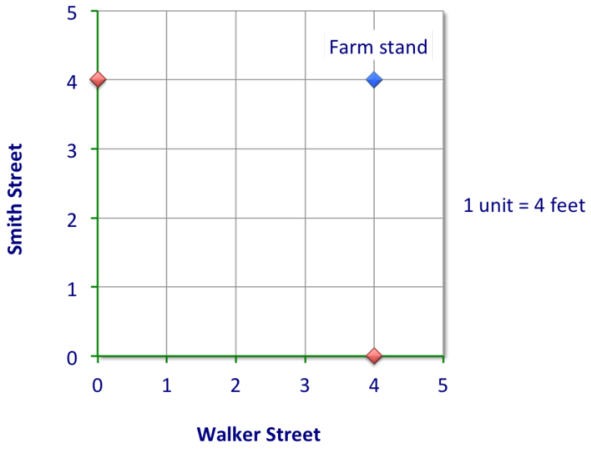


Alex wants to put each sign exactly 16 feet from where the farm stand is located. Each unit or box on the grid represents 4 feet. If he does this, what will be the coordinates of the sign on Walker? What will be the coordinates of the sign on Smith? Figure this out and then draw in the signs on the coordinate grid.

To find the solution, we can count on the vertical axis which is Smith Street. Alex wants to put the sign exactly 16 feet from the farm stand. If we start at the farm stand and count over four units to the vertical axis, we can see that the sign would be placed at (0, 4).

If we count four units to the horizontal axis, which is Walker Street, then Alex would be placing his second sign at (4, 0).

Next, we can use this information to plot the two points. Each point represents a sign that was placed on the neighboring streets.



Circle Graphs and Choosing Displays

Word Problem –Vegetable Statistics

Alex and Tania have this circle graph to represent their vegetable totals. They have already figured out how they can double their production from this season to next and have made predictions based on this information. However, Tania is keeping a journal and wants to write a report on this year's season based on vegetable totals. 400 vegetables were grown in all and she has written down each of the quantities.

Squash = 100

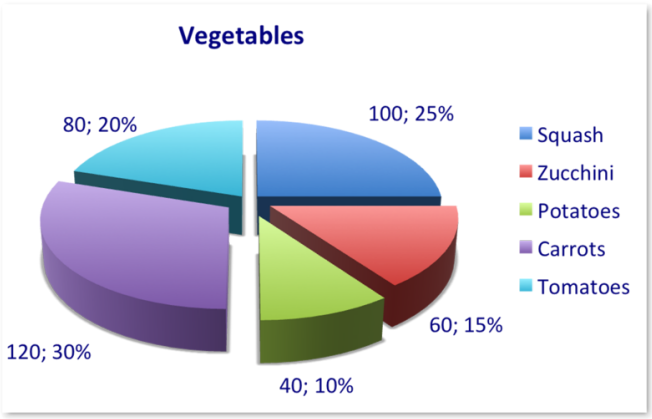
Zucchini = 60

Potatoes = 40

Carrots = 120

Tomatoes = 80

Now Tania has drawn in the circle graph.



Finally, she wants to write each vegetable totals as a fraction and a percent. You can help her to do this work. Remember to write all of your fractions in simplest form.

Solution

Here is the original problem once again. Reread it and then figure out each fraction and percent.

Alex and Tania have this circle graph to represent their vegetable totals. They have already figured out how they can double their production from this season to next and have made predictions based on this information. However, Tania is keeping a journal and wants to write a report on this year's season based on vegetable totals. 400 vegetables were grown in all and she has written down each of the quantities.

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Zucchini = 60

Potatoes = 40

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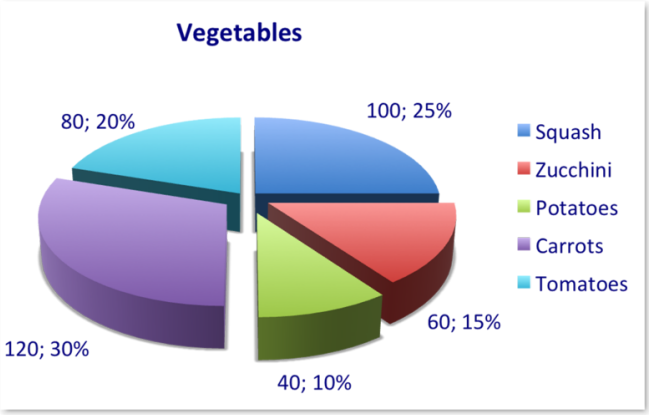

Finally, she wants to write each vegetable totals as a fraction and a percent. You can help her to do this work. Remember to write all of your fractions in simplest form.

To do this, first write each vegetable total as a fraction. The numerator will be the amount of vegetable grown and the denominator will be the total number in all which is 400.

Squash $= \frac{100}{400}$

Zucchini $= \frac{60}{400}$

Potatoes $= \frac{40}{400}$

Carrots $= \frac{120}{400}$

Tomatoes $= \frac{80}{400}$

Now we can simplify each fraction and turn it into a percent.

Squash = 25%

Zucchini = 15%

Potatoes = 10%

Carrots = 30%

Tomatoes = 20%

Mean, Median and Mode

Word Problem –Squash Figures

Tania finished with the carrots and moved on to the squash. She wants to find the mean or average, the median and the mode for the quantities of squash harvested over a nine week period of time. Here are the totals that she calculated.

3, 10, 10, 8, 8, 14, 12, 15, 20

Use what you have learned to find the mean, median and mode for this data set.

Solution

Here is the original problem once again. Reread it and then figure out each measure.

Tania finished with the carrots and moved on to the squash. She wants to find the mean or average, the median and the mode for the quantities of squash harvested over a nine week period of time. Here are the totals that she calculated.

3, 10, 10, 8, 8, 14, 12, 15, 20

Use what you have learned to find the mean, median and mode for this data set.

First, we can find the mean. To do this, we add up all the scores and divide by the number of scores.

$3+10+10+8+8+14+12+15+20=100$

Now we can divide by 9.

$100 \div 9 = 11.1$

The average vegetables harvested during any one week is 11.

To find the median score, we write the scores in order from least to greatest.

3, 8, 8, 10, 10, 12, 14, 15, 20

The median score is 10.

To find the mode, we look at the score that occurs the most often.

In this case, it could be 10 or 8. There are two modes for this data set.

5.3 Addition and Subtraction of Decimals

This third chapter *Addition and Subtraction of Decimals* introduces students to the Decimal System, covering addition and subtraction, rounding and estimation, and ordering decimals as well as introducing stem and leaf Plots. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Decimal Place Value
- Measuring Metric Length
- Ordering Decimals
- Rounding Decimals
- Decimal Estimation
- Adding and Subtracting Decimals
- Stem-and-Leaf Plots
- Use Estimation

Decimal Place Value

Word Problem –Pay Day

While working at the ice cream stand, Julie is excited that she will be getting paid. After her first week of work, she received her pay check. The teller at the bank gave Julie five $20.00 bills, one ten dollar bill, two fives, two ones, three dimes, a nickel and four pennies.

Julie took all of the money home and put it out on the kitchen table. She arranged the money into different money amounts. Julie began to count up all of the money that she received to check it with the amount on the check.

Pretend that you are Julie and you are checking your own money. Given the amount of money described above, how much money does Julie have? Write that amount with a decimal point in the correct place. You will need to use what you have learned about place value to get this done accurately.

Solution

Here is the original problem. Reread the problem and underline the important information.

While working at the ice cream stand, Julie is excited that she will be getting paid. After her first week of work, she received her pay check. The teller at the bank gave Julie five $20.00 bills, one ten dollar bill, two fives, two ones, three dimes, a nickel and four pennies.

Julie took all of the money home and put it out on the kitchen table. She arranged the money into different money amounts. Julie began to count up all of the money that she received to check it with the amount on the check.

Pretend that you are Julie and you are checking your own money. Given the amount of money described above, how much money does Julie have? Write that amount with a decimal point in the correct place. You will need to use what you have learned about place value to get this done accurately.

Now we need to help Julie to count all of her money. Here is the list of money she received.

Five twenty dollar bills = $100.00

One ten dollar bill = $10.00

Two five dollar bills = $10.00

Two one dollar bills = $2.00

Thirty–nine cents = $.39

When we add all of that up, Julie has a total of $122.39.

Measuring Metric Length

Word Problem –Building Picnic Tables

After measuring the area that the picnic tables will go in, Jose began working on designing the construction of the tables themselves. He asked his Dad to help, and Jose's Dad being from Brazil only had metric measures on hand.

Jose took out a ruler and began to work on sketching out the design. This seemed to make sense, but when he began looking at pieces of wood, the ruler didn't seem to make sense anymore. Each piece of wood is 10 feet long. His picnic tables will be 8 feet long or 2.4 meters long.

If Jose is going to work on measuring the wood, does it make more sense for him to use a ruler or a meter stick? Why should he use one tool over the other tool?

Solution

Here is the original problem from the introduction. Think about what you have learned about measuring in metrics and answer the questions at the end of the problem.

After measuring the area of that the picnic tables will go in, Jose began working on designing the construction of the tables themselves. He asked his Dad to help, and Jose's Dad being from Brazil only had metric measures on hand.

Jose took out a ruler and began to work on sketching out the design. This seemed to make sense, but when he began looking at pieces of wood, the ruler didn't seem to make sense anymore. Each piece of wood is 10 feet long. His picnic tables will be 8 feet long or 2.4 meters long.

If Jose is going to work on measuring the wood, does it make more sense for him to use a ruler or a meter stick? Why should he use one tool over the other tool?

If Jose was doing a drawing, then it would make sense for him to use ruler and measure the design in centimeters. However, Jose is working with actual wood. Think of how many ruler lengths it would be if Jose was working with wood pieces that are ten feet long. Jose needs to measure 2.4 meters, and his measuring must be accurate. Since the thing that Jose is measuring is larger than a book or a piece of paper, Jose should use a meter stick to measure the length of the picnic table.

Ordering Decimals

Word Problem –Ice Cream Containers

Mr. Harris called Julie into her office.

"Julie, I need your help. I need you to order new To Go containers for the ice cream sandwiches," Mr. Harris said.

"Alright, I'd be happy to help," Julie said.

"Great. Here is the catalog. Please look up the containers and choose the largest one. The diameter is the important

measurement. Let me know what you choose," Mr. Harris handed Julie the catalog and a piece of paper.

Julie took the catalog out to one of the picnic benches. She turned to the page where the To Go containers were for the ice cream sandwiches. There were three different types of containers.

Container 1 has a diameter of $5.67''$

Container 2 has a diameter of $3.38''$

Container 3 has a diameter of $5.59''$

Which one should Julie choose? You will need to use what you have learned about comparing decimals to select the best one.

Solution

Here is the original problem. Reread it and underline all of the important information.

Mr. Harris called Julie into her office.

"Julie, I need your help. I need you to order new To Go containers for the ice cream sandwiches," Mr. Harris said.

"Alright, I'd be happy to help," Julie said.

"Great. Here is the catalog. Please look up the containers and choose the largest one. The diameter is the important measurement. Let me know what you choose," Mr. Harris handed Julie the catalog and a piece of paper.

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Container 1 has a diameter of $5.67''$

Container 2 has a diameter of $3.38''$

Container 3 has a diameter of $5.59''$

Which one should Julie choose? You will need to use what you have learned about comparing decimals to select the best one.

Julie knew that she needed to choose the largest container and that the diameter would be the important measurement to pay attention to.

First, Julie looked at the one that would be the smallest. Container 2 has a diameter of $3.38''$. Since the whole number 3 is smaller than either 5 it is not the correct choice.

Next, she looked at the last two containers. The first number in both, 5, is the same, so Julie had to compare the decimal parts of the measure.

.67 and .59

.67 is greater than .59

She chose Container 1 with a diameter of 5.67. This is the best choice of containers.

Rounding Decimals

Word Problem –New Dimensions

Jose worked on the sign all day and redesigned it according to the directions Mr. Harris had given him.

The original sign was $4.25' \times 2.5'$.

The letters were $1.67'$

Mr. Harris had told Jose to round the sign measurements up to the nearest half foot and the sign letters up to the nearest whole foot. Jose did the math and came up with new dimensions for the sign.

The new sign would be $4.5' \times 3'$.

The letters would be $2'$.

Jose drew out the design with colors that he liked. He took the design to Mr. Harris for approval. Mr. Harris hesitated while looking at Jose.

"Don't you like it?" Jose asked.

"It isn't that, I just think it should be a bit larger. Why don't you round everything up another quarter inch," Mr. Harris said.

"Okay," Jose said trying not to sound too disappointed. "Back to the drawing board."

Jose went back outside to start all over again.

You can help Jose to round all of these new measurements once again. Remember that this time you are rounding up another quarter inch.

Solution

Here is the original problem once again. Reread it and then underline the important information.

Jose worked on the sign all day and redesigned it according to the directions Mr. Harris had given him.

The original sign was $4.25' \times 2.5'$.

The letters were $1.67'$

Mr. Harris had told Jose to round the sign measurements up to the nearest half foot and the sign letters up to the nearest whole foot. Jose did the math and came up with new dimensions for the sign.

The new sign would be $4.5' \times 3'$.

The letters would be $2'$.

Jose drew out the design with colors that he liked. He took the design to Mr. Harris for approval. Mr. Harris hesitated while looking at Jose.

"Don't you like it?" Jose asked.

"It isn't that, I just think it should be a bit larger. Why don't you round everything up another quarter inch," Mr. Harris said.

"Okay," Jose said trying not to sound too disappointed. "Back to the drawing board."

Jose went back outside to start all over again.

You can help Jose to round all of these new measurements once again. Remember that this time you are rounding up another quarter inch.

To rework all of the measurements, Jose decided to start with the sign.

$4.5' \times 3'$ **needs to be rounded up another quarter inch.**

A quarter inch is $.25''$**. This needs to be added to both measurements.**

$4.5 + .25 = 4.75'$

$3 + .25 = 3.25'$

The new sign dimensions are $4.75' \times 3.25'$**.**

The letters also need to be rounded up $.25''$**. Since they are** $2'$**, the new letter dimensions would be** $2.25'$**.**

This is the solution to the sign problem.

Decimal Estimation

Word Problem –More Recycling

Julie was so inspired by Jose's efforts with the recycling that she helped him continue the program into September, October and November. While school did start again, Julie and Jose continued working at the ice cream stand after school and on weekends. By the end, they had collected a lot of money.

Here are the records for what they collected in the next three months.

September = $51.25

October = $62.19

November = $38.85

Julie looked at the numbers. Just like Jose in the beginning she kept becoming confused by the decimals.

"I keep getting mixed up," she told Jose.

"You have to round and estimate your answer," Jose said. "That's the best way to figure out the sum."

Jose left the room. Julie looked at the figures again and began trying to figure out the sum.

To help Julie with this problem, you will need round each decimal amount and figure out the sum.

Solution

Here is the original problem once again. Reread it and then underline any important information.

Julie was so inspired by Jose's efforts with the recycling that she helped him continue the program into September, October and November. While school did start again, Julie and Jose continued working at the ice cream stand after school and on weekends. By the end, they had collected a lot of money.

Here are the records for what they collected in the next three months.

September = $51.25

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Julie looked at the numbers. Just like Jose in the beginning she kept becoming confused by the decimals.

"I keep getting mixed up," she told Jose.

"You have to round and estimate your answer," Jose said. "That's the best way to figure out the sum."

Jose left the room. Julie looked at the figures again and began trying to figure out the sum.

To help Julie with this problem, you will need round each decimal amount and figure out the sum.

To figure out the sum, Julie needs to round each number to the nearest whole number. We round each to the nearest whole number.

$51.25 rounds down to $51

$62.19 rounds down to $62

$38.85 rounds up to $39

Then we can add all three.

$51 + 62 + 38 = \$151.00$

The recycling program earned approximately $151 dollars during those three months.

Adding and Subtracting Decimals

Word Problem –Jose and the Broken Cash Register

After Julie went to lunch, Jose took over with the broken cash register. His mental math is strong and he was doing well until a woman showed up with three little kids. She ordered three small cones at $2.25 each, one with sprinkles at .10, one with an extra scoop at .85 and one with caramel at .30.

Then she handed Jose a $20.00 bill.

Jose began to add as quickly as he could. He figured out the sum and gave the woman back her change.

What was the sum? How much money did the woman receive as change?

Using what you have learned about adding and subtracting decimals, find the answer to these two questions.

Solution

Here is the original problem once again. Reread it and then find the answers to the two questions at the end.

After Julie went to lunch, Jose took over with the broken cash register. His mental math is strong and he was doing well until a woman showed up with three little kids. She ordered three small cones at $2.25 each, one with sprinkles at .10, one with an extra scoop at .85 and one with caramel at .30.

Then she handed Jose a $20.00 bill.

Jose began to add as quickly as he could. He figured out the sum and gave the woman back her change.

What was the sum? How much money did the woman receive as change?

Using what you have learned about adding and subtracting decimals, find the answer to these two questions.

First, Jose must find the sum.

Three cones at 2.25 each means $2.25+2.25+2.25=\$6.75$ **for the three cones**

$$\begin{array}{r} 6.75 \\ .30 \\ .10 \\ +.85 \\ \hline 8.00 \end{array}$$

The total cost of the order was $8.00.

The woman handed Jose a twenty dollar bill.

$20-8=12$

Jose gave the woman back $12.00 in change.

Stem-and-Leaf Plots

Word Problem –The Worst Week Ever

Mr. Harris had been so excited about the best week ever in sales. Well, the first week of September, everyone went back to school and he had his worst week ever. Julie and Jose continued to work after school, but they saw Mr.

Harris look worried throughout the week. By Saturday, sales were beginning to turn around and Mr. Harris looked better. Here are the sales for the worst week ever.

Monday –20

Tuesday –15

Wednesday –10

Thursday –12

Friday –26

Saturday –32

Sunday –9

Take these sales and organize them in a stem-and-leaf plot. Then answer the following questions about the data.

1. What is the smallest number of ice cream cones sold?
2. What is the largest number of ice cream cones sold?
3. What is the range in the number of cones sold?
4. What is the interval with the most values in it?
5. What is the mode of this data set?
6. What is the average number of cones sold?

Solution

Here is the original problem. Reread it and underline all of the important information.

Mr. Harris had been so excited about the best week ever in sales. Well, the first week of September, everyone went back to school and he had his worst week ever. Julie and Jose continued to work after school, but they saw Mr. Harris look worried throughout the week. By Saturday, sales were beginning to turn around and Mr. Harris looked better. Here are the sales for the worst week ever.

Monday –20

Tuesday –15

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Take these sales and organize them in a stem-and-leaf plot. Then answer the following questions about the data.

1. What is the smallest number of ice cream cones sold?
2. What is the largest number of ice cream cones sold?
3. What is the range in the number of cones sold?
4. What is the interval with the most values in it?
5. What is the mode of this data set?
6. What is the average number of cones sold?

A stem-and-leaf plot is going to organize the data according to the base ten values listed.

$$\begin{array}{r|l} & 9 \\ 1025 & \\ 206 & \\ 32 & \end{array}$$

Use Estimation

Word Problem –Julie's Shopping Spree

Just like Jose, Julie also earned $2100.00 by working all summer. She also decided to spend some money on clothing that she needed for school. She picked out a bunch of new outfits, including new shoes and a new coat. She decided to start with the two things she wanted the most because her Mom also wants her to save some of the money that she earned.

Julie has picked out shoes that cost $57.25 and a new coat for $89.90. She brought $150.00 with her to the store.

Use estimation to calculate the total of Julie's purchases. Did she bring enough money? If so, what was her approximate change?

Solution

Here is the original problem once again. Reread it and then underline the important information.

Just like Jose, Julie also earned $2100.00 by working all summer. She also decided to spend some money on clothing that she needed for school. She picked out a bunch of new outfits, including new shoes and a new coat. She decided to start with the two things she wanted the most because her Mom also wants her to save some of the money that she earned.

Julie has picked out shoes that cost $57.25 and a new coat for $89.90. She brought $150.00 with her to the store.

Use estimation to calculate the total of Julie's purchases. Did she bring enough money? If so, what was her approximate change?

There are three things that need to be done to solve this problem. First, we need to use estimation to figure out the sum of Julie's purchases. We can round the cost of the shoes and the coat to make the addition simpler.

57.25 rounds down to 57

89.90 round up to 90

Now add.

$57 + 90 = 147$

Julie brought $150.00 with her to the store. This means that she had enough money.

Her approximate change is just under $3.00.

5.4 Multiplication and Division of Decimals

This fourth chapter *Multiplication and Division of Decimals* furthers a student's understanding of decimals by introducing methods of multiplication and division. Also covered are the Distributive Property and the use and conversion of metric units. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Multiplying Decimals and Whole Numbers
- The Distributive Property
- Multiplying Decimals
- Dividing by Whole Numbers
- Multiplying and Dividing Decimals by Powers of Ten
- Dividing Decimals
- Metric Units of Mass and Capacity
- Converting Metric Units

Multiplying Decimals and Whole Numbers

Word Problem –A Change in Admission

All of the planning was complete on the day of the Science Museum. Kyle had a list of each student and how much money each of them would need to pay. He figured that all 22 students would pay $8.95 each and then the class needed to pay for the two additional chaperones too. Kyle figured out the total, and he was ready.

However, when Mrs. Andersen took role call there were two students absent. Kyle had done all of his math based on 22 students and not 20.

"It won't change it that much," Kelly said encouragingly.

"I know, but I still have to do all the math again to be sure that I have the right total."

Kyle sat down at his desk with a piece of paper and a pencil. He began figuring things out.

Now that you understand how to multiply decimals and whole numbers, you can help Kyle. You will need to figure out a product and then add the additional two full price tickets for the chaperones. When finished, you will know how much money Kyle needs to collect.

Solution

Here is the original problem once again. Reread it and then underline all of the important information.

All of the planning was complete on the day of the Science Museum. Kyle had a list of each student and how much money each of them would need to pay. He figured that all 22 students would pay $8.95 each and then the class needed to pay for the two additional chaperones too. Kyle figured out the total, and he was ready.

However, when Mrs. Andersen took role call there were two students absent. Kyle had done all of his math based on 22 students and not 20.

"It won't change it that much," Kelly said encouragingly.

"I know, but I still have to do all the math again to be sure that I have the right total."

Kyle sat down at his desk with a piece of paper and a pencil. He began figuring things out.

Now that you understand how to multiply decimals and whole numbers, you can help Kyle. You will need to figure out a product and then add the additional two full price tickets for the chaperones. Remember, a full price ticket is $12.95. When finished, you will know how much money Kyle needs to collect.

To begin, you will need to figure out the total cost for the students. There are 20 students at $8.95 each. To figure this out quickly, you can multiply.

$8.95 \times 20 = 179.00$

The total cost for the students will be $179.00.

Next, you will need to add the cost for the two chaperones.

$12.95 \times 2 = 25.90$

Finally, add the two products together.

$179.00 + 25.90 = \$204.90$

This is the amount of money that Kyle will need to collect.

The Distributive Property

Word Problem –The Planetarium

When Kyle and his classmates arrived at the Science museum there was so much to see that they didn't know where to begin. Mrs. Andersen said that they would start with the show at the Omni on the rainforest. On the way to the Omni theater, many students noticed that there was also a planetarium show.

"Wow! Can we see that too?" Sarah asked Mrs. Andersen.

"It is on meteors," Kyle said.

"That would be really cool!" Amanda said smiling.

Mrs. Andersen looked at Kyle.

"I know," said Kyle. "More math."

Mrs. Andersen did a count and everyone wanted to attend the planetarium as well as the Omni theater. The Omni cost an additional $2.00 per ticket and the planetarium an additional $2.50 per ticket.

Kyle did some quick figuring and collected the money. Then he paid the cashier for both shows for all 20 students. The chaperones were allowed to see the shows for free.

If Kyle used the Distributive Property, what did his expression look like? What was the final amount of money collected?

Solution

Here is the original problem once again. Reread it and underline all of the important information.

When Kyle and his classmates arrived at the Science museum there was so much to see that they didn't know where to begin. Mrs. Andersen said that they would start with the show at the Omni on the rainforest. On the way to the Omni theater, many students noticed that there was also a planetarium show.

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"I know," said Kyle. "More math."

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Kyle did some quick figuring and collected the money. Then he paid the cashier for both shows for all 20 students. The chaperones were allowed to see the shows for free.

If Kyle used the Distributive Property, what did his expression look like? What was the final amount of money collected?

To use the Distributive Property, Kyle would have written the total number of students outside of the parentheses and the two dollar amounts inside the parentheses.

$20(2.00+2.50)$

Now he would have combined the amounts in the parentheses.

20(4.50)

Next, he would have multiplied.

20(4.50) = $90.00

Kyle collected $90.00 from the students.

Multiplying Decimals

Word Problem –The T–Rex

Once Kara was finished at the triceratops, she wandered over to the large model of the Tyrannosaurus Rex. She craned her neck up to look at the underside of his chin and she walked the entire length of it.

"Wow," Kara said. "That is really something."

"It sure is," the curator at the museum commented.

"A T–Rex was 8 times longer than my height and its height is 2.5 times mine," she said smiling.

The curator also smiled and they continued to talk all about dinosaur features.

Meanwhile Kyle had been listening. He listened to Kara and then when they walked away he stopped her.

"How tall are you?" he asked.

"$5\frac{1}{4}$ *feet* or 5.25 if you want to be more specific," she said.

Kyle stopped and thought about the figures that Kara had told the curator. Then he opened his notebook to figure out the length and height of the T–Rex.

You can figure this out too. Use what you have learned about multiplying decimals to figure out the length and height of the average T–Rex.

Solution

Here is the original problem once again. Reread it and then underline all of the important information.

Once Kara was finished at the triceratops, she wandered over to the large model of the Tyrannosaurus Rex. She

craned her neck up to look at the underside of his chin and she walked the entire length of it.

"Wow," Kara said. "That is really something."

"It sure is," the curator at the museum commented.

"A T–Rex was 8 times longer than my height and it's height is 2.5 times mine," she said smiling.

The curator also smiled and they continued to talk all about dinosaur features.

Meanwhile Kyle had been listening. He listened to Kara and then when they walked away he stopped her.

"How tall are you?" he asked.

"$5\frac{1}{4}$ *feet* or 5.25 if you want to be more specific," she said.

Kyle stopped and thought about the figures that Kara had told the curator. Then he opened his notebook to figure out the length and height of the T–Rex.

You can figure this out too. Use what you have learned about multiplying decimals to figure out the length and height of the average T–Rex.

First, Kyle decided to figure out the length of the T–Rex. It also required multiplying a decimal by a whole number, so he figured this was a good place to begin.

Kara's height = 5.25 feet

The T–Rex was 8 times her height.

$5.25 \times 8 = 42$ *feet*

The length of the average T–Rex is 42 feet.

Then Kyle moved on to the height. The height of the T–Rex is 2.5 times Kara's height.

$2.5 \times 5.25 = 13.1$ *feet*

The average height of a T–Rex is 13 feet.

Dividing by Whole Numbers

Word Problem –An Incorrect Calculation

When the students left the museum, Mrs. Andersen received $35.20 for an unexpected discount. She took that money and divided it by 22 and figured out that each student should receive $1.60 change. Once she had it figured out, Mrs. Andersen called Kyle over to give him the change to hand out.

"So, with 22 students, each will receive $1.60 change," she said.

"But that's not right," Kyle corrected.

"Why not?"

"Because only 20 students attended the field trip, so the discount needs to go to 20 students and not 22," Kyle added.

"Good catch Kyle. Can you figure this out while I take roll call on the bus?"

Kyle began to figure out the change.

While Kyle is working, you can work on this problem too. You will need to use what you have learned about dividing decimals by whole numbers to come up with an accurate amount of change for each student.

Solution

Here is the original problem once again. Reread it and underline only the necessary information.

When the students left the museum, Mrs. Andersen received $35.20 for an unexpected discount. She took that money and divided it by 22 and figured out that each student should receive $1.60 change. Once she had it figured out, Mrs. Andersen called Kyle over to give him the change to hand out.

"So, with 22 students, each will receive $1.60 change," she said.

"But that's not right," Kyle corrected.

"Why not?"

"Because only 20 students attended the field trip, so the discount needs to go to 20 students and not 22," Kyle added.

"Good catch Kyle. Can you figure this out while I take roll call on the bus?"

Kyle began to figure out the change.

While Kyle is working, you can work on this problem too. You will need to use what you have learned about dividing decimals by whole numbers to come up with an accurate amount of change for each student.

Kyle knew that this was a division problem, so he set up the problem like this.

$35.20 \div 20$

or

$$20\overline{)35.20}\quad 1.76$$

Each student should receive $1.76 change from the field trip.

Multiplying and Dividing Decimals by Powers of Ten

Word Problem –The Moon

Kailey took a book out the library on the moon. After seeing a whole lecture on the moon at the Science museum, her curiosity is peaked and she wants to learn all that she can about it. While sitting at home, she writes down that the moon is 3474.8 km in diameter.

While Kailey is learning let's use this number to explore multiplying and dividing with decimal powers of ten. Use what you have learned in this lesson to answer the following questions.

1. What would the diameter be if the moon were 10 times as large?
2. What if it were 10 times smaller?
3. What would the diameter be if the moon were 100 times larger?
4. What would the diameter be if the moon were 1000 times larger?

Use what you have learned to complete this section.

Solution

Here is the original problem and the answers to the questions posed at the end of it.

Kailey took a book out the library on the moon. After seeing a whole lecture on the moon at the Science museum, her curiosity is peaked and she wants to learn all that she can about it. While sitting at home, she writes down that the moon is 3474.8 km in diameter.

While Kailey is learning let's use this number to explore multiplying and dividing with decimal powers of ten. Use what you have learned in this lesson to answer the following questions.

1. What would the diameter be if the moon were 10 times as large?

2. What if it were 10 times smaller?
3. What would the diameter be if the moon were 100 times larger?
4. What would the diameter be if the moon were 1000 times larger?

Use what you have learned to complete this section.

If the diameter of the moon were ten times larger, we would multiply by 10.

3474.8×10

The easiest way to do this is to move the decimal point one place to the right.

The answer is 34,748 km.

If it were ten times smaller, then we would move the decimal point to the left one place.

347.48 km is the answer.

If it were 100 times larger, then we would move the decimal point two places to the right.

3474.8 becomes 347,480 km.

If it were 1000 times larger, then we would move the decimal point three places to the right.

3474.8 becomes 3,474,800 km.

Dividing by Decimals

Word Problem –Hour Glass Additions

When Miles entered the Discovery Center he was immediately overwhelmed with all of the options. After looking around, he finally decided to work on an experiment that involves an hour glass. To complete the experiment, Miles needed to figure out how long it takes 1.25 pounds of sand to go through the hour glass. There was bucket of sand that is 6.25 pounds in front of Miles. He had a scale and another bucket to put the sand he needs for his experiment.

Miles needs to complete the experiment as many times as he can with the 6.25 pound bucket of sand. Miles picks up the scoop and begins to sort out the sand.

After working on this experiment for a little while, Sasha became interested in what Miles was working on. She stood behind him watching and then went and asked the person in charge of the exhibit for another bucket of sand. Then she brought it over to where Miles was working.

“Here Miles, I have another 6.25 pound bucket of sand,” she said smiling.

“Great. So we have 12.5 pounds of sand altogether. I wonder how many times we can do the experiment with that much sand?” Miles wondered.

We are wondering too. This is your task. Use what you have learned about dividing decimals to figure out how many times Miles can do the experiment given the new total of sand.

Solution

Here is the original problem once again. Reread it and then figure out the answer to the problem.

When Miles entered the Discovery Center he was immediately overwhelmed with all of the options. After looking around, he finally decided to work on an experiment that involves an hour glass. To complete the experiment, Miles needed to figure out how long it takes 1.25 pounds of sand to go through the hour glass. There was bucket of sand that is 6.25 pounds in front of Miles. He had a scale and another bucket to put the sand he needs for his experiment.

Miles needs to complete the experiment as many times as he can with the 6.25 pound bucket of sand. Miles picks up the scoop and begins to sort out the sand.

After working on this experiment for a little while, Sasha became interested in what Miles was working on. She stood behind him watching and then went and asked the person in charge of the exhibit for another bucket of sand. Then she brought it over to where Miles was working.

"Here Miles, I have another 6.25 pound bucket of sand," she said smiling.

"Great. So we have 12.5 pounds of sand altogether. I wonder how many times we can do the experiment with that much sand?" Miles wondered.

We are wondering too. This is your task. Use what you have learned about dividing decimals to figure out how many times Miles can do the experiment given the new total of sand.

To figure this out, we will need to write a division problem.

$12.5 \div 1.25$

12.5 is the total amount of sand in two buckets.

1.25 pounds is the amount of sand through the hour glass at one time.

$$1.25\overline{)12.5}^{10}$$

Miles can perform the experiment 10 times.

Metric Units of Mass and Capacity

Word Problem –Soda Solutions

"Look at this display," Sam said to Olivia.

It was a display of a bunch of different items. There was a paper clip, a dollar bill, a bag of sugar, and a can of soda.

"What do you do?" Olivia asked looking at the directions.

At this display, there were different magnetic strips that had the words gram and milligram on them. Students were being asked to choose the correct measurement and put it next to the object. If you were correct, then you received ten points.

Olivia looked at the magnetic strips.

"I can remember the conversions, but which one is smaller a gram or a milligram?"

This is a good question. This lesson is about choosing the correct metric unit of mass or capacity. Olivia needs to do this as well. Therefore, take your time and think about each item. First, answer Olivia's question. Then write down which unit you should use to measure each item in the display.

Solution

Here is the original problem once again. Use what you have learned about metric units of mass to figure out the solution.

"Look at this display," Sam said to Olivia.

It was a display of a bunch of different items. There was a paper clip, a dollar bill, a bag of sugar, and a can of soda.

"What do you do?" Olivia asked looking at the directions.

At this display, there were different magnetic strips that had the words gram and milligram on them. Students were being asked to choose the correct measurement and put it next to the object. If you were correct, then you received ten points.

Olivia looked at the magnetic strips.

"I can remember the conversions, but which one is smaller a gram or a milligram?"

This is a good question. This lesson is about choosing the correct metric unit of mass or capacity. Olivia needs to do this as well. Therefore, take your time and think about each item. First, answer Olivia's question. Then write down which unit you should use to measure each item in the display.

Olivia can't remember which unit is the smallest, but the answer is actually in the name itself. First, let's look at milligram. The prefix "milli" means one thousandth. Because of the "th", we already know that that is small part. Therefore a milligram is smaller than a gram. If a milligram is one–thousandth of a gram, then all of the items here would be too heavy to be measured using milligrams.

These are units of weight. Next, we need to determine which item would be measured by each unit.

Paper clip –gram

Dollar bill –gram

Bag of sugar –gram

Can of soda –gram

Converting Metric Units

Word Problem –Weighing in on Elephants

Before Sam and Olivia left the metric park, they spotted one of the most interesting experiments. It was a display in a glass case with a picture of an elephant above scale. Under the display it said, "I weigh 4,600,000 grams. How many kilograms is that?" There was also a place to enter the answer into the computer to see if the answer is correct.

Olivia looked at Sam and smiled.

"Do you know?" Sam asked.

"I can figure it out," Olivia said.

"I can't remember how many grams are in one kilogram," Sam said.

"Watch this," Olivia said taking a piece of scrap paper from the display.

While Olivia explains how to figure out the weight of the elephant in kilograms, you can do the same. Use what you have learned about metric units of mass to figure out the dilemma.

Solution

Here is the original problem once again. Use what you have learned about metric units of mass to figure out the solution.

Before Sam and Olivia left the metric park, they spotted one of the most interesting experiments. It was a display in a glass case with a picture of an elephant above scale. Under the display it said, "I weigh 4,600,000 grams. How many kilograms is that?" There was also a place to enter the answer into the computer to see if the answer is correct.

Olivia looked at Sam and smiled.

"Do you know?" Sam asked.

"I can figure it out," Olivia said.

"I can't remember how many grams are in one kilogram," Sam said.

"Watch this," Olivia said taking a piece of scrap paper from the display.

While Olivia explains how to figure out the weight of the elephant in kilograms, you can do the same. Use what you have learned about metric units of mass to figure out the dilemma.

First, let's write down what we know. We know that the elephant weighs 4,600,000 grams and that we need to change that to kilograms.

Sam can't remember how many grams are in one kilogram, and remember a kilogram is larger than a gram. Olivia knows the answer to this question.

There are 1000 grams in 1 kilogram.

Now all you need to do is to figure out the conversion from grams to kilograms.

$4,600,000\ grams \div 1000 = 4600\ kilograms$

An elephant weighs 4600 kilograms.

5.5 Number Patterns and Fractions

This fifth chapter *Number Patterns and Fractions* covers prime and composite numbers and factorization. Students will also explore equivalent fractions, how to order fractions, and the relationship between fractions and decimals through methods of conversion. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Prime Factorization
- Greatest Common Factors
- Equivalent Fractions
- Least Common Multiple
- Ordering Fractions
- Mixed Numbers and Improper Fractions
- Changing Decimals to Fractions
- Changing Fractions to Decimals

Prime Factorization

Word Problem –New Additions

Allison and Hector worked hard on figuring out how to arrange all of the students for the sixth grade social. By organizing the students in the two clusters, they figured out that there would be 23 students at each event at each time. This was a very manageable number and one that they were sure the teachers would agree on. Let's review the work they did.

Cluster 6A has 48 students.

Cluster 6B has 44 students.

By figuring out the prime factorization of both numbers, it made sense to organize the classes into the four groups. Just as they were sure that they were finished, their teachers came over to them.

"We have a small change to the social," Mr. Watkins began.

"What is it?" Hector asked.

"Well, we invited the sixth grade from Christophers, private school to join us. It is an addition of 24 students," he said.

Hector and Allison looked at Mr. Watkins and then at each other. How would the new addition of 24 students affect their group size?

"Back to the drawing board," said Allison.

Use what you have learned about prime factorization to figure out how many students would be at each event given the new students from the private school.

Solution

Here is the original problem once again. Reread it and then figure out how many students will be attending each event at the social.

Allison and Hector worked hard on figuring out how to arrange all of the students for the sixth grade social. By organizing the students in the two clusters, they figured out that there would be 23 students at each event at each time. This was a very manageable number and one that they were sure the teachers would agree on. Let's review the work they did.

Cluster 6A has 48 students.

Cluster 6B has 44 students.

By figuring out the prime factorization of both numbers, it made sense to organize the classes into the four groups. Just as they were sure that they were finished, their teachers came over to them.

"We have a small change to the social," Mr. Watkins began.

"What is it?" Hector asked.

"Well, we invited the sixth grade from Christophers, private school to join us. It is an addition of 24 students," he said.

Hector and Allison looked at Mr. Watkins and then at each other. How would the new addition of 24 students affect their group size?

"Back to the drawing board," said Allison.

Use what you have learned about prime factorization to figure out how many students would be at each event given the new students from the private school.

We know that each group at the social has 23 students in it right now. Well, Hector and Allison now have to figure in the students from Christophers. There are 24 students in the sixth grade. Because of this, we will need to factor 24.

24

1×24

2×12

3×8

4×6

There are four groups at the social since there are four events. Therefore, there will be an addition of 6 students to each of those groups. That will divide the students from Christophers evenly.

23 original students + 6 from Christophers = 29 students

There will be 29 students at each event at the social.

Greatest Common Factors

Word Problem –The Basketball Scrimmage

The sixth grade basketball team is playing a scrimmage with two neighboring schools. Each school will band together and form four new teams. The new schools are very excited. The first new school, we can call it school A has 24 students who are going to play. The other school, we can call it school B has 36 students who are going to play. Then there are two clusters, 6A and 6B already playing.

Here are the numbers of students who are going to play.

6A has 48 students.

6B has 44 students.

School A has 24 students.

School B has 36 students.

To work this out, you will need to figure out how many students from each of the four groups will play on each of four teams. To figure this out, you will need to use what you have learned about greatest common factors.

Solution

Here is the original problem once again. Reread it and then underline the important information before solving it.

The sixth grade basketball team is playing a scrimmage with two neighboring schools. Each school will band together and form four new teams. The new schools are very excited. The first new school, we can call it school A has 24 students who are going to play. The other school, we can call it school B has 36 students who are going to play. Then there are two clusters, 6A and 6B already playing.

Here are the numbers of students who are going to play.

6A has 48 students.

6B has 44 students.

School A has 24 students.

School B has 36 students.

To work this out, you will need to figure out how many students from each of the four groups will play on each team. To figure this out, you will need to use what you have learned about greatest common factors. The greatest common factor will help us to figure out how many teams there should be.

Let's write out all of the factors for each group of students.

$6A = 48$

1×48

2×24

4×12

$6B = 44$

1×44

2×22

4×11

School $A = 24$

1×24

2×12

3×8

4×6

School $B = 36$

1×36

2×18

3×12

4×9

The greatest common factor of all four groups is 4. Therefore, there will be four teams of students playing in the scrimmage.

We can divide each number of students by 4 to figure out how many from each group will be on each team.

6A will have 12 students on each team.

6B will have 11 students on each team.

School A will have 6 students on each team.

School B will have 9 students on each team.

This is our answer.

Equivalent Fractions

Word Problem –Let's Eat Cake

When the first round of cake eating was complete, two other teams of students decided to play the games to try to win cake. Since the first round was over, the teachers brought out two new cakes, these were large sheet cakes with vanilla icing and raspberry toppings. Each cake was divided into 24 pieces.

The two groups were thrilled to play for the cake. One group was all girls and one group was all boys. The boys thought that they would rather have $\frac{2}{3}$ of the cake than one–half.

"Why do you want that amount?"asked one of the girls.

"We aren't happy with one–half. Two–thirds is a greater amount of cake," one of the boys responded.

"Maybe you are being greedy," another girl commented.

"Nope, just hungry," the second boys said.

If the boys do get $\frac{2}{3}$ of the cake, how many slices will they receive?

How many will the girls receive if they get $\frac{1}{2}$ of the cake?

How many more pieces will one or both of them receive?

Solution

Here is the original problem once again. Reread it and then answer the questions by using what you have learned about equivalent fractions.

When the first round of cake eating was complete, two other teams of students decided to play the games to try to win cake. Since the first round was over, the teachers brought out two new cakes, these were large sheet cakes with vanilla icing and raspberry toppings. Each cake was divided into 24 pieces.

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"We aren't happy with one–half. Two–thirds is a greater amount of cake," one of the boys responded.

"Maybe you are being greedy," another girl commented.

"Nope, just hungry," the second boys said.

If the boys do get $\frac{2}{3}$ of the cake, how many slices will they receive?

How many will the girls receive if they get $\frac{1}{2}$ of the cake?

How many more pieces will one or both of them receive?

We know that the cake has 24 slices in it. We need to figure out the $\frac{2}{3}$ for the boys first.

$$\frac{2}{3} = \frac{x}{24}$$

Here is our equivalent fraction set–up. Now we can solve for x.

The boys will receive 16 pieces of cake.

Now let's figure out the girls' portion of the cake.

$$\frac{1}{2} = \frac{x}{24}$$

The girls will receive 12 pieces of cake.

The boys will receive 4 extra pieces of cake by requesting $\frac{2}{3}$ of the cake instead of one–half.

Least Common Multiple

Word Problem –Woodshop Woes

Mr. Caron shared his strategy about the art room with Mr. Webber the woodshop teacher. The two of them were sitting in the teacher's room having a cup of coffee when the conversation about shared classrooms came up.

"I figured it out mathematically," said Mr. Caron sipping his coffee.

"I should do the same thing. You see, I have 6A every three days and 6B every four days. Then there is a day every now and again that they are in the shop together. If I planned it out, then I would know that this would be a good day to do painting because the group is so large. It could be our finishing day. I need to figure this out right away," Mr. Webber said.

This is where you come in. You can use what you have learned about least common multiples to help Mr. Webber. Then use what you figure out to answer these questions.

What is the first day that the two classes will both be in the woodshop?

On what day of the week will the classes be together for the second time?

If one Monday was off, what would the second day be?

Solution

Here is the original problem once again. Use what you have learned about least common multiples and then answer the questions at the end.

Mr. Caron shared his strategy about the art room with Mr. Webber the woodshop teacher. The two of them were sitting in the teacher's room having a cup of coffee when the conversation about shared classrooms came up.

"I figured it out mathematically," said Mr. Caron sipping his coffee.

"I should do the same thing. You see, I have 6A every three days and 6B every four days. Then there is a day every now and again that they are in the shop together. If I planned it out, then I would know that this would be a good day

to do painting because the group is so large. It could be our finishing day. I need to figure this out right away," Mr. Webber said.

This is where you come in. You can use what you have learned about least common multiples to help Mr. Webber. Then use what you figure out to answer these questions.

What is the first day that the two classes will both be in the woodshop?

On what day of the week will the classes be together for the second time?

If one Monday was off, what would the second day be?

First, we can use least common multiples to figure out what days they will both be in the woodshop.

6A every three days.

3 6 9 12 15 18 21 24

6B every four days.

4 8 12 16 20 24

They will meet for the first time on day 12.

Now count off days of the week to figure out the day that they will meet together for the second time. You can use a calendar for this or just a list.

The answer is that the two classes will meet up on a Thursday.

If Monday is a day off, then they will meet up on a Friday.

Ordering Fractions

Word Problem –Cake Clean–Up

When the students went to clean up the cake activity, they discovered that there were several pieces of cake left over. They sat around trying to figure out who should get which leftovers for a long time. Finally, Amanda spoke up.

"I am willing to take the smallest part," she said and she picked up $\frac{1}{10}$ of a vanilla cake.

"What about the rest?" Josh said smiling.

They looked at the cake slices left. Here is what was left.

$\frac{2}{6}$ of a raspberry cake

$\frac{5}{8}$ of a chocolate cake

$\frac{1}{2}$ of a banana cake

Before the students decide who should eat which cake left–overs, they need to figure them out according to size. Even though you can't see the cakes, you can figure this out by looking at the fractions of cake that is left. Use what you have learned about ordering fractions to write them in order from least to greatest. Be sure to include the cake that Amanda took.

Solution

Here is the original problem once again. Reread it and then underline the important information.

When the students went to clean up the cake activity, they discovered that there were several pieces of cake left over. They sat around trying to figure out who should get which leftovers for a long time. Finally, Amanda spoke up.

"I am willing to take the smallest part," she said and she picked up $\frac{1}{10}$of a vanilla cake.

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Before the students decide who should eat which cake left–overs, they need to figure them out according to size. Even though you can't see the cakes, you can figure this out by looking at the fractions of cake that is left. Use what you have learned about ordering fractions to write them in order from least to greatest. Be sure to include the cake that Amanda took.

Now we need to order the fractions from least to greatest. Let's see them written out first.

$$\frac{1}{10}, \frac{1}{2}, \frac{5}{8}, \frac{2}{6}$$

We know that one–tenth is the smallest because Amanda told us that when she made her selection. So now we need to work with the others.

Let's rewrite them with a common denominator as equivalent fractions.

$$\frac{1}{2} \textit{ becomes } \frac{12}{24}$$
$$\frac{2}{6} \textit{ becomes } \frac{8}{24}$$
$$\frac{5}{8} \textit{ becomes } \frac{15}{24}$$

Now we can write them in order from least to greatest.

$$\frac{1}{10}, \frac{2}{6}, \frac{1}{2}, \frac{5}{8}$$

This is our answer.

Mixed Numbers and Improper Fractions

Word Problem –Pizza Leftovers

At the social, the students decided to serve pizzas. Mr. Caron ordered them so that they were divided into 8 pieces. The pieces were a little larger so everyone got a bit more pizza. The pizza sales were a huge success and the sixth grade class was able to earn some extra money for their fundraiser.

When the social was over, there was some pizza left over. Travis was in charge of tallying the amount of pizza sold and the amount left over.

Mr. Caron had purchased 24 pizzas. They were each divided into 8 slices. When Travis tallied up what was left, he discovered that there was 15 slices left.

$\frac{35}{8}$ was left.

How many whole pizzas were actually left?

How many extra slices?

You can use what you have learned about mixed numbers and improper fractions to figure out the answers to these questions.

Solution

Here is the original problem once again. Reread it and then underline the important information.

At the social, the students decided to serve pizzas. Mr. Caron ordered them so that they were divided into 8 pieces. The pieces were a little larger so everyone got a bit more pizza. The pizza sales were a huge success and the sixth grade class was able to earn some extra money for their fundraiser.

When the social was over, there was some pizza left over. Travis was in charge of tallying the amount of pizza sold and the amount left over.

Mr. Caron had purchased 24 pizzas. They were each divided into 8 slices. When Travis tallied up what was left, he discovered that there was 35 slices left.

$\frac{35}{8}$ was left.

How many whole pizzas were actually left?

How many extra slices?

You can use what you have learned about mixed numbers and improper fractions to figure out the answers to these questions.

To figure out the answers to these questions, first let's look at the left over pizza.

$$\frac{35}{8}$$

This represents an improper fraction.

8 slices make a whole, so we can divide 35 slices by 8 to figure out the number of whole pizzas left.

$35 \div 8 = 4$

With 3 slices left over.

Four whole pizzas were left, and three extra slices.

As a mixed number, we can write $4\frac{3}{8}$ ***pizza was left.***

Changing Decimals to Fractions

Word Problem –The Drill

When Aaron looked at Mr. Jones' toolbox, he was amazed at all of the different sized drill bits and center bits in the case.

"Each one of those drills a different size hole," Mr. Jones said to Aaron.

Aaron was amazed. He had never seen so many different sized parts in his life.

"There must be one hundred of these," he exclaimed.

"Maybe, but I often use only the same ones. The kit comes with many," Mr. Jones explained.

Aaron looked again and saw the measurements of three different bits.

0.010 inches

0.015 inches

0.020 inches

Aaron isn't very good at reading decimals yet, so he wants to change each of these to fractions so he can understand the size a bit better. You can use what you have learned to help Aaron with this work.

Solution

Here is the original problem once again. Reread it and then convert each decimal to a fraction equivalent.

When Aaron looked at Mr. Jones' toolbox, he was amazed at all of the different sized drill bits and center bits in the case.

"Each one of those drills a different size hole," Mr. Jones said to Aaron.

Aaron was amazed. He had never seen so many different sized parts in his life.

"There must be one hundred of these," he exclaimed.

"Maybe, but I often use only the same ones. The kit comes with many," Mr. Jones explained.

Aaron looked again and saw the measurements of three different bits.

0.010 inches

0.015 inches

0.020 inches

Aaron isn't very good at reading decimals yet, so he wants to change each of these to fractions so he can understand the size a bit better. You can use what you have learned to help Aaron with this work.

Aaron wants to convert the following three decimals to fractions.

0.010 inches

0.015 inches

0.020 inches

To do this, he is going to need to think in terms of place value. When thinking about place value, Aaron will know what denominator is needed for each fraction.

Each decimal has three decimal places after the decimal point, so Aaron knows that the decimal is written in thousandths. That is the denominator that is needed.

$0.010\ inches = \frac{10}{1000}\ inches$

$0.015\ inches = \frac{15}{1000}\ inches$

$0.020\ inches = \frac{20}{1000}\ inches$

These fractions and decimals are equivalent, so our work is done. Aaron can't believe how small those bits actually are. To think that a drill bit can drill $\frac{15}{1000}$ ths of an inch. Wow!

Changing Fractions to Decimals

Word Problem –Adding to the Survey

Wendy decided to include the students from Christopher's private school in her survey. She explained her plan to Sarah at lunch.

"I figure that since they came too, we should ask them what they thought," she told Sarah.

After lunch, Wendy called over to Christopher's and talked to the class teacher about the survey. He said he would ask his students and get back to Wendy the next day.

So the next morning, Wendy was overjoyed to discover that Christopher's had sent over their survey results. They had had 20 out of 24 students attend the sixth grade social.

$\frac{18}{20}$ said that they had a terrific time.

"This is great," Wendy told Sarah. "Now I just have to figure this out."

She began to work on doing the math to figure out the decimal equivalent for the fraction $\frac{18}{20}$.

Solution

Here is the original problem once again. Reread it and then help Wendy with her equivalent decimal.

Wendy decided to include the students from Christopher's private school in her survey. She explained her plan to Sarah at lunch.

"I figure that since they came too, we should ask them what they thought," she told Sarah.

After lunch, Wendy called over to Christopher's and talked to the class teacher about the survey. He said he would ask his students and get back to Wendy the next day.

So the next morning, Wendy was overjoyed to discover that Christopher's had sent over their survey results. They had had 20 out of 24 students attend the sixth grade social.

$\frac{18}{20}$ said that they had a terrific time.

"This is great," Wendy told Sarah. "Now I just have to figure this out."

She began to work on doing the math to figure out the decimal equivalent for the fraction $\frac{18}{20}$.

To change the fraction to the decimal, we can either divide or use an equivalent fraction. Let's use an equivalent fraction with a denominator of 100.

$$\frac{18}{20} = \frac{x}{100}$$

20 goes into 100 five times.

We can multiply 18×5 **to get the numerator.**

$18 \times 5 = 90$

$$\frac{90}{100} = .90$$

This is our answer. Our work is done.

5.6 Addition and Subtraction of Fractions

This sixth chapter *Addition and Subtraction of Fractions* further develops a student's capability with fractions by introducing methods of addition and subtraction. Also covered are box-and-whisker plots, solving problems using diagrams, and basic problems involving time. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Fraction Estimation
- Adding and Subtracting Fractions with Like Denominators
- Adding and Subtracting Fractions with Different Denominators
- Adding and Subtracting Mixed Numbers
- Subtracting Mixed Numbers by Renaming
- Elapsed Time
- Box-and-Whisker Plots
- Problem-Solving Strategy-Draw a Diagram

Fraction Estimation

Word Problem –The Extra Board

When Travis finished working, he figured out that the extra $10''$ of board would probably work for a go–cart which he was building at home.

"Can I take this piece?" he asked his Uncle.

"Sure."

Travis took the wood home. He has $10''$ of extra board. Travis took out a measuring tape and measured the space in the floor boards of his go–cart. It measured $6\frac{3}{4}''$. Travis took out a paper and a pencil. He wanted to estimate the difference between the board length and the space that it needed to fill.

You can do this by working with fraction estimation. About how much board will Travis have left after he fixes the floor board of the go–cart?

Solution

Here is the original problem once again. Reread it and then underline the important information.

When Travis finished working, he figured out that the extra $10''$ of board would probably work for a go–cart which he was building at home.

"Can I take this piece?" he asked his Uncle.

"Sure."

Travis took the wood home. He has $10''$ of extra board. Travis took out a measuring tape and measured the space in the floor boards of his go–cart. It measured $6\frac{3}{4}''$. Travis took out a paper and a pencil. He wanted to estimate the difference between the board length and the space that it needed to fill.

You can do this by working with fraction estimation. About how much board will Travis have left after he fixes the floor board of the go–cart?

To figure this problem out, Travis will need to use estimation and subtraction. First, let's look at the fractional space of the floor board of the go–cart.

$$6\frac{3}{4}'' \textit{ is the space}$$

Six and three–fourths is close to 7, so it would be best to round up. Remember, we are estimating so the answer does not have to be exact.

Now we can find the difference between 7 and 10. Difference means subtraction, so that is the operation that we will use.

$10 - 7 = 3$

There will be approximately three inches of board left after Travis fixes the floor board.

Adding and Subtracting Fractions with Like Denominators

Word Problem –A Measuring Error

Travis was very proud of his work. His measuring was accurate, and his uncle had gotten a lot done because of him. They were finally on the last bracket.

"Travis, this is great work," his uncle commented.

Travis grinned looking at where he had made the measurement right up to the corner of the wall. In this corner, about a foot away from the bracket, a window was scheduled to go.

"We have to fix the measurement on this bracket. I didn't realize that it would come so close to the corner and the window will be in the way," Uncle Larry said.

"Did I make the mistake?" Travis inquired.

"Oh no, it was my error. This last measurement is at $\frac{6}{8}''$ I think if you take off $\frac{1}{4}$ *or* $\frac{2}{8}''$ it will be perfect. Can you do that?" Uncle Larry asked.

"Of course," Travis said taking the ruler and pencil in hand as Uncle Larry walked off to look at something else.

After he was gone, Travis began thinking. In the last few measurements, he had needed to add the measurements. But this is taking away, so Travis will need to subtract. You can use what you have learned about subtracting fractions with like denominators to help Travis figure out where to put the new mark.

Solution

Here is the original problem once again. Reread it and then solve it for the new measurement.

Travis was very proud of his work. His measuring was accurate, and his uncle had gotten a lot done because of him. They were finally on the last bracket.

"Travis, this is great work," his uncle commented.

Travis grinned looking at where he had made the measurement right up to the corner of the wall. In this corner, about a foot away from the bracket, a window was scheduled to go.

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"Oh no, it was my error. This last measurement is at $\frac{6}{8}''$ I think if you take off $\frac{1}{4}$ *or* $\frac{2}{8}''$ it will be perfect. Can you do that?" Uncle Larry asked.

"Of course," Travis said taking the ruler and pencil in hand as Uncle Larry walked off to look at something else.

After he was gone, Travis began thinking. In the last few measurements, he had needed to add the measurements. But this is taking away, so Travis will need to subtract. You can use what you have learned about subtracting fractions with like denominators to help Travis figure out where to put the new mark.

Travis knows that this is a subtraction problem. He is going to need the original measurement and the measurement his uncle wants him to take away.

The original measurement was placed at $\frac{6}{8}''$.

Travis needs to take off $\frac{2}{8}''$.

That is the subtraction that is needed. Here is how we can set up the problem.

$$\frac{6}{8} - \frac{2}{8}$$

Now because the denominators are the same, we can subtract the numerators.

Our answer is $\frac{4}{8}''$.

If you have learned to simplify, you could also write this as $\frac{1}{2}''$.

Adding and Subtracting Fractions with Different Denominators

Word Problem –The Roof

Before Travis had finished his visit with Uncle Larry, he was given a research job. It seems that the owner of the house wanted to put a slate roof on the house. Uncle Larry wonders how thick it will be if they use slate verses regular shingle.

"I need you to do a little research for me, Travis," Uncle Larry said.

"Alright, I would be happy to," Travis answered.

"Good. I need to know how thick roofing felt is combined with slate shingles so that I can figure out if this makes any sense," Uncle Larry said.

"I am on it," Travis said going directly to the computer.

Travis began his research. He found two different measurements for the thickness of slate tile.

One tile is $\frac{3}{16}''$ thick.

The other tile is $\frac{1}{4}''$ thick.

The roofing felt is $\frac{1}{8}''$ thick.

"I wonder which tile he will want?" Travis thinks to himself.

He decides to do two different calculations, so that each slate tile is represented.

How can Travis do this? The denominators are different on all of these fractions. To figure out two different sums, Travis will need to add fractions with different denominators. This is where you come in. Your task is to help Travis with this work.

Solution

Here is the original problem once again. Reread it and then solve it for TWO different sums. Each sum will represent the two different tiles plus the roofing felt.

Before Travis had finished his visit with Uncle Larry, he was given a research job. It seems that the owner of the house wanted to put a slate roof on the house. Uncle Larry wonders how thick it will be if they use slate verses regular shingle.

"I need you to do a little research for me, Travis," Uncle Larry said.

"Alright, I would be happy to," Travis answered.

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The roofing felt is $\frac{1}{8}''$ thick.

"I wonder which tile he will want?" Travis thinks to himself.

He decides to do two different calculations, so that each slate tile is represented.

How can Travis do this? The denominators are different on all of these fractions. To figure out two different sums, Travis will need to add fractions with different denominators. This is where you come in. Your task is to help Travis with this work.

Travis decides to take the first tile and add it with the roofing felt for the first sum. This will give his uncle one measure of thickness.

$$\frac{3}{16}+\frac{1}{8}$$

Now Travis will need to find the least common denominator and rename fractions. The least common denominator between 16 and 8 is 16. He can rename the second fraction in terms of sixteenths.

$$\frac{1}{8}=\frac{2}{16}$$

Here is the new problem.

$$\frac{3}{16}+\frac{2}{16}=\frac{5}{16}''$$

This is the first measure of thickness.

Now Travis can add the other two measurements. Here is our second problem.

$$\frac{1}{4}+\frac{1}{8}$$

Travis will need the least common denominator for 4 and 8. The least common denominator is 8.

$$\frac{1}{4} = \frac{2}{8}$$

Now he can add the two fractions.

$$\frac{1}{8} + \frac{2}{8} = \frac{3''}{8}$$

This is the second measure of thickness.

Adding and Subtracting Mixed Numbers

Word Problem –From Window to Ceiling

Travis and his uncle completed the measurements from the bottom of the sill of the window to the floor. Travis began to think about the window space to the ceiling. Once they put the window in, how much space would there be from the top of the window to the ceiling. When Travis and his uncle did the measurement from floor to window opening, they forgot about the trim, so the original measurement changed a bit.

The original measurement from floor to top of window was $64\frac{5}{8}''$. This measurement changed to $64\frac{3}{8}''$. to allow for the trim.

The ceiling is $8\frac{1}{2}\ ft$. Travis knows that this must be in inches to work with it.

The ceiling is $96\frac{6}{12}''$ from the floor.

Now Travis can figure out the difference between the top of the window and the ceiling. You can help him do this by subtracting mixed numbers. Because there is a "difference", we know that the needed operation is subtraction.

Solution

Here is the original problem once again. Reread it and the figure out the distance from the top of the window to the ceiling.

Travis and his uncle completed the measurements from the bottom of the sill of the window to the floor. Travis began to think about the window space to the ceiling. Once they put the window in, how much space would there be from the top of the window to the ceiling. When Travis and his uncle did the measurement from floor to window opening, they forgot about the trim, so the original measurement changed a bit.

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The ceiling is $96\frac{6}{12}''$ from the floor.

Now Travis can figure out the difference between the top of the window and the ceiling. You can help him do this by subtracting mixed numbers. Because there is a "difference", we know that the needed operation is subtraction.

First, let's think about what Travis knows. He knows that from the floor to the top of the window measures $64\frac{5}{8}''$.

He knows that the height of the ceiling is $96\frac{1}{2}''$.

He can subtract one from the other to figure out the measurement of the space.

$$96\frac{1}{2} - 64\frac{5}{8}$$

Uh oh, these mixed numbers have different denominators. We will need to rename them to work with them.

We can rename by finding the least common denominator between 2 and 8, which is 8.

$96\frac{1}{2} = 96\frac{4}{8}$

Now we can subtract.

$$96\frac{4}{8} - 64\frac{3}{8} = 32\frac{1}{8}''$$

This is the measurement of the space from the top of the window to the ceiling.

Subtracting Mixed Numbers by Renaming

Word Problem –The Door

Uncle Larry pulled Travis aside and showed him a beautiful wooden door. It had been made for this house and Travis was in awe of the detail and craftmanship of the door.

"Our client paid to have this made," Uncle Larry explained.

"Wow, it is beautiful," Travis said running his fingers over some of the detail work.

"Yes, now we are going to figure out where it will fit. I need the distance from the top of the door to the ceiling. Can you figure that out?" Uncle Larry asked.

"Sure. Can you tell me the measurements?"

"Of course. The ceiling is $8\frac{1}{2}'$ and the door height is $6\frac{8}{12}'$."

Travis took out a piece of paper and a pencil. He began to work through the math.

To figure out this problem, you will need to subtract and rename mixed numbers. Use what you have learned to figure out the solution to this problem.

Solution

Here is the original problem once again. Reread it and then figure out the solution.

Uncle Larry pulled Travis aside and showed him a beautiful wooden door. It had been made for this house and Travis was in awe of the detail and craftmanship of the door.

"Our client paid to have this made," Uncle Larry explained.

"Wow, it is beautiful," Travis said running his fingers over some of the detail work.

"Yes, now we are going to figure out where it will fit. I need the distance from the top of the door to the ceiling. Can you figure that out?" Uncle Larry asked.

"Sure. Can you tell me the measurements?"

"Of course. The ceiling is $8\frac{1}{2}'$ and the door height is $6\frac{8}{12}'$."

Travis took out a piece of paper and a pencil. He began to work through the math.

To figure out this problem, you will need to subtract and rename mixed numbers. Use what you have learned to figure out the solution to this problem.

To figure this out, we can begin by writing a problem.

$$8\frac{1}{2} - 6\frac{8}{12}$$

We can rename the feet in terms of inches so that it will be easier to work with.

$8 = 96''$

$6 = 72''$

Next, we need to rename the fraction part so that the denominators are the same.

$$96\frac{1}{2} = 96\frac{6}{12}$$

Now we can rewrite the subtraction problem.

$$96\frac{6}{12} - 72\frac{8}{12}$$

Uh oh, we can't take 8 away from 6, so we will have to rename. We can borrow one from the 96, make it 95 and add $\frac{12}{12}$ to the fraction.

Here is our new problem.

$$95\frac{18}{12} - 72\frac{8}{12}$$

Now we can subtract to find the difference.

$23\frac{10}{12} = 23\frac{5}{6}''$

This is our answer. The door will be a little less than 2 feet from the ceiling.

Elapsed Time

Word Problem –The Lunch Break

Travis is given a one hour lunch break. He figures that this is enough time to ride to the park, eat lunch with his friends who are hanging out there and get back. His lunch break starts at noon and he leaves right away.

It takes Travis 15 minutes to ride to the park. When he gets there, the fun is underway. His friends are riding skateboards and bikes and Travis sits to eat lunch and gets talking and hanging out too.

He is having such a great time, that instead of 30 minutes, he spends 40 minutes at the park. Realizing he is cutting it close, Travis rides back to the work site. It takes him another 15 minutes to ride back.

"You're late," his uncle says when he arrives.

How late is he? What time did Travis arrive back at the work site? Use what you have learned about elapsed time to figure out the solution to this problem.

Solution

Here is the original problem once again. Reread it and underline all of the important information. Then solve it by answering the two questions at the end of the problem.

Travis is given a one hour lunch break. He figures that this is enough time to ride to the park, eat lunch with his friends who are hanging out there and get back. His lunch break starts at noon and he leaves right away.

It takes Travis 15 minutes to ride to the park. When he gets there, the fun is underway. His friends are riding skateboards and bikes and Travis sits to eat lunch and gets talking and hanging out too.

He is having such a great time, that instead of 30 minutes, he spends 40 minutes at the park. Realizing he is cutting it close, Travis rides back to the work site. It takes him another 15 minutes to ride back.

"You're late," his uncle says when he arrives.

How late is he? What time did Travis arrive back at the work site? Use what you have learned about elapsed time to figure out the solution to this problem.

First, we can figure out the sum of the time that Travis spent.

He spent 15 minutes riding to the park, and 15 minutes riding back for a total of 30 minutes.

Then he spent 40 minutes at the park with his friends.

$30 + 40 = 70$ *minutes*

There are 60 minutes in one hour.

Travis was 10 minutes late.

He arrived back at work at 1:10 pm instead of 1:00 pm.

Box-and-Whisker Plots

Word Problem –Comparing House Sales

After doing the research in the current neighborhood, Uncle Larry and Travis took a drive to a new development about 5 miles from where the new house is being built.

"Let's go see what sales were like in this new development," Uncle Larry said.

When they arrived, they went to the sales office and talked with a manager there. Uncle Larry received a list of homes and the number of days that it took to sell them.

"These sold a lot faster," Uncle Larry told Travis.

Here is the data collected.

20 days

10 days

5 days

3 days

15 days

18 days

14 days

11 days

"Can I take these to draw it out?" Travis asked.

"Sure. I would love to see it drawn out so that we can compare it with the data from our area."

When they arrive back at the work site, Travis began to work on figuring out the data. He decided that a box-and-whisker plot for this data would be the best way to compare and contrast the two sets of information.

Solution

Here is the original problem once again. Reread it and then figure out all of the information so that a box-and-whisker plot can be constructed.

After doing the research in the current neighborhood, Uncle Larry and Travis took a drive to a new development about 5 miles from where the new house is being built.

"Let's go see what sales were like in this new development," Uncle Larry said.

When they arrived, they went to the sales office and talked with a manager there. Uncle Larry received a list of homes and the number of days that it took to sell them.

"These sold a lot faster," Uncle Larry told Travis.

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5 days

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18 days

14 days

11 days

"Can I take these to draw it out?" Travis asked.

"Sure. I would love to see it drawn out so that we can compare it with the data from our area."

When they arrive back at the work site, Travis began to work on figuring out the data. He decided that a box-and-whisker plot for this data would be the best way to compare and contrast the two sets of information.

First, Travis will need the median of the data. To do this, he will need to list all of the data in order from least to greatest.

3, 5, 10, 11, 14, 15, 18, 20

The median is between 11 and 14. To figure this out we need to average the two values.

$11 + 14 = 25 \div 2 = 12.5$

The median is 12.5.

Next, we figure out the lower quartile.

3, 5, 10, 11 are the data values.

We average 5 and 10 to find the median of these values which is 7.5.

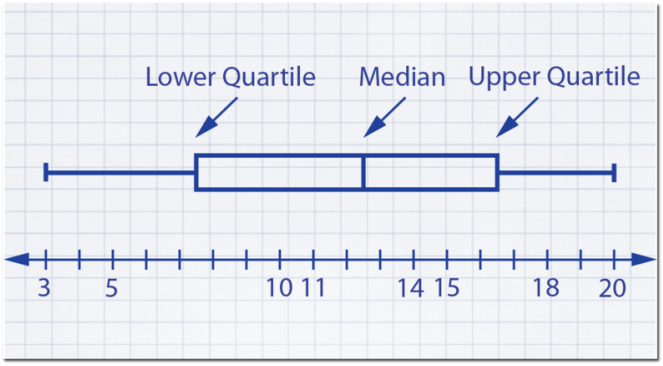
The LQ is 7.5.

The upper quartile involves the values 14, 15, 18 and 20

We average 15 and 18 and get a median value of 16.5.

The UQ is 16.5

Now we can create box-and-whisker plot of the data.



Problem-Solving Strategy-Draw a Diagram

Word Problem –Tiling Trouble

Well, Travis finished the math and thought that he would be finished when lunch arrived. However, he hadn't figured that he would get tired and so he would work slower. Much to his dismay, when lunch arrived at 2 pm, he still wasn't finished.

"Stop and eat, it will help you finish," Uncle Larry said.

Travis stopped and grabbed a few pieces of pizza.

"You've done a great job. Just a bit more to go," Uncle Larry said smiling.

Travis has three–fourths of the floor finished. Remember that it is divided into 12 sections. Based on this, how many sections does he have left to do? You can draw a diagram to help you to figure out the solution to this problem.

Solution

Here is the original problem once again. Reread it and then figure out how many sections are left to finish.

Well, Travis finished the math and thought that he would be finished when lunch arrived. However, he hadn't figured that he would get tired and so he would work slower. Much to his dismay, when lunch arrived at 2 pm, he still wasn't finished.

"Stop and eat, it will help you finish," Uncle Larry said.

Travis stopped and grabbed a few pieces of pizza.

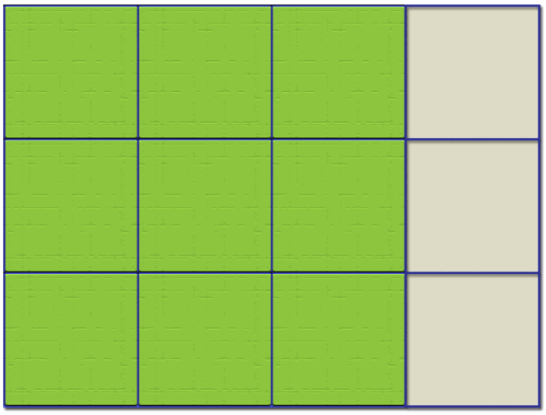
"You've done a great job. Just a bit more to go," Uncle Larry said smiling.

Travis has three–fourths of the floor finished. Remember that it is divided into 12 sections. Based on this, how many sections does he have left to do? You can draw a diagram to help you to figure out the solution to this problem.

First, let's write down what we know. We know that the floor is divided into 12 sections. Three–fourths of the sections are finished. We need to change $\frac{3}{4}$ in terms of 12ths.

$$\frac{3}{4} = \frac{9}{12}$$

Now we can draw a diagram to see if we are correct.



The diagram can be divided into four sections, so we can see that there will be three squares in each of the four sections. We know that three–fourths is done, so we can shade in those boxes.

There are three boxes not filled in. Travis has three sections out of 12 left to finish after lunch.

5.7 Multiplication and Division of Fractions

This seventh chapter *Multiplication and Division of Fractions* covers the methods needed to multiply and divide fractions and mixed numbers. Students are also introduced to using and converting customary units. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Multiplying Fractions and Whole Numbers
- Multiplying Fractions
- Multiplying Mixed Numbers
- Dividing Fractions
- Dividing Mixed Numbers
- Customary Units of Weight and Capacity
- Converting Customary Units
- Problem-Solving Strategy: Choose an Operation

Multiplying Fractions and Whole Numbers

Word Problem –More Rain in the Rainforest

Once Julie and Camilla had figured out the initial rain totals for the rainforest, they began to focus on some other rainforest facts. But then Julie discovered another statistic different from her initial one.

"Hey Camilla," she said calling her friend over.

Camilla came over to where Julie was reading.

"This page says that some places get up to 430 inches of rain per year. That's an average of $\frac{3}{10}''$ per day," Julie stated.

"Wow, that's a lot of rain. How much is that per month?" Camilla asked.

Julie paused and looked at her friend.

"I bet I can multiply for this one too," Julie said.

"Yes, you can. You can think about 30 days in a month," Camilla began.

"Don't tell me, I think I can figure it out," Julie instructed.

Can you figure it out? Use the information in the problem and what you have learned about multiplying fractions and whole numbers to figure out the average rainfall for one month.

Solution

Here is the original problem once again. Reread it and then solve it for the total rainfall for one month.

Once Julie and Camilla had figured out the initial rain totals for the rainforest, they began to focus on some other rainforest facts. But then Julie discovered another statistic different from her initial one.

"Hey Camilla," she said calling her friend over.

Camilla came over to where Julie was reading.

"This page says that some places get up to 430 inches of rain per year. That's an average of $\frac{3}{10}''$ per day," Julie stated.

"Wow, that's a lot of rain. How much is that per month?" Camilla asked. Julie paused and looked at her friend.

"I bet I can multiply for this one too," Julie said.

"Yes, you can. You can think about 30 days in a month," Camilla began.

"Don't tell me, I think I can figure it out," Julie instructed.

Can you figure it out? Use the information in the problem and what you have learned about multiplying fractions and whole numbers to figure out the average rainfall for one month.

Julie knows that the average rain per day in this part of the rainforest is $\frac{3}{10}''$ per day. She also knows that there are 30 days average in a month. She can use that and multiplication to figure out the average rainfall.

$$\frac{3}{10} \times 30$$

To do this, we must write the whole number as a fraction and multiply across.

$$\frac{3}{10} \times \frac{30}{1} = \frac{90}{10}$$

We can simplify this fraction to 9.

The average rainfall given these numbers is 9 inches per month.

Multiplying Fractions

Word Problem –Rainforest Plants

After finishing with the rain, Julie moved on to looking at plants. She found some terrific places on the internet to read about all of the different types of plants and their many uses. She copied a bunch of information and brought it home with her. While her Mom cooked dinner, Julie spread her papers out on the table for closer examination.

"Wow, listen to this Mom," Julie said. Two–thirds of the world's plants are found in the rainforests. Of those two–thirds, one–fourth of the plants we need for medicine are found there."

"That is so interesting," her Mom responded.

Julie couldn't help wondering what part of the two–thirds was one–fourth. She started out trying to divide, but then remembered the rain and multiplication. So instead, she wrote a multiplication problem.

Using what you have learned about multiplication and fractions, help Julie with this solution.

Solution

Here is the original problem once again. Reread it and then find the solution.

After finishing with the rain, Julie moved on to looking at plants. She found some terrific places on the internet to read about all of the different types of plants and their many uses. She copied a bunch of information and brought it home with her. While her Mom cooked dinner, Julie spread her papers out on the table for closer examination.

"Wow, listen to this Mom," Julie said. "Two–thirds of the world's plants are found in the rainforests. Of those two–thirds, one–fourth of the plants we need for medicine are found there."

"That is so interesting," her Mom responded.

Julie couldn't help wondering what part of the two–thirds was the one–fourth. She started out trying to divide, but then remembered the rain and multiplication. So instead, she wrote a multiplication problem.

Using what you have learned about multiplication and fractions, help Julie with this solution.

Julie began by writing a problem using fractions and multiplication.

$$\frac{2}{3} \times \frac{1}{4}$$

She wanted to figure out one–fourth of two–thirds. Now we can multiply across and simplify.

$$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

Our answer is that one–sixth of the plants found are used for medicines.

Multiplying Mixed Numbers

Word Problem –Rainforest Woes

Julie couldn't focus through the rest of her class. After figuring out that the rainforest is losing so many acres every second, the rest of class didn't seem to matter. She sat there figuring out how many acres would be lost in one forty-five minute class period. It was maddening. She tried not to be distracting, but couldn't help jotting down figure after figure as she worked it out.

$1\frac{1}{2}$ *acres per second*.

45 minute class

This is where you come in. To figure out how many acres are lost in one class period, you will need to do some figuring. You will also need to multiply by mixed numbers. Use what you have learned to help Julie with her dilemma.

Solution

Here is the original problem once again. Reread it and then figure out the solution.

Julie couldn't focus through the rest of her class. After figuring out that the rainforest is losing so many acres every second, the rest of class didn't seem to matter. She sat there figuring out how many acres would be lost in one forty-five minute class period. It was maddening. She tried not to be distracting, but couldn't help jotting down figure after figure as she worked it out.

$1\frac{1}{2}$ *acres per second*.

45 minute class

This is where you come in. To figure out how many acres are lost in one class period, you will need to do some figuring. You will also need to multiply by mixed numbers. Use what you have learned to help Julie with her dilemma.

First, we will need to convert minutes to seconds. Why? Because the measure of acreage lost is in seconds, we will need all our units to match.

There are 60 seconds in one minute.

$45 \times 60 = 2700$ *seconds*

Now we can multiply.

$$1\frac{1}{2} \times 2700$$

Next, we convert the mixed number to an improper fraction.

$$1\frac{1}{2} = \frac{3}{2}$$
$$\frac{3}{2} \times \frac{2700}{1} = \frac{8100}{2} = 4{,}050 \text{ } acres$$

According to these numbers, 4,050 acres are lost in one 45 minute class period.

Dividing Fractions

Word Problem –The Rainforest Game

Julie thought she was done with her game and creating her cards, but when she double checked the paper she was using she found that it had writing on the back. Julie was discouraged, but found a sheet of blank poster board in the back of her closet at home.

Julie needs to create 25 cards and they each need to be $\frac{3}{4}''$. The poster board has different dimensions from her original paper. She has discovered that it's width is $22''$ instead of the original $20''$. Given these dimensions, how many cards can Julie create using this poster board?

Solution

Here is the original problem once again. Reread it and then figure out the answer to the question at the end of the problem.

Julie thought she was done with her game and creating her cards, but when she double checked the paper she was using she found that it had writing on the back. Julie was discouraged, but found a sheet of blank poster board in the back of her closet at home.

Julie needs to create 25 cards and they each need to be $\frac{3}{4}''$. The poster board has different dimensions from her original paper. She has discovered that it's width is $22''$ instead of the original $20''$.

Given these dimensions, how many cards can Julie create using this posterboard?

Julie will be using division to figure out the solution to this problem. She needs to divide $22''$ by $\frac{3}{4}''$. Why? She is doing this because each card is $\frac{3}{4}''$ and the width of the poster board that she is working with is $22''$.

$22 \div \frac{3}{4}$

Now we need to change this to a multiplication problem since multiplication is the inverse of division.

$22 \times \frac{4}{3}$

Next we write the whole number as a fraction over 1.

$$\frac{22}{1} \times \frac{3}{4} = \frac{88}{3} = 29\frac{1}{3} \text{ } cards$$

Julie can make 29 cards and have one–third of a card left over.

Dividing Mixed Numbers

Word Problem –Snake Snares

Julie became so interested in the anaconda that she began to research other snakes that could be found in the rainforest. She became enamored with the boa constrictor after discovering that it could stretch itself and eat many things by swallowing them whole.

"The average boa constrictor is between 6 and 8 feet," she told her friend Camilla. "This one is $6\frac{1}{3}\ feet$."

"Wow, how big is the anaconda?" Camilla asked.

"It's average is $13\frac{1}{2}\ feet$."

That got Julie thinking. How many boa constrictors would equal one length of an anaconda? She made a few notes on a piece of paper and began to write a division problem.

"This will be just like the garter snake problem," she said to herself.

Do you remember the garter snake problem from the text? Review it and then use what you learn to figure out a solution to Julie's question in this problem.

Solution

Here is the original problem once again. Reread it and then find a solution.

Julie became so interested in the anaconda that she began to research other snakes that could be found in the rainforest. She became enamored with the boa constrictor after discovering that it could stretch itself and eat many things by swallowing them whole.

"The average boa constrictor is between 6 and 8 feet," she told her friend Camilla. "This one is $6\frac{1}{3}\ feet$."

"Wow, how big is the anaconda?" Camilla asked.

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That got Julie thinking. How many boa constrictors would equal one length of an anaconda? She made a few notes on a piece of paper and began to write a division problem.

"This will be just like the garter snake problem," she said to herself.

Do you remember the garter snake problem from the text? Review it and then use what you learn to figure out a solution to Julie's question in this problem.

Using what Julie did in the problem with the garter snake, you can write a division problem to be solved.

$$13\frac{1}{2} \div 6\frac{1}{3}$$

Next, convert both mixed numbers to improper fractions.

$$\frac{27}{2} \div \frac{19}{3}$$

Now we can change this to a multiplication problem since multiplication is the inverse of division.

$$\frac{27}{2} \times \frac{3}{19}$$

Multiply across and solve.

$$\frac{27}{2} \times \frac{3}{19} = \frac{81}{38} = 2.1$$

Two boa constrictors and a little bit more equals the length of one anaconda.

Customary Units of Weight and Capacity

Word Problem –The Water Bottle

"That is one big water bottle," Julie's brother Chris commented when he came into the kitchen.

Julie was unwrapping a late birthday present from her aunt and uncle. She had unwrapped a water bottle that was the biggest one that she had ever seen. She smiled at Chris.

"I guess they think that I get thirsty," she said laughing.

"How many ounces are in that thing?" Chris asked.

Julie turned the green and blue bottle in her hand. At the bottom of the bottom she saw a little tag.

"It has 64 ounces in it," she said. "I wonder how many cups that is?"

Julie went to the drawer to get a measuring cup.

"Hey goofball, you can figure it out with math and not a measuring cup," Chris said shaking his head and leaving the room.

Julie stopped to think about this. Within a minute she knew that Chris was right, but she wasn't sure how to do the math.

"I know that there are 8 ounces in one cup," she thought to herself.

This is where you can do some math with Julie. How many cups are there in her water bottle? When you figure that out, how many pints are there?

Solution

Here is the original problem once again. Reread it and then underline the important information.

"That is one big water bottle," Julie's brother Chris commented when he came into the kitchen.

Julie was unwrapping a late birthday present from her aunt and uncle. She had unwrapped a water bottle that was the biggest one that she had ever seen. She smiled at Chris.

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"I know that there are 8 ounces in one cup," she thought to herself.

This is where you can do some math with Julie. How many cups are there in her water bottle? When you figure that out, how many pints are there?

To solve this problem, we need to help Julie to figure out how many cups there are in her 64 ounce water bottle. Then we will need to figure out the number of pints in it too.

First, here are some equivalent measures.

8 ounces = 1 cup

2 cups = 1 pint

This is really all that we need to help us figure out the problem. When going from small units to larger units, we divide. Ounces are smaller than cups, so we can divide.

$64 \div 8 = 8$

There are 8 cups in the water bottle.

Now we can do the same thing with pints. Two cups equals one pint.

$8\ cups \div 2 = 4$

There are 4 pints in the water bottle.

Converting Customary Units

Word Problem –More Amazon Jokes

After Jacob had been so tricky with Julie, she decided to try to stump him with some Amazon facts. Julie spent all afternoon trying to figure out what she would say to Jacob to really trick him. In the end, she went to the computer and did some research on the internet. When finished, she was ready.

The next day Julie ran into Jacob outside class.

"I have one for you," she said. "The Amazon is 3600 inches deep."

"I know," Jacob grinned. "It is 300 feet deep."

Julie grinned back.

Are these numbers correct? To figure it out, you will need to convert customary units of measure to figure out whether or not 3600 inches is the same as 300 feet. Be prepared to prove your answer.

Solution

Here is the original problem once again. Reread it and then prove your answer.

After Jacob had been so tricky with Julie, she decided to try to stump him with some Amazon facts. Julie spent all afternoon trying to figure out what she would say to Jacob to really trick him. In the end, she went to the computer and did some research on the internet. When finished, she was ready.

The next day Julie ran into Jacob outside class.

"I have one for you," she said. "The Amazon is 3600 inches deep."

"I know," Jacob grinned. "It is 300 feet deep."

Julie grinned back.

Are these numbers correct? To figure it out, you will need to convert customary units of measure to figure out whether or not 3600 inches is the same as 300 feet. Be prepared to prove your answer.

These two values are equivalent.

3600 inches = 300 feet

Now to prove it, we can show how we can convert customary units of measurement. In this case, we are converting units that we use to measure length.

First, Julie gave Jacob the following measurement.

The Amazon is 3600 inches deep.

We can convert that to feet to see if she is correct. There are 12 inches in one foot, so we divide by 12.

$3600 \div 12 = 300$

Jacob said the Amazon is 300 feet deep.

These answer are equivalent, and we have proven it with mathematics.

Problem-Solving Strategy: Choose an Operation

Word Problem –Comparing More Beetles

After comparing the Goliath beetle with the dung beetle, Julie wondered how a Goliath beetle compared with a regular Japanese beetle like the ones that she found in her garden last summer. To figure this out, she went to the library and took out a book on beetles. In her research, she found out the following information about Japanese beetles.

The Japanese beetle is less than $\frac{1}{2}$ *inch* long. In fact, it is about dung beetle, Julie wondered how a Goliath beetle compared with a regular Japanese beetle like the ones that she found in her garden last summer. To figure this out, she went to the library and took out a book on beetles. In her research, she found out the following information about Japanese beetles.

The Japanese beetle is less than $\frac{1}{2}$ *inch* long. In fact, it is about $\frac{6}{10}''$ long and $\frac{4}{10}''$ wide.

A Goliath beetle is $4\frac{1}{2}''$ long.

How many Japanese beetles would equal the length of one Goliath beetle?

Solution

Here is the original problem once again. Reread it and then choose an operation to solve it.

After comparing the Goliath beetle with the dung beetle, Julie wondered how a Goliath beetle compared with a regular Japanese beetle like the ones that she found in her garden last summer. To figure this out, she went to the library and took out a book on beetles. In her research, she found out the following information about Japanese beetles.

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The Japanese beetle is less than $\frac{1}{2}$ *inch* long. In fact, it is about $\frac{6}{10}''$ long and $\frac{4}{10}''$ wide.

A Goliath beetle is $4\frac{1}{2}''$ long.

How many Japanese beetles would equal the length of one Goliath beetle?

The key to solving this problem is in choosing the correct operation to solve the problem. We know that we need to figure out how many Japanese beetles are equal to one Goliath beetle. We could do this by continuing to add the Japanese beetles lengths until it equals $4\frac{1}{2}''$ long.

$\frac{6}{10} + \frac{6}{10} + \frac{6}{10}$ **etc.**

This is a long way to work. Instead, we can divide.

Here is what the problem and solution would look like.

$$4\frac{1}{2} \div \frac{6}{10} = \frac{9}{2} \div \frac{6}{10} = \frac{9}{2} \times \frac{10}{6} = \frac{3}{1} \times \frac{5}{2} = \frac{15}{2} = 7\frac{1}{2}$$

It would take $7\frac{1}{2}$ Japanese beetles to equal one Goliath beetle.

5.8 Ratios, Proportions and Percents

This eighth chapter *Ratios, Proportions and Percents* introduces students to ratios and rates, basic uses of proportions including understanding scale drawings, and percents. Also explored is the relationship among percents, decimals, and fractions. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

Ratios

Word Problem –The Egg Dilemma

When Casey gets home with the milk, she sees that her Mom is making breakfast for dinner and is in the middle of making the scrambled eggs.

“I love breakfast for dinner,” she says smiling. “Can I help?”

“Sure, why don’t you take over on eggs,” her Mom says.

Casey opens up the egg carton and sees that there are 8 eggs left in the carton.

“Eight eggs for four people,” Casey thinks. “Should I use them all?”

She knows that the people in her family love eggs. Casey is sure that she can write this comparison between eggs and people as a ratio. She also knows that it will help her to figure out how many eggs each person will receive.

You can do this too. Use what you have learned to write a ratio that compares eggs to people. Then use this ratio to figure out how many eggs each person will receive.

Solution

Here is the original problem once again. Reread it and then work on the solution.

When Casey gets home with the milk, she sees that her Mom is making breakfast for dinner and is in the middle of making the scrambled eggs.

“I love breakfast for dinner,” she says smiling. “Can I help?”

“Sure, why don’t you take over on eggs,” her Mom says.

Casey opens up the egg carton and sees that there are 8 eggs left in the carton.

"Eight eggs for four people," Casey thinks. "Should I use them all?"

She knows that the people in her family love eggs. Casey is sure that she can write this comparison between eggs and people as a ratio. She also knows that it will help her to figure out how many eggs each person will receive.

You can do this too. Use what you have learned to write a ratio that compares eggs to people. Then use this ratio to figure out how many eggs each person will receive.

First, we will need to write a ratio that compares eggs to people.

There are 8 eggs left in the carton, and there are four people in Casey's family.

8 : 4 or 8 to 4 or $\frac{8}{4}$

Next, we can use the fraction form of the ratio to figure out how many eggs per person. This is a bit of a preview for the next lesson on rates. To figure this out, let's simplify the ratio.

$$\frac{8}{4} = \frac{2}{1}$$

Do you see that the two lines up with the 8, this is the number of eggs per person. The denominator was our people, so this is one. Therefore, each person in Casey's family will receive two eggs.

Rates

Word Problem –More Nuts

At the end of her day at the supermarket, Kiley decided to go and purchase some cashews for herself. The ones that the customer had ordered earlier in the day continued to be on her mind for the rest of her shift. So before going home, she went to get some cashews for herself.

Kiley scooped the cashews into a bag. Then she reached into her pocket and took out a ten dollar bill.

"Only ten dollars," Kiley said to herself.

She remembered that cashews are $3.29 per pound. How many pounds can Kiley buy with her ten dollars?

Solution

Here is the original problem once again. Reread it and then solve the problem.

At the end of her day at the supermarket, Kiley decided to go and purchase some cashews for herself. The ones that the customer had ordered earlier in the day continued to be on her mind for the rest of her shift. So before going home, she went to get some cashews for herself.

Kiley scooped the cashews into a bag. Then she reached into her pocket and took out a ten dollar bill.

"Only ten dollars," Kiley said to herself.

She remembered that cashews are $3.29 per pound. How many pounds can Kiley buy with her ten dollars?

You will need to use what you have learned about rates and ratios to solve this problem. As with any word problem, begin with what you know.

We know that cashews are $3.29 per pound. That means that $3.29 is being compared to 1.

$$\frac{3.29}{1}$$

Kiley had $10.00 to spend. We are trying to figure out how many pounds she can buy with that amount of money. Let's set up the other ratio.

$$\frac{3.29}{1} = \frac{10.00}{x}$$

Now we will need to figure out how many times 3.29 goes into 10. We can divide or use guess and check to figure this out.

Let's try guess and check. We can try multiplying 3.29 by 3.

$3.29 \times 3 = 9.87$

If Kiley buys 3 pounds of cashews, it will cost her $9.87. This is our solution.

Solving Proportions

Word Problem –Nuts per Pound Another Way

In the last lesson, Kiley figured out that she could buy 3 pounds of cashews with her ten dollars. The total cost would be $9.87. We used guess and check to solve this problem. Now that you have learned how to solve proportions, you can use what you have learned to solve this problem another way. Look at the proportion from the last lesson.

$$\frac{3.29}{1} = \frac{10.00}{x}$$

Now use what you have learned about solving proportions to figure out this solution.

Solution

Here is the original problem once again. Reread it and then solve the proportion.

In the last lesson, Kiley figured out that she could buy 3 pounds of cashews with her ten dollars. The total cost would be $9.87. We used guess and check to solve this problem. Now that you have learned how to solve proportions, you can use what you have learned to solve this problem another way. Look at the proportion from the last lesson.

$$\frac{3.29}{1} = \frac{10.00}{x}$$

Now use what you have learned about solving proportions to figure out this solution.

Kiley's cashews form a proportion because we know that $3.29 is the cost for one pound of cashews. We also know that Kiley had $10.00 to spend on cashews. Notice that the proportion has units that match.

$$\frac{dollars}{pounds} = \frac{dollars}{pounds}$$

When you write a proportion, the units must always match.

Now we know some of the values in this proportion.

$$\frac{3.29}{1} = \frac{10.00}{x}$$

The x represents that unknown number of pounds Kiley can purchase.

Next, we cross multiply and solve.

$3.29x = 10$

We can divide 10 by 3.29.

$10 \div 3.29 = 3.03$ *pounds*

Kiley can buy three pounds and a little more for her money.

Proportions and Scale Drawings

Word Problem –The Outdoor Display

Once Jessica's sign was finished, she received many, many compliments. Then one day she was called into a manager's meeting. Jessica was a little nervous. She wondered what all of the managers could possibly want with her.

"Jessica, we love your display. In fact we would love you to double the size of it and make one for in front of the store. You draw up the design and we will send it out to be created," the store manager Holly said.

"I would love to," Jessica said smiling.

Jessica left that meeting feeling terrific. It was a huge honor to have been asked to do this for the outside of the store. All of the people driving by would see it too!

Jessica decided to work on the new design right away. She took out her original sketch.

The original sketch is $8'' \times 5''$ with a scale of $1'' : 6''$.

Jessica isn't sure, but she thinks that she can use the same drawing. She needs to double it to be sure that the sign designers make the new sign large enough.

How can she do it? Is she correct? What does she need to change on her original drawing?

Solution

Here is the original problem once again. Reread it and then answer the questions at the end.

Once Jessica's sign was finished, she received many, many compliments. Then one day she was called into a manager's meeting. Jessica was a little nervous. She wondered what all of the managers could possibly want with her.

"Jessica, we love your display. In fact we would love you to double the size of it and make one for in front of the store. You draw up the design and we will send it out to be created," the store manager Holly said.

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Jessica isn't sure, but she thinks that she can use the same drawing. She needs to double it to be sure that the sign designers make the new sign large enough.

How can she do it? Is she correct? What does she need to change on her original drawing?

Jessica thinks that she can use her original drawing, and she is correct. She doesn't need to change the original

drawing, but she will need to change the scale.

With the old scale, the poster created was $4\ feet \times 2.5\ feet$. **The managers asked to have the sign doubled.**

If Julie were making the sign herself, she could do this by multiplying all of her final dimensions by 2. However, Julie isn't going to make the sign. The store managers are going to send it out to a designer. Therefore, Julie will need to change the scale of the drawing.

If it is going to be double the original, then the scale will be changed to double as well.

$1'' : 12''$ **or** $1'' : 1\ foot$

Given this scale, the new poster will be $8' \times 5'$.

Understanding Percent

Word Problem –Cereal Statistics

After completing the reorder, Kevin was asked to conduct a survey of customers who shopped at the supermarket. The management team wanted to figure out how many people bought cereal when they came to the grocery store. Kevin was assigned the task of handing out comment cards so that people could check off a box if they had bought cereal or not. There was also a place for them to write in any specific brands if they chose to.

At the end of the day, Kevin collected 500 comment cards. He sat at a desk to figure out the statistics. Out of the 500 cards, 335 people had purchased cereal.

Kevin wrote the following fraction on his paper.

$$\frac{335}{500}$$

Now he needs to change this fraction to a percent so that he can give the statistic in a fraction and in a percent.

Kevin is confused on how to do it. He knows that he can simplify the fraction since both the numerator and the denominator can be divided by 5, but that is as far as he has gotten.

This is where you come in. Use what you have learned about percents to help Kevin rewrite his statistic as a percent.

Solution

Here is the original problem once again. Reread it and then convert the fraction to a percent.

After completing the reorder, Kevin was asked to conduct a survey of customers who shopped at the supermarket. The management team wanted to figure out how many people bought cereal when they came to the grocery store. Kevin was assigned the task of handing out comment cards so that people could check off a box if they had bought cereal or not. There was also a place for them to write in any specific brands if they chose to.

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This is where you come in. Use what you have learned about percents to help Kevin rewrite his statistic as a percent.

Kevin knows that both the numerator and denominator can be divided by 5. We can start by simplifying the fraction.

$$\frac{335}{500} \div \frac{5}{5} = \frac{67}{100}$$

Now that the fraction is written in terms of hundredths, it is easy to convert it to a percent. Percent means "out of 100".

$$\frac{67}{100} = 67\%$$

67% of the people surveyed purchased cereal.

Percents, Decimals and Fractions

Word Problem –Cleaning Up

When Sam finished cleaning the floor, she went to put the jug of cleaner back on the shelf and realized that she had used quite a lot of it washing up.

Sam looked at the jug and estimated that about one–third of the cleaner was left.

"I am going to have to tell Holly, the manager to order more," Sam said.

She looked at the jug again and noticed that she mixed the cleaning solution incorrectly. That is why she had used so much of it. She should have used one–fourth solution to three–fourths water. Instead, she had mixed it the other way around.

"Well, I won't do that again," Sam said.

If the jug Sam used has one–third left, what fraction of the cleaner did she use?

What is this value as a percent?

What percent of the cleaner is left?

To answer these questions, you will need to use what you have learned about converting fractions, decimals and percents.

Solution

Here is the original problem once again. Reread it and then answer the questions at the end of the problem. Be sure to use only the fractions that you need.

When Sam finished cleaning the floor, she went to put the jug of cleaner back on the shelf and realized that she had used quite a lot of it washing up.

Sam looked at the jug and estimated that about one–third of the cleaner was left.

"I am going to have to tell Holly, the manager to order more," Sam said.

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"Well, I won't do that again," Sam said.

If the jug Sam used has one–third left, what fraction of the cleaner did she use?

What is this value as a percent?

What percent of the cleaner is left?

To answer these questions, you will need to use what you have learned about converting fractions, decimals and percents.

First, we know that one–third of the cleaner is left in the jug.

$$\frac{1}{3}\ left$$

If we subtract one–third from one whole, then we are left with two-thirds left.

Sam used $\frac{2}{3}$ of the cleaner.

As a percent, we will need to figure this out by dividing 2 by 3.

$2 \div 3 = .666$

We can say that 66.6 % of the cleaner was used.

Now let's write a percent for the amount of cleaner that is left.

$\frac{1}{3} = 1 \div 3 = .333 = 33.3\%$ **of the cleaner is left in the jug.**

Finding a Percent of a Number

Word Problem –A Broken Register

Discount day is a huge success, except for Cameron. Just as he has a huge order to process, his register breaks down. It gives Cameron the total before going on the fritz, but that doesn't help him.

The customer has purchased $568.00 worth of groceries before the discount.

Cameron turns on his register light to let Mr. Kemp know that he needs help. Mr. Kemp looks over and signals to Cameron that it is going to be a few minutes. Cameron looks at the customer, he is getting impatient.

Cameron thinks he can figure out the new total without the register. He knows the total and that there is a 15% discount on the sum of all purchases.

Cameron needs to figure out 15% of $568.00. This will give him the discount.

You can use what you have learned to help Cameron. First, you will need to find the amount of the discount, and then you will need to figure out how much the customer will need to pay.

Solution

Here is the original problem once again. Reread the problem and then find the solution. Remember, there are two parts to the answer.

Discount day is a huge success, except for Cameron. Just as he has a huge order to process, his register breaks down. It gives Cameron the total before going on the fritz, but that doesn't help him.

The customer has purchased $568.00 worth of groceries before the discount.

Cameron turns on his register light to let Mr. Kemp know that he needs help. Mr. Kemp looks over and signals to Cameron that it is going to be a few minutes. Cameron looks at the customer, he is getting impatient.

Cameron thinks he can figure out the new total without the register. He knows the total and that there is a 15% discount on the sum of all purchases.

Cameron needs to figure out 15% of $568.00. This will give him the discount.

You can use what you have learned to help Cameron. First, you will need to find the amount of the discount, and then you will need to figure out how much the customer will need to pay.

First, Cameron needs to figure out the amount of the discount.

15% of 568

To do this, he will first need to change 15% to a decimal.

15% becomes .15

Then he will need to multiply this amount by 568. Remember that the word "of" is a key word which means multiplication.

$568 \times .15 = \$85.20$

This is the amount of the discount.

To figure out how much the customer needs to pay, Cameron will need to subtract the discount from the total.

$568.00 - 85.20 = 482.80$

The customer will need to pay $482.80.

Problem-Solving Strategy: Use a Proportion

Word Problem –Eating like a Polar Bear

Tim is very excited to share with his sister Karen what he has discovered about hopping like a frog. When he comes in the house, he runs over to tell her. Karen is immersed in a book herself. She is reading a book on polar bears. Tim shares about his frog and Karen smiles.

"That's cool. But listen to this about polar bears," she says. "A polar bear can hold 20% of its body weight in its stomach. That means that a 1100 pound polar bear can hold 220 pounds of food in its stomach."

"Wow, that is a lot of food," Tim comments looking at the book with her.

"Yup. Imagine if you were a polar bear," she states.

Tim starts to think about this. If he was a polar bear, how much food could he fit in his stomach? He weighs 100 pounds.

As Tim thinks about this, you think about it too. You can use a proportion to figure it out. Think about what you have learned in this lesson, then use what you have learned to figure out how many pounds of food would fit in Tim's stomach if he were a polar bear.

Solution

Here is the original problem once again. Reread it and then find the solution.

Tim is very excited to share with his sister Karen what he has discovered about hopping like a frog. When he comes in the house, he runs over to tell her. Karen is immersed in a book herself. She is reading a book on polar bears. Tim shares about his frog and Karen smiles.

"That's cool. But listen to this about polar bears," she says. "A polar bear can hold 20% of its body weight in its stomach. That means that a 1100 pound polar bear can hold 220 pounds of food in its stomach."

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As Tim thinks about this, you think about it too. You can use a proportion to figure it out. Think about what you have learned in this lesson, then use what you have learned to figure out how many pounds of food would fit in Tim's stomach if he were a polar bear.

First, we need to write a proportion. Think about what we are comparing. We are comparing weight to pounds of food.

$$\frac{weight}{pounds} = \frac{weight}{pounds}$$

Now let's fill in what we know.

$\frac{1100}{220}$ **The polar bear weighs 1100 pounds and can hold 220 pounds of food in his stomach. This is our first ratio.**

$$\frac{1100}{220} = \frac{100}{x}$$

We know Tim's weight, but we need to figure out the number of pounds of food he can hold. This is our unknown.

Next, we cross multiply and solve.

$22000 = 1100x$

$x = 20\ pounds\ of\ food$

If Tim were a polar bear, he could hold 20 pounds of food in his stomach.

5.9 Geometric Figures

This ninth chapter *Geometric Figures* covers basic principles of geometry. Students are introduced to angles, triangles, quadrilaterals, and polygons. Also explored are congruent and similar figures, line symmetry, and strategies for using Venn Diagrams to solve problems. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Introduction to Geometry
- Classifying Angles
- Classifying Triangles
- Classifying Quadrilaterals
- Classifying Polygons
- Congruent and Similar Figures
- Line Symmetry
- Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Introduction to Geometry

Word Problem –Skateboard Ramps

Marc and Isaac finished their design for the skateboard park and sat back to review it. Once they had the initial ground plan done, they began tried to figure out where each features of the skateboard park would be placed. This was fascinating to them.

“We are going to need a bunch of ramps. I want there to be so many cool ramps to ride,” Isaac said.

As Isaac and Marc think about ramps, it is time for you to think about a little geometry. Go back and review each of the terms in the lesson. Then make a list of the different geometric terms that would be necessary when thinking about building ramps.

Solution

Here is the original problem once again. Use the vocabulary from the lesson to write about ramps.

Marc and Isaac finished their design for the skateboard park and sat back to review it. Once they had the initial ground plan done, they began tried to figure out where each features of the skateboard park would be placed. This was fascinating to them.

“We are going to need a bunch of ramps. I want there to be so many cool ramps to ride,” Isaac said.

As Isaac and Marc think about ramps, it is time for you to think about a little geometry. Go back and review each of the terms in the lesson. Then make a list of the different geometric terms that would be necessary when thinking about building ramps.

First, let’s make a list of the geometric terms from the lesson.

Point

Ray

Line

Line Segment

Point of Intersection

Intersecting Lines

Parallel Lines

Angle

Vertex

There are many of these words which would apply to building a ramp. First, each side or edge of a ramp can be considered a line segment. The line segments of a ramp will have end points and be straight. The sides of the ramp will be parallel and some of the corners will intersect or meet at a point.

A key to building a ramp is the angle of the ramp. This is what will give the angle its slant. The riding surface of the ramp, depending on how steep it is, will be given by the angle created by its supports.

Classifying Angles

Word Problem –More Angle Connections

Once Isaac and Marc had designed their park, they began thinking about all of the angles that they would need to work with. Marc took a book out of the library on building ramps and the two spent Saturday pouring over the instructions.

"It says here that all of the ramp angles are acute," Marc told Isaac.

"Yes, and that they increase in steepness in 10° increments so that the ramp doesn't get too steep too quickly," Isaac added.

The word "acute" is new to Marc and Isaac, but they know that it has to do with steepness and with the angle that is part of the ramp. To understand this word, you will need to learn about different angles. These angles are all described in the lesson. When finished, you will be able to define the three different types of angles, including acute angles. Use what you have learned to do this.

Solution

Here is the original problem once again. Reread it and then define the three types of angles described in the lesson.

Once Isaac and Marc had designed their park, they began thinking about all of the angles that they would need to work with. Marc took a book out of the library on building ramps and the two spent Saturday pouring over the instructions.

"It says here that all of the ramp angles are acute," Marc told Isaac.

"Yes, and that they increase in steepness in 10° increments so that the ramp doesn't get too steep too quickly," Isaac added.

The word "acute" is new to Marc and Isaac, but they know that it has to do with steepness and with the angle that is part of the ramp. To understand this word, you will need to learn about different angles. These angles are all described in the lesson. When finished, you will be able to define the three different types of angles, including acute angles. Use what you have learned to do this.

Angles are defined by the number of degrees that they measure. The three different types of angles are acute,

obtuse and right angles.

An acute angle is one that measures less than $90°$**. The ramps that Isaac and Marc want to build have acute angles in them.**

An obtuse angle is an angle that measures more than $90°$**.**

A right angle measures exactly $90°$**.**

Classifying Triangles

Word Problem –The Dragon's Triangle

"Look at this!" exclaimed Marc. "There's another one besides Bermuda."

He shoved a book in front of Isaac which was turned to a page on The Dragon's Triangle. The Dragon's Triangle is found in the Phillipine Sea off the eastern coast of China. It has as much mystery around it as the Bermuda triangle. People in the area believe that the area has dragons underneath the sea that come to the surface and capture anything in its path.

"This is awesome!" Isaac said.

"I think we have to put one of these in the skatepark too," Marc said. "It can be even steeper than the Bermuda triangle, but we can use similar angles."

In a flash, Marc began to draw the triangle. He drew the first angle at $55°$ and the second angle at $70°$. Then he stopped before drawing the third angle. He took out his protractor.

What will be the angle measure of this third angle in the triangle?

What type of triangle will it be?

Solution

Here is the original problem once again. Reread it and then answer the two question at the end.

"Look at this!" exclaimed Marc. "There's another one besides Bermuda."

He shoved a book in front of Isaac which was turned to a page on The Dragon's Triangle. The Dragon's Triangle is found in the Phillipine Sea off the eastern coast of China. It has as much mystery around it as the Bermuda triangle. People in the area believe that the area has dragons underneath the sea that come to the surface and capture anything in its path.

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What will be the angle measure of this third angle in the triangle?

What type of triangle will it be?

To figure this out, we have to rely on information learned in the lesson. We know that the sum of the interior angles of a triangle is $180°$**. We have been given the first two angle measures.**

55 and 70

If we add these together, then we will be able to figure out the third angle. Let's write an equation to solve this.

$55 + 70 + x = 180$

$125 + x = 180$

$180 - 125 = x$

55°

The final angle of the triangle will be 55°.

Because all of the angles of this triangle are less than 90°, it is an acute triangle.

Classifying Quadrilaterals

Word Problem –Bermuda Trapezoid?

"Hey I think we made a mistake," Marc said to Isaac one afternoon.

The two boys were deep in the design phase of their skatepark and were getting together to put some new components on the paper.

"What do you mean?" Isaac asked.

"Well, I was online and saw a think on wikipedia that said that the Bermuda Triangle actually has four sides not three. It stretches from the Straits of Florida to the Bahamas to the Caribbean Islands to the Atlantic eastern coast. Some say it goes all the way to the Azores. It can vary according to the author," Marc explained.

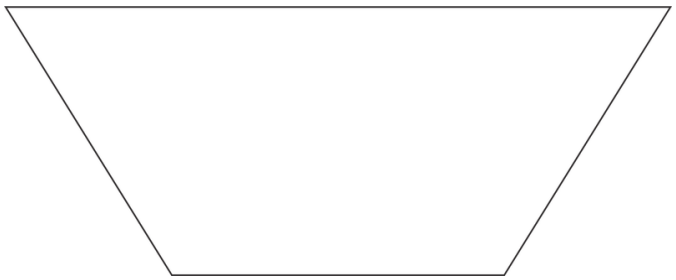
"So?"

"So, we want to be accurate, right? Maybe we should change our design to show this," Marc said.

"I don't know. We are almost finished. What does it look like on the map?" Isaac asked.

"I brought a sketch I made with me."

Marc took out a piece of paper and drew the following figure.



"Okay, we could include this figure," Isaac said smiling. "It might make the area even more interesting."

Marc has drawn a four sided quadrilateral. This quadrilateral has a more specific name and can be described because of its properties. Use what you have learned in this lesson to define the figure and to name the properties connected to it.

Solution

Here is the original problem once again. Reread it and then answer the questions at the end of it.

"Hey I think we made a mistake," Marc said to Isaac one afternoon.

The two boys were deep in the design phase of their skatepark and were getting together to put some new components on the paper.

"What do you mean?" Isaac asked.

"Well, I was online and saw a think on wikipedia that said that the Bermuda Triangle actually has four sides not three. It stretches from the Straits of Florida to the Bahamas to the Caribbean Islands to the Atlantic eastern coast. Some say it goes all the way to the Azores. It can vary according to the author," Marc explained.

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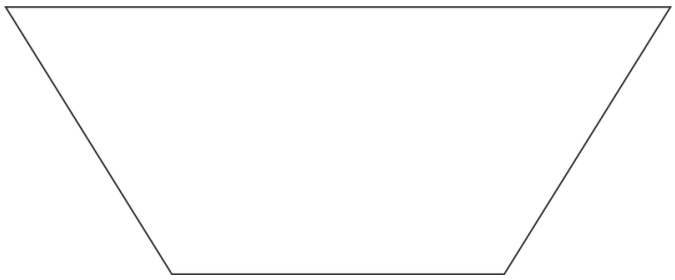


"Okay, we could include this figure," Isaac said smiling. "It might make the area even more interesting."

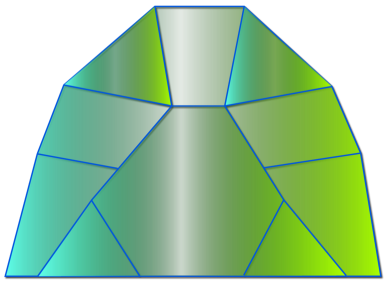
Marc has drawn a four sided quadrilateral. This quadrilateral has a more specific name and can be described because of its properties. Use what you have learned in this lesson to define the figure and to name the properties connected to it.

This four sided quadrilateral is called a trapezoid. It has one pair of parallel sides and four acute angles. We can see in the drawing that the top and bottom sides are parallel, but not congruent. This means that they are not the same length. The opposite two sides are congruent, the same length, but not parallel.

Classifying Polygons

Word Problem –The Sculpture Sign

After Marc, Isaac and Isabelle decided on the structure for their sculpture, they figured out that they could put the sign for the skatepark in the center of the hexagon on the front. Here is their design once again.



"I wish that hexagon was a regular hexagon," Marc said.

"I know, it seems off every time I see it," Isabelle agreed.

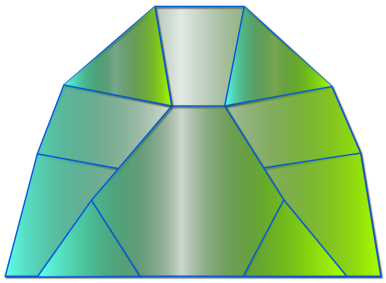
"What do you mean "off"?" Isaac asked.

Marc and Isabelle are referring to something specific when they say "regular hexagon". What do they mean? Use what you have learned from the lesson to help them to explain this figure to Isaac.

Solution

Here is the original problem once again. Use what you have learned to explain a regular hexagon.

After Marc, Isaac and Isabelle decided on the structure for their sculpture, they figured out that they could put the sign for the skatepark in the center of the hexagon on the front. Here is their design once again.



"I wish that hexagon was a regular hexagon," Marc said.

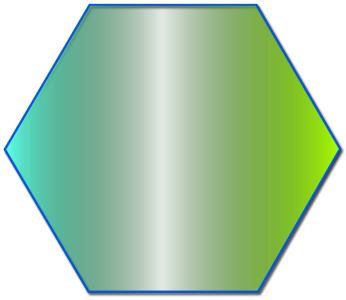
"I know, it seems off every time I see it," Isabelle agreed.

"What do you mean "off"?" Isaac asked.

Marc and Isabelle are referring to something specific when they say "regular hexagon". What do they mean? Use what you have learned from the lesson to help them to explain this figure to Isaac.

The hexagon in the sculpture is not a "regular" hexagon because the side lengths are not all the same. The hexagon in the sculpture is a hexagon because it has six sides to it. For it to be a regular hexagon, all of the side lengths would have to be the same length. This would mean that the sculpture would be redesigned to include this feature.

Here is a picture of a regular hexagon.



Congruent and Similar Figures

Word Problem –The Flagpole

Isaac is five feet tall. As the skatepark was being constructed, he walked outside to check on the progress. Everything seemed to be going along perfectly. It was a beautiful sunny day and he loved looking across at the construction knowing that he had a part in what was happening.

He looked down at the ground and noticed that his shadow stretched out in front of him. Next to his own shadow, the shadow of the flagpole stretched out too. It stretched much farther than Isaac's did.

When Marc came out to join him, Isaac asked Marc to measure his shadow and that of the flagpole.

"Why do you want to do that?" Marc asked stretching out the tape.

"Because I want to know how tall the flagpole is," Isaac explained.

Marc did as he was asked and wrote the following figures on a piece of paper.

Isaac's shadow 8 feet.

Flagpole's shadow 32 feet.

Using what you have learned about indirect measurement, figure out the height of the flagpole.

Solution

Here is the original problem once again. Reread it and underline all of the important information.

Isaac is five feet tall. As the skatepark was being constructed, he walked outside to check on the progress. Everything seemed to be going along perfectly. It was a beautiful sunny day and he loved looking across at the construction knowing that he had a part in what was happening.

He looked down at the ground and noticed that his shadow stretched out in front of him. Next to his own shadow, the shadow of the flagpole stretched out too. It stretched much farther than Isaac's did.

When Marc came out to join him, Isaac asked Marc to measure his shadow and that of the flagpole.

"Why do you want to do that?" Marc asked stretching out the tape.

"Because I want to know how tall the flagpole is," Isaac explained.

Marc did as he was asked and wrote the following figures on a piece of paper.

Isaac's shadow 8 feet.

Flagpole's shadow 32 feet.

Using what you have learned about indirect measurement, figure out the height of the flagpole.

Now we can write a pair of ratios to show that we are comparing Isaac's height with the height of the flagpole and the shadow length of Isaac with the shadow length of the flagpole.

$$\frac{Isaac's\ Height}{Flagpole's\ Height} = \frac{Isaac's\ Shadow}{Flagpole's\ Shadow}$$

Next we can fill in the information that we know.

$$\frac{5\ feet}{x} = \frac{8\ feet}{32\ feet}$$

Indirect measurements form a proportion, so we can cross multiply and solve this proportion.

$$8x = 160$$
$$x = 20$$

The flagpole's height is 20 feet.

Line Symmetry

Word Problem –The Sculpture

“I want to paint each area of the hexagon a different color,” Isabelle told Marc and Isaac.

“You mean on the sign?” Isaac asked.

“Yes, if we divide the sign along its lines of symmetry, it will be perfectly divided. Then I can paint each section a different color. We can put the writing on the sign,” Isabelle explained.

“How do you know it will be even,” Marc asked.

“Because of the lines of symmetry,” Isabelle said smiling.

“Well, I can divide it into many more parts than that,” Isaac said. “There are many lines of symmetry in a regular hexagon.”

Is Isaac correct? Use what you have learned to show all of the lines of symmetry in a regular hexagon. Be sure to draw them in to demonstrate them.

Solution

Here is the original problem once again. Reread it and then demonstrate a solution.

“I want to paint each area of the hexagon a different color,” Isabelle told Marc and Isaac.

“You mean on the sign?” Isaac asked.

“Yes, if we divide the sign along its lines of symmetry, it will be perfectly divided. Then I can paint each section a different color. We can put the writing on the sign,” Isabelle explained.

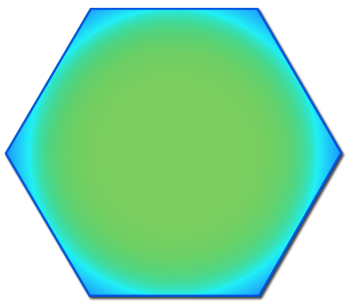
“How do you know it will be even,” Marc asked.

“Because of the lines of symmetry,” Isabelle said smiling.

“Well, I can divide it into many more parts than that,” Isaac said. “There are many lines of symmetry in a regular hexagon.”

Is Isaac correct? Use what you have learned to show all of the lines of symmetry in a regular hexagon. Be sure to draw them in to demonstrate them.

To demonstrate a solution, we will first need a regular hexagon.

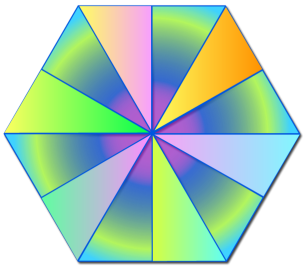


Now, we can draw in lines of symmetry. Because this is a regular hexagon, all of the lines are the same length. We can divide this hexagon vertically with a line of symmetry. This will create a vertical mirror image.

We can also divide this hexagon horizontally with a line of symmetry creating a horizontal mirror image.

Next, notice that we can draw in lines of symmetry connecting every vertex with the vertex opposite it.

Finally, we can draw in two last lines of symmetry which divide this regular hexagon into 12 even parts.



Problem-Solving Strategy: Look for a Pattern; Use a Venn Diagram

Word Problem –Work Teams

Isaac, Marc and Isabelle received some much needed help one afternoon from two seventh grade classes. Each class decided to band together to help with the skatepark. There was every kind of job to be done, from gardening to painting. One class had 32 students in it and the other class had 28 students in it.

"Let's mix up the classes," Marc suggested. "With an equal number of kids from each class on each team."

"How many teams can we make if we do this?" Isabelle asked.

There are several different strategies for solving this problem. Think back to the lesson for the two ways to solve the problem.

Solution

Here is the original problem once again. Reread it and then select a way to solve the problem. You can use either a pattern or a Venn Diagram.

Isaac, Marc and Isabelle received some much needed help one afternoon from two seventh grade classes. Each class decided to band together to help with the skatepark. There was every kind of job to be done, from gardening to painting. One class had 32 students in it and the other class had 28 students in it.

"Let's mix up the classes," Marc suggested. "With an equal number of kids from each class on each team."

"How many teams can we make if we do this?" Isabelle asked.

There are several different strategies for solving this problem. Think back to the lesson for the two ways to solve the problem.

To solve this problem, we can begin by using number patterns and factorization. We will be looking for the greatest common factor of the two numbers, 32 and 28, to determine the number of teams possible.

32

8×4

$4 \times 2 \times 2 \times 2$

$2 \times 2 \times 2 \times 2 \times 2$

28

74

Already, we can see that the greatest common factor is 4.

There are 4 teams possible.

5.10 Geometry and Measurement

This tenth chapter *Geometry and Measurement* further covers geometric principles through methods for finding the area of parallelograms, triangles, and circles. The student is also introduced to the concepts of solid figures, surface areas, and volumes of prisms and cylinders. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Area of Parallelograms
- Area of Triangles
- Circumference of Circles
- Area of Circles
- Classifying Solid Figures
- Surface Area and Volume of Prisms
- Surface Area and Volume of Cylinders
- Problem-Solving Strategy: Solve a Simpler Problem

Area of Parallelograms

Word Problem –A Parallelogram Design

Jillian is working on her quilt, but at the same time, she has decided to create a diagram of what the quilt pattern will look like. Specifically, she wants to create a design inside the parallelogram by combining different swatches of color.

"I think it is going to be too complicated," her grandmother warned when Jillian shared her plan.

"I want to try it anyway. I think if I create a drawing of the parallelogram double the size of the ones on the quilt, that it will be perfect. Then I can see all of the details," Jillian explained.

The original parallelogram has a base of $3\frac{1}{2}''$ and a height of $2''$. The side length is $3''$. Jillian wants to create a drawing where the parallelogram is double the size of the one on the quilt. To do this, she took each measurement and multiplied it by two.

Her drawing will have a parallelogram with the following measurements.

Base $= 7''$

Height $= 4''$

Slanted side length $= 6''$

If the parallelogram in Jillian's drawing has these dimensions, what will the area of this parallelogram be?

Use what you have learned to figure this out.

Solution

Here is the original problem once again. Reread it and then underline all of the important information.

Jillian is working on her quilt, but at the same time, she has decided to create a diagram of what the quilt pattern

will look like. Specifically, she wants to create a design inside the parallelogram by combining different swatches of color.

"I think it is going to be too complicated," her grandmother warned when Jillian shared her plan.

"I want to try it anyway. I think if I create a drawing of the parallelogram double the size of the ones on the quilt, that it will be perfect. Then I can see all of the details," Jillian explained.

The original parallelogram has a base of $3\frac{1}{2}''$ and a height of $2''$. The side length is $3''$. Jillian wants to create a drawing where the parallelogram is double the size of the one on the quilt. To do this, she took each measurement and multiplied it by two.

Her drawing will have a parallelogram with the following measurements.

Base $= 7''$

Height $= 4''$

Slanted side length $= 6''$

If the parallelogram in Jillian's drawing has these dimensions, what will the area of this parallelogram be?

Use what you have learned to figure this out.

Now we will need to find the area of the parallelogram in the drawing. To do this, we can use the formula that was taught in the chapter.

$A = bh$

Next, we fill in all of the measurements that we know.

$$A = (7)(4)$$
$$A = 28 \; sq. \; inches$$

This is our answer. The side length of the parallelogram isn't used to find the area of the parallelogram, but Jillian will need it when she creates the actual drawing.

Area of Triangles

Word Problem –Enlarging Triangles

Jillian is very pleased with her drawing of the new parallelogram design. It has several colors in it. Now she is on to creating a drawing of the triangles.

"I think I can do these in the same way," she told her grandmother.

"Really?"

"Yes, if I double the measurement of the triangle, then I can draw a triangle with the parallelogram and work on the pattern that I want to be in the triangle. I want to create small triangles inside the large one on the actual quilt," Jillian explained.

"Why not have the triangles be all solid colors?" her grandmother asked.

"Oh, gram, that would be too boring," Jillian stated smiling.

The original triangle on the quilt is a right triangle with two side lengths of $3''$ each. The slant isn't measured because Jillian figures that if she is accurate on the side lengths, then it is just a matter of connecting them together for the slanted side of the triangle.

If Jillian doubles both side lengths of the triangle for the drawing, what will the area of the triangle be that she draws?

Use what you have learned about finding the area of a triangle to figure this out.

Solution

Here is the original problem once again. Reread it and then underline all of the important information.

Jillian is very pleased with her drawing of the new parallelogram design. It has several colors in it. Now she is on to creating a drawing of the triangles.

"I think I can do these in the same way," she told her grandmother.

"Really?"

"Yes, if I double the measurement of the triangle, then I can draw a triangle with the parallelogram and work on the pattern that I want to be in the triangle. I want to create small triangles inside the large one on the actual quilt," Jillian explained.

"Why not have the triangles be all solid colors?" her grandmother asked.

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The original triangle on the quilt is a right triangle with two side lengths of $3''$ each. The slant isn't measured because Jillian figures that if she is accurate on the side lengths, then it is just a matter of connecting them together for the slanted side of the triangle.

If Jillian doubles both side lengths of the triangle for the drawing, what will the area of the triangle be that she draws?

Use what you have learned about finding the area of a triangle to figure this out.

To double the measurements of the triangle, Jillian multiplies both of the side lengths by 2.

This means that each side length goes from $3''$ to $6''$. Now she can find the area of the triangle. We can also find the area of the triangle by using the formula from the lesson.

$$A = \frac{1}{2}\,bh$$
$$A = \frac{1}{2}\,(6)(6)$$
$$A = \frac{1}{2}\,(36)$$
$$A = 18\ sq.\ inches$$

The area of the triangle in Jillian's drawing will be 18 square inches.

Circumference of Circles

Word Problem –Table Problems

On the day that the quilters are scheduled to arrive, Jillian's Mom decides to move them to a round table on the porch so that she can use the table in the kitchen for food.

"I don't think that is going to work," Jillian told her Mother.

"Why not?"

"Well, I did the math on the other table. There are eight quilters coming and each quilter needs 2 feet of space to work. We can fit 9 people at the table in the kitchen, so that one is perfect," Jillian explained.

"You just said you can fit 9 at the table in the kitchen, well, the table on the porch isn't that much smaller. See if it will work," her Mom said.

Jillian went out to the porch with a measuring tape. She measured that the diameter of the table is 5 feet.

Will this table work for the 8 quilters?

Use what you have learned about the circumference of a circle to figure out and justify your answer.

Solution

Here is the original problem once again. Reread it and then underline all of the important information.

On the day that the quilters are scheduled to arrive, Jillian's Mom decides to move them to a round table on the porch so that she can use the table in the kitchen for food.

"I don't think that is going to work," Jillian told her Mother.

"Why not?"

"Well, I did the math on the other table. There are eight quilters coming and each quilter needs 2 feet of space to work. We can fit 9 people at the table in the kitchen, so that one is perfect," Jillian explained.

"You just said you can fit 9 at the table in the kitchen, well, the table on the porch isn't that much smaller. See if it will work," her Mom said.

Jillian went out to the porch with a measuring tape. She measured that the diameter of the table is 5 feet.

Will this table work for the 8 quilters?

Use what you have learned about the circumference of a circle to figure out and justify your answer.

To figure out if this table will work, Jillian will first need to find the circumference of the table. Here is the formula for finding the circumference of a circle.

$$C = \pi d$$
$$C = (3.14)(5)$$
$$C = 15.7 \; inches$$

Now that Jillian has the circumference, she knows that each quilter needs 2 feet of space. If she divided the circumference by the two feet, she will know how many people can fit at the table.

$15.7 \div 2 = 7.8$

The table on the porch can only hold 7.8 people. Someone won't have 2 feet of work space.

After Jillian discovered this, she told her Mom. They decided to use the porch table for food and let the quilters work in the kitchen.

Area of Circles

Word Problem –Area of the Table

Jillian is curious about which table has the most area, the one on the porch or the one in the kitchen. It seems likely that the one in the kitchen would because it has a greater diameter. The table in the kitchen has a diameter of 6 feet, while the one on the porch has an area of 5 feet.

Jillian has decided to try to prove this fact. While Jillian is tackling this problem, we will tackle it too. Use what you have learned about finding the area of a circle to determine which table has the larger area and how much larger one

table is compared with the other.

Solution

Here is the original problem once again. Reread it and then figure out the two parts of the solution.

Jillian is curious about which table has the most area, the one on the porch or the one in the kitchen. It seems likely that the one in the kitchen would because it has a greater diameter. The table in the kitchen has a diameter of 6 feet, while the one on the porch has an area of 5 feet.

Jillian has decided to try to prove this fact. While Jillian is tackling this problem, we will tackle it too. Use what you have learned about finding the area of a circle to determine which table has the larger area and how much larger one table is compared with the other.

First, we need to find the area of both tables. Let's start with the one in the kitchen.

Kitchen

$$A = \pi r^2$$
$$A = (3.14)(6^2)$$
$$A = 3.14(36)$$
$$A = 113.04 \; sq. \; feet$$

Porch

$$A = \pi r^2$$
$$A = (3.14)(5^2)$$
$$A = 3.14(25)$$
$$A = 78.5 \; sq. \; feet$$

Jillian was correct. The table in the kitchen has a greater surface area than the one on the porch.

How much larger is it? To figure this out, we can subtract one area from the other.

$113.04 - 78.5 = 34.54 \; sq. \; feet$

This is the difference in size between the two tables.

Classifying Solid Figures

Word Problem –Cubes and Prisms

Jillian's sewing box is definitely a prism. She has decided to make it a rectangular prism, but just as she is finishing the design, her brother Ryan decided to put in his two cents.

"Why don't you make it a cube?" he asked.

"I am making it a prism," she said.

"Well, a cube is a type of prism," Ryan responded.

That made Jillian think for a minute. Is Ryan correct? Is a cube a type of prism?

You are going to figure this out by using what you have learned in the lesson. After you have written down whether a cube is a prism or not, use what you have learned to justify your answer.

Solution

Here is the original problem once again. Reread it and then figure out the solution.

Jillian's sewing box is definitely a prism. She has decided to make it a rectangular prism, but just as she is finishing the design, her brother Ryan decided to put in his two cents.

"Why don't you make it a cube?" he asked.

"I am making it a prism," she said.

"Well, a cube is a type of prism," Ryan responded.

That made Jillian think for a minute. Is Ryan correct? Is a cube a type of prism?

You are going to figure this out by using what you have learned in the lesson. After you have written down whether a cube is a prism or not, use what you have learned to justify your answer.

A cube is type of prism. Why?

To answer this question, we have to think about the characteristic of a prism. A prism has two parallel congruent bases and the base can be any polygon. A cube has parallel congruent bases and the bases are squares. Therefore, a cube is a prism.

Surface Area and Volume of Prisms

Word Problem –The Sewing Box

Jillian has figured out how much material she will need to cover the outside of her sewing box. Now she wants to figure out the volume of the sewing box. Here are the measurements of the box itself.

$7'' \times 6'' \times 4''$

She knows that the length of the box is 7 inches. The width of the box is 6 inches, and the height of the box is 4 inches.

Jillian is sure that this is all that she needs to find the volume of the box. She just can't remember how to do it.

This is where you come in. Use what you have learned from the lesson to figure out the volume of Jillian's sewing box.

Solution

Here is the original problem once again. Reread it and underline any important information.

Jillian has figured out how much material she will need to cover the outside of her sewing box. Now she wants to figure out the volume of the sewing box. Here are the measurements of the box itself.

$7'' \times 6'' \times 4''$

She knows that the length of the box is 7 inches. The width of the box is 6 inches, and the height of the box is 4 inches.

Jillian is sure that this is all that she needs to find the volume of the box. She just can't remember how to do it.

This is where you come in. Use what you have learned from the lesson to figure out the volume of Jillian's sewing box.

To find the volume of the sewing box, we will need to use the formula for finding the volume of a prism. This is the same formula that was presented in the lesson.

$V = Bh$

In this example, the capital "B" refers to the area of the base. We will need to find that out first. The base is a rectangle, so we can use the formula for finding the area of a rectangle.

$$A = 7(6)$$
$$A = 42 \; sq. \; inches$$

Now we can substitute that into the formula for finding volume along with the height of the box.

$$V = (42)(4)$$
$$V = 168 \; inches^3$$

Remember that because we multiply three different units together, the measurement of volume is always measured in cubic units. In this case, it is inches cubed.

Surface Area and Volume of Cylinders

Word Problem –Replacing a Jar of Beans

After Jillian's grandmother made those delicious baked beans, Jillian decided to buy a jar like the first one to send home with her grandmother. Her Mom remembered which store she bought the jar in, so she and Jillian went to the kitchen store and bought the tall jar for her grandmother.

Then Jillian filled it with beans. Finally, she decided to cover the jar with wrapping paper and she made a beautiful bow for the top of it.

The jar she purchased was just like the other jar. It had a diameter of 8 inches and a height of 16 inches.

As Jillian prepared to wrap the jar, she began to think about how much wrapping paper she would need. She chose a beautiful gold paper. The roll says that it has 12.5 square feet on it. Jillian wondered if it would be big enough.

This is where you come in. You will need to figure out the surface area of the jar and then decide if Jillian has enough wrapping paper to cover it.

Solution

Here is the original problem once again. Reread it and then underline all of the important information.

After Jillian's grandmother made those delicious baked beans, Jillian decided to buy a jar like the first one to send home with her grandmother. Her Mom remembered which store she bought the jar in, so she and Jillian went to the kitchen store and bought the tall jar for her grandmother.

Then Jillian filled it with beans. Finally, she decided to cover the jar with wrapping paper and she made a beautiful bow for the top of it.

The jar she purchased was just like the other jar. It had a diameter of 8 inches and a height of 16 inches.

As Jillian prepared to wrap the jar, she began to think about how much wrapping paper she would need. She chose a beautiful gold paper. The roll says that it has 12.5 square feet on it. Jillian wondered if it would be big enough.

This is where you come in. You will need to figure out the surface area of the jar and then decide if Jillian has enough wrapping paper to cover it.

First, we will need to figure out the surface area of the jar. To do this, we can use the formula presented in the lesson.

$SA = 2\pi r^2 + 2\pi rh$

Next, substitute in the given values. We know the diameter of the jar is 8 inches, so the radius is half of that.

$$SA = 2(3.14)(4^2) + 2(3.14)(4)(16)$$
$$SA = 100.48 + 401.92$$
$$SA = 502.4 \; square \; inches$$

We know that the wrapping paper is measured in feet. It has 12.5 square feet on it. We now need to change our surface area from inches to feet. We can do that by dividing by 12.

$502.4 \div 12 = 41.86 \; sq.feet$

Jillian will need 41.86 sq. feet of paper to wrap the jar. The gold paper won't be large enough.

Problem-Solving Strategy: Solve a Simpler Problem

Word Problem –The Quilt

Jillian is having a wonderful time learning about quilts. After designing her second quilt, she went to the library and checked out a book on quilts. She found one that she really liked, but it looked too complicated to draw out.

The quilt had 20 quilt squares. Each quilt square alternated in its pattern. The first one had two squares and one triangle, then the next one had two triangles and one square and so on until there was a pattern with 20 quilt squares.

Jillian is stuck. She wants to figure out how many squares and triangles will be needed for the entire quilt. She knows that she could sit and count them all, but she is sure that the is a pattern or table she could use.

You are going to help Jillian with this dilemma. By breaking down her problem into smaller parts, you will be able to make a table and help Jillian figure out how many triangles and squares are needed.

Solution

Here is the original problem once again. Reread it and then find the solution.

Jillian is having a wonderful time learning about quilts. After designing her second quilt, she went to the library and checked out a book on quilts. She found one that she really liked, but it looked too complicated to draw out.

The quilt had 20 quilt squares. Each quilt square alternated in its pattern. The first one had two squares and one triangle, then the next one had two triangles and one square and so on until there was a pattern with 20 quilt squares.

Jillian is stuck. She wants to figure out how many squares and triangles will be needed for the entire quilt. She knows that she could sit and count them all, but she is sure that the is a pattern or table she could use.

You are going to help Jillian with this dilemma. By breaking down her problem into smaller parts, you will be able to make a table and help Jillian figure out how many triangles and squares are needed.

To simplify this problem, we can use a table. Then we can fill in the information for each of the quilt squares. In the end, all we will need to do is to add up the columns.

TABLE 5.1:

#	Triangles	Squares
1	1	2
2	2	1
3	1	2

TABLE 5.1: (continued)

#	Triangles	Squares
4	2	1
5	1	2
6	2	1
7	1	2
8	2	1
9	1	2
10	2	1

Because there are 20 quilt squares, we can add these columns and double each for the total number of squares and triangles needed in the quilt.

15 triangles = 30 triangles

15 squares = 30 squares

Jillian will need 30 squares and 30 triangles for her quilt.

5.11 Integers

In this eleventh chapter *Integers* students will learn about positive and negative numbers, adding numbers, sums and differences of integers, multiplication, division, coordinates to name locations, transformations, and all about data displays. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

- Comparing Integers
- Adding Integers
- Subtracting Integers
- Multiplying Integers
- Dividing Integers
- The Coordinate Plane
- Transformations
- Surveys and Data Displays

Comparing Integers

Word Problem –Sea Level

After thinking about temperature, the students began exploring sea level. They discovered that New York City is 409.8 feet or 124.9 meters above sea level. Kelsey began looking up the stats on Auckland.

"Listen to this," she said in class. "Auckland's sea level goes from 0 at it's lowest point to 196 meters at its highest point which is along a magnificent nature preserve called the Waitakere Range."

She held up a page from the book so that the others in her class could see the pictures.

"Looks like something out of "Lord of the Rings"," Jesse commented.

"Well, it's time to do some math with these figures," Mrs. Harris said. "Begin by figuring out the difference between the lowest points and the highest points. Write your answers as integers."

The students are off to work and so are you. Begin by thinking about integers and review defining integers from the lesson. Then write your answers. You should have two different answers when finished.

Solution

Here is the original problem once again. Reread it and then figure out the answers.

After thinking about temperature, the students began exploring sea level. They discovered that New York City is 409.8 feet or 124.9 meters above sea level. Kelsey began looking up the stats on Auckland.

"Listen to this," she said in class. "Auckland's sea level goes from 0 at it's lowest point to 196 meters at its highest point which is along a magnificent nature preserve called the Waitakere Range."

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"Looks like something out of "Lord of the Rings"," Jesse commented.

"Well, it's time to do some math with these figures," Mrs. Harris said. "Begin by figuring out the difference between the lowest points and the highest points. Write your answers as integers."

The students are off to work and so are you. Begin by thinking about integers and review defining integers from the lesson. Then write your answers. You should have two different answers when finished.

First, we can find the difference between the lowest points of each place.

NYC is 124.9 meters above sea level

Auckland's lowest point is at 0 sea level.

There is a difference of -124.9 from NYC to Auckland.

Now the highest points.

NYC is 124.9 meters above sea level

Auckland's highest point is at 196 meters above sea level.

Auckland has a difference of +71.1 compared with NYC.

Adding Integers

Word Problem –More Time Zone Trouble

Cooper thinks he has this time zone thing down now. After working through the math with Emma, he decides to try again later in the afternoon.

At 4 pm, Cooper sits down at his kitchen table so that he can figure out what time it is in Auckland New Zealand. He knows that there is a 16 hour difference, and that Auckland is 16 hours ahead of New York.

He write +16 on a piece of paper.

Then he writes the following down.

$4 + -16 = -12$

This would mean that it is midnight in Auckland.

Cooper looks at this again. It doesn't look right. He is sure that he has done something wrong.

"Oh, I wrote a negative instead of a positive," he says to himself.

He changes the problem.

$4 + 16 = 20$

Now he is really confused. There isn't a 20 on the clock. Cooper needs some help. Using what you have learned about adding integers, figure out what time it is in Auckland New Zealand if it is 4 pm in New York City.

Solution

Here is the original problem once again. Reread it and then help Cooper to figure out the time in Auckland.

Cooper thinks he has this time zone thing down now. After working through the math with Emma, he decides to try again later in the afternoon.

At 4 pm, Cooper sits down at his kitchen table so that he can figure out what time it is in Auckland New Zealand. He knows that there is a 16 hour difference, and that Auckland is 16 hours ahead of New York.

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Cooper looks at this again. It doesn't look right. He is sure that he has done something wrong.

"Oh, I wrote a negative instead of a positive," he says to himself.

He changes the problem.

$4 + 16 = 20$

Now he is really confused. There isn't a 20 on the clock. Cooper needs some help. Using what you have learned about adding integers, figure out what time it is in Auckland New Zealand if it is 4 pm in New York City.

Time zones can be tricky because we have to keep in mind the current time and the difference between the time zones. Then there is calculating using a clock to include as well.

First, let's write down what we know.

We know that it is 4 pm in NYC. We know that it is a 16 hour difference between Auckland and NYC.

The word "difference" threw Cooper off. He thought it meant negative, so he wrote -16. However, because Auckland is ahead of NYC it is $a + 16$.

That is why Cooper wrote:

4 + 16

However, we are adding hours not numbers. If we start at 4 pm, we can easily add 12 hours. That would take us to 4 am.

But there is a 16 hour difference.

$16 - 12 = 4$

We have to add another 4 hours to 4 am.

When it is 4 pm in NYC, it is 8 am in Auckland New Zealand. This is our answer.

Subtracting Integers

Word Problem –Rugby

After Sarah sent Emily the letter about the football game, Sarah received a letter from Emily. Emily wrote Sarah all about a rugby game. Sarah doesn't know anything about rugby, so she went online so that she could understand Emily's letter. In the letter, Emily wrote that her favorite city team had won the most recent game, but that it had been a close call.

There had been three trys scored, but then the referee said that the player hadn't touched the ball, so a try had been taken away. Then it was re-awarded.

The whole thing sounded very exciting to Sarah, especially after she figured out the rules for rugby. Here is what she learned.

A try is worth 5 points and must be scored by a player touching the ground.

A conversion can be scored with a kick, and it is worth 2 points.

Sarah wrote down the following math problem based on Emily's letter.

5 + 5 + 5 for the three trys

Then + -5 for the one taken away.

Then $a+5$ for the one reawarded.

Based on these figures, how many points had Emily's team earned when they won the game?

Solution

Here is the original problem once again. Reread it and then figure out the answer to the question at the end of the problem.

After Sarah sent Emily the letter about the football game, Sarah received a letter from Emily. Emily wrote Sarah all about a rugby game. Sarah doesn't know anything about rugby, so she went online so that she could understand Emily's letter. In the letter, Emily wrote that her favorite city team had won the most recent game, but that it had been a close call.

There had been three trys scored, but then the referee said that the player hadn't touched the ball, so a try had been taken away. Then it was re-awarded.

The whole thing sounded very exciting to Sarah, especially after she figured out the rules for rugby. Here is what she learned.

A try is worth 5 points and must be scored by a player touching the ground.

A conversion can be scored with a kick, and it is worth 2 points.

Sarah wrote down the following math problem based on Emily's letter.

5 + 5 + 5 for the three trys

Then + -5 for the one taken away.

Then $a+5$ for the one reawarded.

Based on these figures, how many points had Emily's team earned when they won the game?

First, we can add the three fives.

$5+5+5=15$

Next, we can add the -5 for the try that was taken away.

$15+-5=10$

Now we can take away the -5 that was re–awarded.

$10--5=15$

The team won with 15 points.

Multiplying Integers

Word Problem –Amelia's Ride

Amelia went bike riding again with Jafakids. She wrote Tyler about her second ride with the group by sending it over email. Tyler loved this because he could research Jafakids while still learning about Amelia and the things that she was discovering.

On this second ride, Amelia said it was 77° when the group began. At the end of the ride, the temperature was 61°.

Tyler read this and couldn't help wondering how far the group had gone. If the statistic of a -2° temperature change per kilometer still applied, can you figure out how far Amelia rode with Jafakids? Use what you have learned about multiplying integers to help you to figure this out.

Solution

Here is the original problem once again. Reread it carefully because finding the solution could be a bit tricky.

Amelia went bike riding again with Jafakids. She wrote Tyler about her second ride with the group by sending it over email. Tyler loved this because he could research Jafakids while still learning about Amelia and the things that she was discovering.

On this second ride, Amelia said it was 77° when the group began. At the end of the ride, the temperature was 61°.

Tyler read this and couldn't help wondering how far the group had gone. If the statistic of a -2° temperature change per kilometer still applied, can you figure out how far Amelia rode with Jafakids? Use what you have learned about multiplying integers to help you to figure this out.

To figure this out, let's begin by writing down what we know.

It was 77° at the start of the ride.

It was 61° at the end of the ride.

If we subtract these two values, then we can see that it is a difference of 16° from the start to the end of the ride.

Each kilometer has a -2° change in temperature.

We don't know how far Amelia rode, but we do know that we could multiply that by -2 to find out the temperature. We need to find the number of kilometers. Let's use mental math to figure this out.

What number times -2 equals -16?

We know that 8 times 2 is 16, so 8 times -2 equals -16.

Amelia and her group rode for 8 kilometers.

Dividing Integers

Word Problem –Rethinking Amelia's Ride

When looking at the last problem, we used mental math to solve it. However, we could also have used division. Let's look at it once again.

Amelia went bike riding again with Jafakids. She wrote Tyler about her second ride with the group by sending it over email. Tyler loved this because he could research Jafakids while still learning about Amelia and the things that she was discovering.

On this second ride, Amelia said it was 77° when the group began. At the end of the ride, the temperature was 61°.

Tyler read this and couldn't help wondering how far the group had gone. If the statistic of a -2° temperature change per kilometer still applied, can you figure out how far Amelia rode with Jafakids?

Now think about how you could solve this problem using division. You will need to write an equation with an unknown and then use division to solve it. You should get the same answer as before.

Solution

Here is the problem once again. Reread it and then solve it using an equation and division.

Amelia went bike riding again with Jafakids. She wrote Tyler about her second ride with the group by sending it over email. Tyler loved this because he could research Jafakids while still learning about Amelia and the things that she was discovering.

On this second ride, Amelia said it was 77° when the group began. At the end of the ride, the temperature was 61°.

Tyler read this and couldn't help wondering how far the group had gone. If the statistic of a -2° temperature change

per kilometer still applied, can you figure out how far Amelia rode with Jafakids?

Now think about how you could solve this problem using division. You will need to write an equation with an unknown and then use division to solve it. You should get the same answer as before.

To figure this out, let's begin by writing down what we know.

It was 77° at the start of the ride.

It was 61° at the end of the ride.

If we subtract these two values, then we can see that it is a difference of 16° from the start to the end of the ride.

Each kilometer has a -2° change in temperature.

We know that our unknown is the number of kilometers traveled. We can call that unknown x.

Next, let's write an equation.

$$-2x = -16$$

This says that the -2° change in temperature times the number of kilometers will equal a temperature change of -16 degrees.

To find the unknown, we divide -16 by -2.

$$x = -\frac{16}{-2} = 8$$

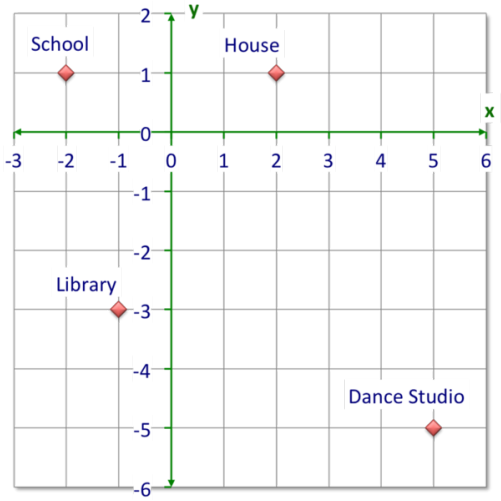
Amelia and the group rode 8 kilometers.

You can see that our answer is the same as when we used mental math, but we could use division to solve this problem as well.

The Coordinate Plane

Word Problem –Charlotte's Map

After Kevin had sent his map to Charlotte, she sent a similar map back to him. Here is Charlotte's map.



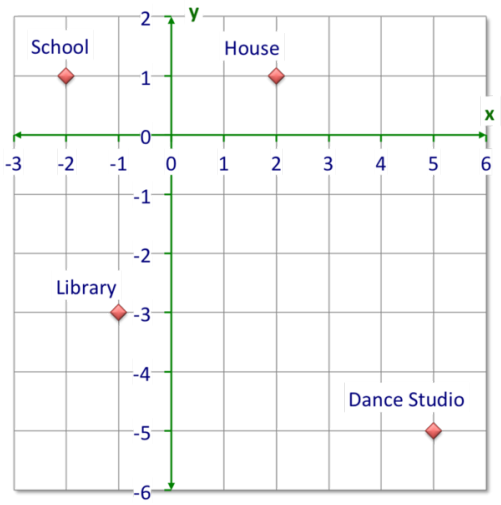
Kevin was amazed that Charlotte lived so close to her school and the library. Each was only a few blocks away. Even though they lived so far from each other, Kevin's neighborhood wasn't very different from Charlotte's neighborhood. This made him smile.

Look at the map of Charlotte's neighborhood. Use what you have learned about the coordinate plane to make a note of each location. Be sure to use ordered pairs to describe each location.

Solution

Here is the original problem once again. Reread it and then use what you have learned about the coordinate plane to write each set of coordinates.

After Kevin had sent his map to Charlotte, she sent a similar map back to him. Here is Charlotte's map.



Kevin was amazed that Charlotte lived so close to her school and the library. Each was only a few blocks away. Even though they lived so far from each other, Kevin's neighborhood wasn't very different from Charlotte's neighborhood. This made him smile.

Look at the map of Charlotte's neighborhood. Use what you have learned about the Coordinate Plane to make a note of each location. Be sure to use ordered pairs to describe each location.

Charlotte has labeled four different things on her map. Each one has different coordinates which can be used to indicate each location.

Here are the coordinates.

House (2, 1)

School (-2, 1)

Library (-1, -3)

Dance Studio (5, -5)

Now our work is finished.

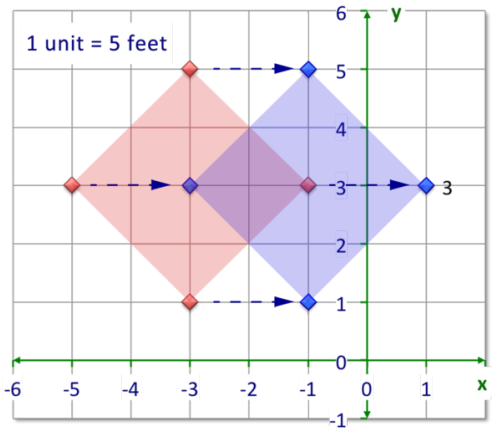
Transformations

Word Problem –The Clubhouse

Cody created his clubhouse and drew the design for it. He wanted his Dad to see that the clubhouse had been moved, so he drew it as a translation. The clubhouse needed to move 10 feet to the right. Cody took each coordinate of the

clubhouse and did this. Since his scale was that each unit equals 5 feet, moving the coordinates two units on the coordinate plane made the most sense.

Here is his final map.



The coordinates changed in this way.

(-3, 5) became (-1, 5)

(-3, 1) became (-1, 1)

(-5, 3) became (-3, 3)

(-1, 3) became (1, 3)

There is a way to figure out new coordinates through mathematics. You don't even need a coordinate grid. Because these coordinates were moved to the right horizontally, notice that only the x value was altered.

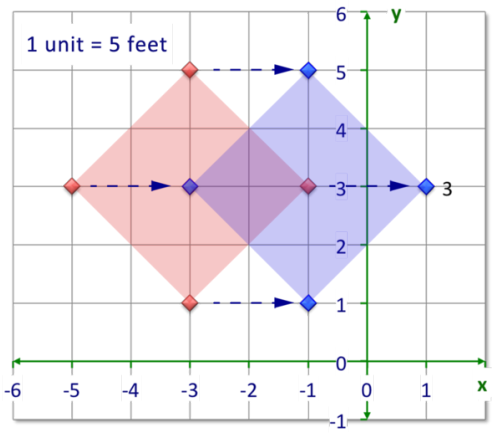
Can you find the pattern? Based on the pattern, what would happen if Cody's Dad wanted the clubhouse moved 15 feet instead of 10? Can you write the new coordinates?

Solution

Here is the original problem once again. Reread it and then figure out the new coordinates given the questions at the end of the problem.

Cody created his clubhouse and drew the design for it. He wanted his Dad to see that the clubhouse had been moved, so he drew it as a translation. The clubhouse needed to move 10 feet to the right. Cody took each coordinate of the clubhouse and did this. Since his scale was that each unit equals 5 feet, moving the coordinates two units on the coordinate plane made the most sense.

Here is his final map.



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(-3, 5) became (-1, 5)

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(-5, 3) became (-3, 3)

(-1, 3) became (1, 3)

There is a way to figure out new coordinates through mathematics. You don't even need a coordinate grid. Because these coordinates were moved to the right horizontally, notice that only the x value was altered.

Can you find the pattern? Based on the pattern, what would happen if Cody's Dad wanted the clubhouse moved 15 feet instead of 10? Can you write the new coordinates?

To figure this out, you have to decipher the pattern. Because each unit on the coordinate grid was worth 5 feet and Cody's Dad wanted the clubhouse moved 10 feet, each x value was taken and two was added to it. The two represents the two units on the grid.

Notice -3 became -1 etc.

Now the new dilemma posed at the end of this problem is that the clubhouse would be moved 15 feet. That would be three units instead of two.

We can figure out the new coordinates by adding three to the original x values.

(-3, 5) becomes (0, 5)

(-3, 1) becomes (0, 1)

(-5, 3) becomes (-2, 3)

(-1, 3) becomes (2, 3)

This is the solution to the problem.

Surveys and Data Displays

Word Problem –Line Graph

Kelly created a circle graph and a double bar graph to show the data from her survey. Remember that Kelly had taken a survey of other students to figure out what their bedtimes were so that she could convince her parents that

she needed a later bedtime.

Now that she has done that, she wants to create a line graph with the same data. Kelly is having a difficult time with this. She can't seem to make the figures work.

Why do you think this is happening? What kind of data does she need to make a line graph? Write an example of data that would fit for this type of graph.

Solution

Here is the original problem once again. Reread it and then use what you have learned about data displays to answer the questions at the end of the lesson.

Kelly created a circle graph and a double bar graph to show the data from her survey. Remember that Kelly had taken a survey of other students to figure out what their bedtimes were so that she could convince her parents that she needed a later bedtime.

Now that she has done that, she wants to create a line graph with the same data. Kelly is having a difficult time with this. She can't seem to make the figures work.

Why do you think this is happening? What kind of data does she need to make a line graph? Write an example of data that would fit for this type of graph.

To use a line graph, Kelly would need data that has changed over time. Her bedtime survey only recorded current data. It did not record how the data has changed. Therefore, a line graph won't work as a data display for this set of data.

Given this information, Kelly could create a line graph if she had written down how her bedtime had changed over time. Then she could show how times had increased as she had gotten older.

5.12 Equations and Functions; Probability

In this twelfth chapter *Equations and Functions; Probability* students will begin learning all about expressions and equations. They will also learn about solving equations, single-variable equations, graphing functions, probability, and how to calculate the probability of independent events. In this flexbook, titled Problem Solving, you will be given an additional story problem for the students. These story problems will continue with the theme of the chapter and can be used as homework problems, classwork or as a test or quiz problem. A solution has also been provided.

Lessons

The following lessons are part of this chapter.

Writing Expressions and Equations

Word Problem –The Rides

Now that the trip is all arranged for the amusement park, Lennox is trying to figure out how many rides he can go on in the time allowed. First, he needs to figure out the times connected with the trip.

He knows that the bus is planning to leave at 8 am. It is a two hour ride there, and the bus will be leaving to come home after 5 hours at the park.

What time will the bus leave to come home? Use what you have learned to write expressions and/or equations for Lennox.

Solution

Here is the original problem once again. Reread it and then answer the questions at the end of the problem.

Now that the trip is all arranged for the amusement park, Lennox is trying to figure out how many rides he can go on in the time allowed. First, he needs to figure out the times connected with the trip.

He knows that the bus is planning to leave at 8 am. It is a two hour ride there, and the bus will be leaving to come home after 5 hours at the park.

What time will the bus leave to come home? Use what you have learned to write expressions and/or equations for Lennox.

First, let's write down what we know. There are several expressions and equations that can be written for this situation.

The bus leaves at 8 am. It's a two hour ride to the amusement park.

$8+2=y$

$y = 10\ am$

The bus will arrive at 10 am.

The students will stay at the amusement park for five hours. What time will they leave to go home?

$10\ am + 2\ hours = 12\ Noon$

$12\ Noon + 3\ hours = x$

$x = 3\ pm$

The bus will leave at 3 pm and arrive back at 5 pm.

In this example, the unknown was the result of addition. Sometimes, you will also need to solve an equation where the unknown is in the middle of the problem. You will see how to do this in the next lesson.

Solving Equations Using Addition and Subtraction

Word Problem –Amusement Park Spending

The bus ended up costing each student an additional $3.50. Carl was very proud of his efforts considering that his first estimate was that it would cost each student $5.50 plus their admission ticket of $14.50.

Carl has $28.50 to spend on the trip to the amusement park. With that money, he needs to purchase his ticket and pay for the bus. With whatever is leftover, Carl will be able to buy food and souvenirs.

Use what you have learned to write an equation and solve it for the amount of money that Carl can spend at the amusement park.

Solution

Here is the original problem once again. Reread it and then write an equation that expresses the situation. Finally, solve it for the amount of money Carl can spend at the park.

The bus ended up costing each student an additional $3.50. Carl was very proud of his efforts considering that his first estimate was that it would cost each student $5.50 plus their admission ticket price of $14.50.

Carl has $28.50 to spend on the trip to the amusement park. With that money, he needs to purchase his ticket and pay for the bus. With whatever is leftover, Carl will be able to buy food and souvenirs.

Use what you have learned to write an equation and solve it for the amount of money that Carl can spend at the amusement park.

First, let's add up the amount Carl will spend on admission and the bus.

$14.50 + $3.50 = $18.00

He had $28.50 to spend for everything.

Here is the equation.

$\$18.00 + x = \$28.50$

The unknown is the amount of spending money Carl will have.

We can solve this by using subtraction.

$$x = 28.50 - 18.00$$

Carl will have $10.50 spending money.

Solving Equations Using Multiplication and Division

Word Problem –Riding the Rides

Now that Carl has figured out the bus and his money, he is ready to think about the rides. The night before the big trip, Carl downloaded a map of the amusement park from the computer. He began looking at each of the rides and trying to figure out how many rides he can go on during the trip.

"I would love to go on many rides," he said aloud.

"What are you doing?" his sister Karen asked.

"Well, I'm trying to figure out how many rides I can go on while we are at the amusement park," he explained.

"How are you going to do this?" Karen asked.

"I have a plan. I figure each ride is about 5 minutes long, then I figure it is an average of 15 minutes in line for each ride, maybe less if I am lucky. Using these times, I am going to be able to figure out how many rides I can ride in 5 hours," Carl explained.

"I don't get it," Karen said.

Do you get it? Using the information Carl has given, write an equation and figure out how many rides Carl can go on given the time allotted.

Solution

Here is the original problem once again. Reread it and then underline the important information.

Now that Carl has figured out the bus and his money, he is ready to think about the rides. The night before the big trip, Carl downloaded a map of the amusement park from the computer. He began looking at each of the rides and trying to figure out how many rides he can go on during the trip.

"I would love to go on many rides," he said aloud.

"What are you doing?" his sister Karen asked.

"Well, I'm trying to figure out how many rides I can go on while we are at the amusement park," he explained.

"How are you going to do this?" Karen asked.

"I have a plan. I figure each ride is about 5 minutes long, then I figure it is an average of 15 minutes in line for each ride, maybe less if I am lucky. Using these times, I am going to be able to figure out how many rides I can ride in 5 hours," Carl explained.

"I don't get it," Karen said.

Do you get it? Using the information Carl has given, write an equation and figure out how many rides Carl can go on given the time allotted.

Carl will need to write an equation. The unknown quantity is the number of rides.

$y =$ number of rides

The time to wait and ride is what we need to multiply by the unknown number of rides. If the time doesn't exceed 5 hours, then that value will work.

5 minutes on a ride + 15 minutes waiting = 20 minutes

$20y = 5$ *hours*

Uh oh, our time is in minutes and hours. We have to change the hours to minutes.

$5 \times 60 = 300$ *minutes*

$20y = 300$ *minutes*

Now we can solve the equation by using division.

$y = 300 \div 20 = 15$

Based on these numbers, Carl can ride 15 rides in the given time.

Functions

Word Problem –Car Wash Tips

After four hours, the students began adding up their money. They had already figured out how many cars had been washed each hour. Toby had taken the data and written it in a table. Here is the table.

TABLE 5.2:

Hours	*Cars*
0	0
1	5
2	10
3	15
4	20

After looking at the chart and counting, the class figured out that there was an additional car for every hour that the students washed cars.

Toby looked at his original equation of $5x$.

This would no longer work given the new information. How does Toby need to change the equation to allow for this extra car?

Solution

Here is the original problem once again. Reread it and then write a new equation.

After four hours, the students began adding up their money. They had already figured out how many cars had been washed each hour. Toby had taken the data and written it in a table. Here is the table.

TABLE 5.3:

Hours	*Cars*
0	0
1	5
2	10
3	15
4	20

After looking at the chart and counting, the class figured out that there was an additional car for every hour that the students washed cars.

Toby looked at his original equation of $5x$ where the 5 represents the 5

This would no longer work given the new information. How does Toby need to change the equation to allow for this extra car?

To do this, Toby can begin with the original equation he wrote.

$$5x$$

Since there is an additional car per hour, we can change this equation by adding 1.

$$5x+1$$

This is our answer.

Graphing Functions

Word Problem –Car Wash Graph

After the car wash was over, the students decided to graph their data. They knew that the equation that they had written and their data was accurate. Here is the equation that they wrote about their data.

$$5x+1$$

With this equation, the students know that they can calculate the number of cars washed for every hour that they worked. Here is a table of the data.

TABLE 5.4:

Hours	Cars
1	6
2	11
3	16
4	21
5	26
6	31
7	36
8	41

Here is the data for the number of cars washed each hour.

Using what you have learned about graphing functions, take this data and graph the function. Is it linear or non–linear?

Solution

Here is the original problem once again. Reread it and then graph the function.

After the car wash was over, the students decided to graph their data. They knew that the equation that they had written and their data was accurate. Here is the equation that they wrote about their data.

$$5x+1$$

With this equation, the students know that they can calculate the number of cars washed for every hour that they

worked. Here is a table of the data.

TABLE 5.5:

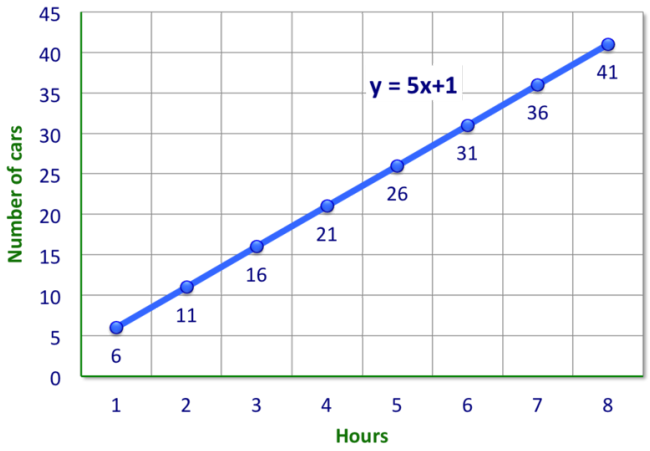
Hours	Cars
1	6
2	11
3	16
4	21
5	26
6	31
7	36
8	41

Here is the data for the number of cars washed each hour.

Using what you have learned about graphing functions, take this data and graph the function. Is it linear or non–linear?

To graph this function, we can take the hour or input as the x value and the cars as the y value.

Here is our graph.



Now when looking at this graph, we can see that it is a linear graph.

Introduction to Probability

Word Problem –Doubling the Odds

Just as Keith was about to spin the spinner at the archery game, the man running the booth had important information.

"If you spin now young man, I will double your chances," he said.

"What do you mean?" Keith asked.

"I mean that I will take the number you spin and double it. That means your chances will be better than ever," the man explained.

Keith agreed and spun a 4 out of 10 possible spaces. He will get 8 tries to shoot the target. What are Keith's chances of hitting the target on the 4, 5, 6 or 7^{th} attempt?

Write the probability as a fraction and a percent.

Solution

Here is the original problem once again. Reread it and then answer the questions at the end.

Just as Keith was about to spin the spinner at the archery game, the man running the booth had important information.

"If you spin now young man, I will double your chances," he said.

"What do you mean?" Keith asked.

"I mean that I will take the number you spin and double it. That means your chances will be better than ever," the man explained.

Keith agreed and spun a 4 out of 10 possible spaces. He will get 8 tries to shoot the target. What are Keith's chances of hitting the target on the 4, 5, 6 or 7^{th} attempt?

Write the probability as a fraction and a percent.

Now we can write the probability and the percent.

Kevin has a 4 out of 8 chance of hitting the target on the 4, 5, 6 or 7^{th} attempt. We can write that as a fraction.

$$\frac{4}{8} = \frac{1}{2}$$

Notice that we simplified the fraction to one–half.

Now we can convert one–half to a percent.

$$\frac{1}{2} = \frac{50}{100} = 50\%$$

This is the answer.

Finding Outcomes

Word Problem –Ferris Wheel Repeats

Now that the girls, Sarah, Julie and Maggie have all ridden the Ferris wheel six times, Chris is decided to join in the fun.

This time, the four friends are going to repeat the pattern of riding that they did earlier. Each person will take turns sitting with someone else until everyone has had a chance to ride with everyone.

"This means that we will ride 6 times again," Maggie said.

"No it doesn't because of me," Chris said.

Chris is right. The number will change. Given this new addition to the Ferris wheel, how many times will the friends ride the Ferris wheel? Use what you have learned about probability and outcomes to figure out the answer to this question.

Solution

Here is the original problem once again. Reread it and then answer the question at the end of the problem.

Now that the girls, Sarah, Julie and Maggie have all ridden the Ferris wheel six times, Chris is decided to join in the fun.

This time, the four friends are going to repeat the pattern of riding that they did earlier. Each person will take turns sitting with someone else until everyone has had a chance to ride with everyone.

"This means that we will ride 6 times again," Maggie said.

"No it doesn't because of me," Chris said.

Chris is right. The number will change. Given this new addition to the Ferris wheel, how many times will the friends ride the Ferris wheel? Use what you have learned about probability and outcomes to figure out the answer to this question.

Four friends are going to ride the Ferris Wheel. They are going to ride it in pairs so that each person has the chance to ride with everyone else. We could create an elaborate drawing of this to show all of the possible riding partners. However, we don't need to know that. We only need to know the total number of rides the four friends will do on the Ferris Wheel.

$P(4,2)$

This is the probability of four friends riding two at a time.

We can multiply to find the total number of rides.

$4 \times 2 = 8$

The friends will ride the Ferris wheel eight times in a row.

Probability of Independent Events

Word Problem –Riding the Rides

Remember Carl and his question about how many rides he could ride in the five hour period? Well, on the way back from the trip, Carl conducted his own survey. He asked girls and boys separately how many of them had gotten to ride at least 10 rides. Carl himself had ridden 15, so he took himself out of the survey.

Here are his results.

$$\frac{9}{12} \textit{ boys rode at least } 10 \textit{ rides}$$
$$\frac{10}{12} \textit{ girls rode at least } 10 \textit{ rides}$$

What is the probability of girls and boys riding at least 10 rides?

Solution

Here is the original problem once again. Reread it and then solve it for a percentage.

Remember Carl and his question about how many rides he could ride in the five hour period? Well, on the way back from the trip, Carl conducted his own survey. He asked girls and boys separately how many of them had gotten to ride at least 10 rides. Carl himself had ridden 15, so he took himself out of the survey.

Here are his results.

$$\frac{9}{12} \textit{ boys rode at least } 10 \textit{ rides}$$
$$\frac{10}{12} \textit{ girls rode at least } 10 \textit{ rides}$$

What is the probability of girls and boys riding at least 10 rides?

To figure this out, we first need to find a decimal for each fraction.

$$\frac{9}{12} \textit{ becomes } .75$$
$$\frac{10}{12} \textit{ becomes } .83$$

Now we can multiply those two decimals together.

$.75 \times .83 = .6225$

We move the decimal two places for the percent.

62.25% or $62\frac{1}{4}\%$ are the chances of boys and girls riding at least 10 rides.